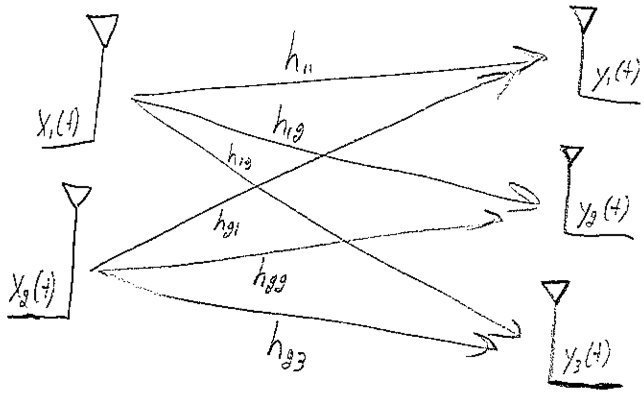
EE16B Discussion 3B

HW 2 Problem 2



We are in the <u>time domain</u>.

Think of H as some attenuation applied to each signal.

$$\begin{bmatrix} h_{11} & h_{21} \\ h_{12} & h_{22} \\ h_{13} & h_{23} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix}$$

a) We want to recover $x(t)$ after $y(t)$ is received. To do this, we will multiply $y(t)$ by some matrix A:

$$x(t) = A\,y(t)$$

Use SVD to decompose H and find A.

"Use SVD" is just a matrix technique!

$$\underset{3\times 2}{H} = \underset{3\times 3}{U}\,\underset{3\times 2}{\Sigma}\,\underset{2\times 2}{V^T} \qquad \underset{2\times 2}{H^T H} \qquad \underset{3\times 3}{H H^T}$$

Dimensions?

How to find U, Σ, V^T given H?

So:

$$U \Sigma V^T \mathbf{x} = y$$

Solve for y...

<u>Note</u>: This assumes we know the channel characteristics! What if we don't?

- Could guess (AWGN, additive white gaussian noise)
- Channel estimation: Send a known signal every so often, estimate $\hat{H}$ (e.g., with least squares)

b) How is (a) related to least squares?

Well, least squares tells us that the solution to $A\underline{x} = \underline{b} + \underline{e}$ that minimizes $\underline{e}$ is $\underline{x} = (A^T A)^{-1} A^T \underline{b}$

The variables mean somewhat different things in this case, but the structure is the same and so is the solution:

$$Hx = y$$

$$x = (H^T H)^{-1} H^T y$$

We want to go from this to match our solution from (a): $x = f(U, \Sigma, V)\, y$

<u>Hint</u>: First step: $(H^T H) x = H^T y$

(This will make the math easier!)