Discussion 8

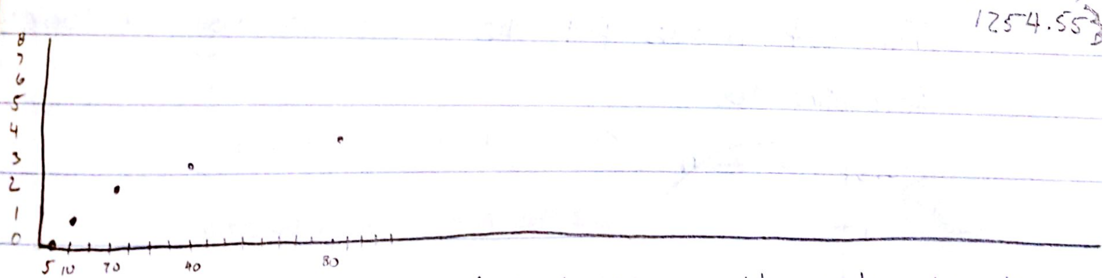
Things we will do today:

- Plotting Frequency, details & how to
- Filter transfer function, examples & practice
- Interpreting frequency plots of transfer functions & how it relates to filter design.

Frequency Plotting:

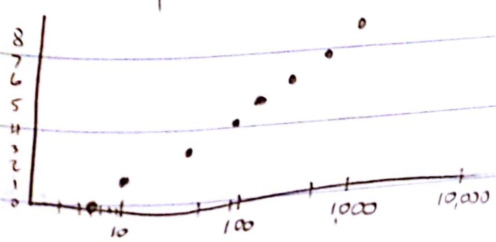
Lets look at musical notes, plot 9 octaves of A

A = {4.9, 9.8, 19.6, 39.2, 78.41, 156.82, 313.64, 627.27, 1254.55}



↳ got in trouble quickly plotting like this

Turns out, for just about every frequency application, what we really care about is log(f)

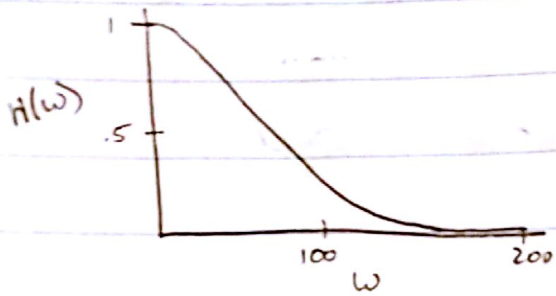


← much easier to plot

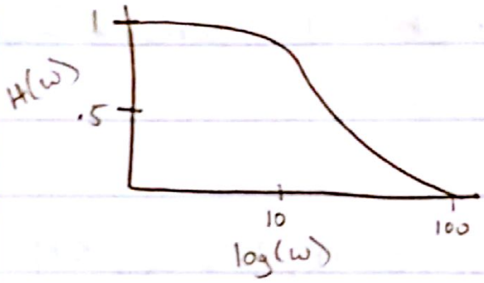
Now, what about amplitude?

If we take a transfer function & plot it, what will it look like?

Low Pass

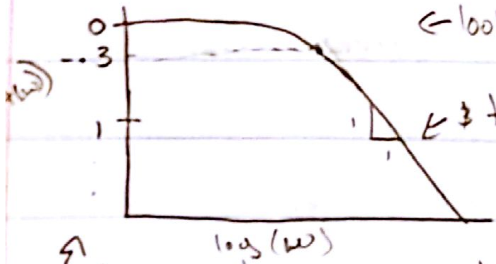


First plot on log(f)



← still curvy & wobbly

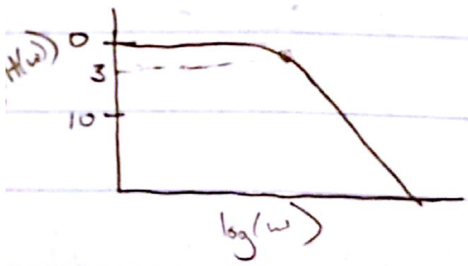
Turns out if we plot the y axis in log, it looks friendlier too



← look at that pretty corner

← & the nice straight slope down

however, this axis, not a fan of the .3, so multiply by 10



This is almost the graph you drew in class.
In class you plotted in dB

$$dB \rightarrow 20\log_{10}$$

Aside

if this doesn't make sense don't worry, it is extra

So, where does the factor of 2 come in?

What are the units of amplitude in the transfer function?

Voltage

Turns out, voltage is not really what we want to plot, b/c current matters too. Power is what we really want.

$$H = 10\log_{10}\left(\frac{P_o}{P_i}\right)$$
$$= 10\log_{10}\left(\frac{V_o}{V_i}\right)^2$$
$$= 20\log_{10}\left(\frac{V_o}{V_{in}}\right)$$

← so that's where the 2 comes from, it actually lets you forget about the power stuff

Now, how about this magical 3dB point?
It has to do w/ power again.

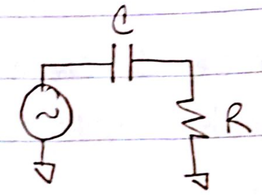
3dB is the cutoff freq, also known as the half-power point.

$$10\log_{10}\left(\tfrac{1}{2}\right) \cong -3dB$$
$$20\log_{10}\left(\tfrac{1}{\sqrt{2}}\right) \cong -3dB$$

This is a really useful trick to know, that the 3dB point is $V_{pass}/\sqrt{2}$

Filter Transfer Function:

You are all pros at solving RC diffeqs now, right?



1. set up the diff eq
2. Replace V_{out} with HV_{in}, V_{in} with $e^{j\omega t}$

   transfer function ↗   ↖ eigen function

For example,

$$V_{out} + RC\frac{dV_{out}}{dt} = RC\frac{dV_{in}}{dt}$$

Becomes,

$$He^{j\omega t} + RC\frac{dHe^{j\omega t}}{dt} = RC\frac{de^{j\omega t}}{dt}$$

$$H\cancel{e^{j\omega t}} + HRCj\omega \cancel{e^{j\omega t}} = RCj\omega \cancel{e^{j\omega t}}$$

$$H + HRCj\omega = RCj\omega$$

3. Solve for H

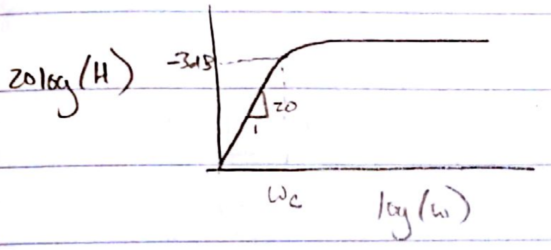
$$H = \frac{RCj\omega}{1 + RCj\omega}$$

What is this function?

$$H(\omega) = \frac{j\omega RC}{1 + j\omega RC} \Leftarrow \text{High Pass}$$

- How can we tell?
- What are some reasons to use a high pass?

Graph the high pass:



$\omega_c = \frac{1}{RC}$

Note, that plotted in dB, the stuff before the cutoff has a slope of 20dB/decade.

What if we want 40dB/decade?

Cascade two High Pass w/ same ω_c