1. Anti-Aliasing Filters

We want to digitize a signal $x_c(t) = \cos(2\pi 200t) + \cos(2\pi 300t)t + \cos(2\pi 400t) + \cos(2\pi 500t)$.

(a) What is the maximum sampling period T_s that will allow us to perfectly reconstruct the signal?

(b) If $T_s = 1/1200$ seconds, draw the spectrum $X_{d0}(f)$ of the sampled signal and label the key frequencies. Does aliasing occur? Why or why not?

(c) If $T_s = 1/900$ seconds, draw the spectrum $X_{d1}(f)$ of the sampled signal and label the key frequencies. Does aliasing occur? Why or why not?

(d) Using the T_s from part (c), what filter $H_{a1}(f)$ could be applied **after** sampling to only preserve the frequencies of X_{d1} that are not "corrupted" by aliasing? (Any frequency information that does not match the original signal is considered corrupted.) Draw or write an equation to describe the filter. Out of the eight non-zero frequencies present in the original signal, how many are preserved?

(e) Using the T_s from part (c), explain how you could apply a filter $H_{a0}(f)$ **before** sampling to recover more of the original signal. Draw or write an equation to describe the filter. Then draw the spectrum $X_{d2}(f)$ of the sampled signal and label the key frequencies. Out of the eight non-zero frequencies present in the original signal, how many are preserved?

2. Rocket ~~Science~~ Engineering

A vibration sensor mounted on a rocket during a test firing produces an output $x_c(t)$ that is proportional to the acceleration. The sensor output $x(t)$ is sampled at a rate of $f_s = 1000$Hz and a total of 10,000 samples are recorded. The sequence $x_d(n) = x_c(nT_s)$ is then processed using the Numpy `fft.fft` algorithm to compute $X[k]$, where $k = 1, 2, ..., 10000$.

(a) If $X[500]$, $X[3250]$, $X[6750]$, and $X[9500]$ are non-zero, and the remaining $X[k]$ are zero, sketch the spectrum of $x_d(n)$. Assuming no aliasing occurred, what frequencies were present in $x_c(t)$? Write a possible expression for $x_c(t)$.

(b) An engineer famliar with the dynamics of the rocket says that the highest frequency component determined in part (a) is to be exected but the lowest frequency component does not make sense. Further, a frequency component somewhere in the range of $500 < f < 1000$ Hz should be present. Given this information, what frequencies were actually present in $x_c(t)$? Write a more accurate expression for $x_c(t)$.