

1. DFT

Determine the DFT for each signal described below.

(a) $x(n) = \sin\left(\frac{2\pi}{5}n\right) + \cos\left(\frac{4\pi}{5}n\right)$, $\forall n$, with window of length 5

(b) $x(n) = \delta(n) - \delta(n-2)$, with window of length 6

2. Matrix Multiplication Suppose $x(n)$ is the input signal applied to a linear time-invariant (LTI) system characterized by the impulse response $h : \mathbb{Z} \rightarrow \mathbb{R}$ and corresponding frequency response H , where

$$H(\omega) = \sum_{n=-\infty}^{\infty} h(n) e^{-i\omega n}, \quad \forall \omega$$

Let $y(n)$ be the corresponding output signal. If

$$h(n) = \frac{\delta(n) + \delta(n - 10)}{2}$$

and

$$x(n) = \sin\left(\frac{2\pi}{10}n\right) + \cos\left(\frac{2\pi}{45}n\right),$$

determine an appropriate DFT window size that is the same length as the period of $x(n)$. Then find the output DFT coefficients Y_k in terms of the input DFT coefficients X_k .

3. Symmetric Matrices

Prove the following: For any symmetric matrix A , any two eigenvectors corresponding to distinct eigenvalues of A are orthogonal.

Hint: Use the definition of an eigenvalue to show that $\lambda_1(v_1 \cdot v_2) = \lambda_2(v_1 \cdot v_2)$.