

**This homework is due November 5, 2015, at 5PM.**

**1. Practice with Phase Response**

As we discussed in lecture, the phase response of our filters can be very important in terms of how it affects the shape of the time domain waveforms coming out of these filters. As we will see through a simplified example in this problem, phase can also play a very important role when we are building feedback control loops. First, however, let's get some practice with deriving the phase response of a few of the filters we've looked at so far.

For each of the following filters, plot its magnitude  $\|H(\omega)\|$  (in dB) and phase response  $\angle H(\omega)$  (in radians) vs. the log of  $\omega$ .

(a)  $H(\omega) = \frac{1}{1+j\omega(10ns)}$

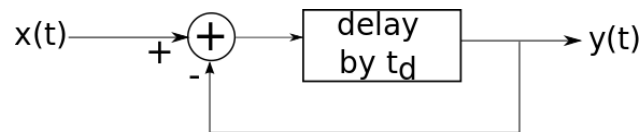
(b)  $H(\omega) = \frac{j\omega(10ns)}{1+j\omega(10ns)}$

Now let's look at a slightly strange filter - one which is simply a delay. That is, the filter is a system whose output  $y(t)$  is simply:

$$y(t) = x(t - t_d)$$

(c) For  $x(t) = e^{i\omega t}$ , what is  $H_{td}(\omega)$ ? (Plot  $\|H(\omega)\|$  in dB and  $\angle H(\omega)$  in radians vs. the log of  $\omega$ )

(d) Now let's consider the block diagram shown below; this block diagram represents a highly simplified model of a control loop where all we are trying to do is get the output to track the input, but where we have a delay between when we detect the difference between the output and the input and when we can re-apply a new output.



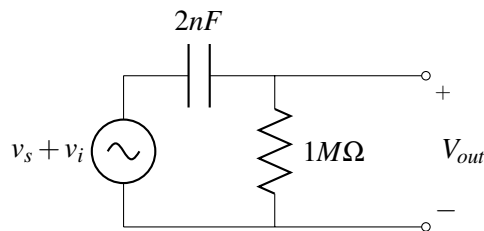
Derive  $H_{fb}(\omega)$  for the block diagram shown above. Assuming that  $t_d = 10ns$ , plot  $\|H_{fb}(\omega)\|$  in dB vs. the log of  $\omega$ .

(e) You should have noticed a "strange" behavior at one particular  $\omega$  in your plot from part (e) - explain intuitively what is going on that causes this behavior, and what the implication would be if you tried to use this particular block in a real application where  $x(t)$  might at some point have a non-zero component at this "strange"  $\omega$ .

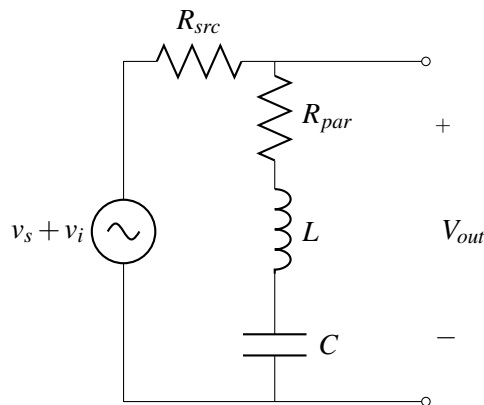
## 2. Dealing with Power-Line Interference

In many sensor applications, one of the biggest sources of potential interference is due to the fact that many AC power lines delivering electricity to our plugs operate at 60Hz. For this problem, let's assume that the signal we are actually interested in detecting has frequency content from 90Hz to 400Hz, and that we sample at 1.2kHz. We will thus be examining how well a number of different types of filters perform in attenuating this potential interference from the power lines.

- (a) For the continuous-time high-pass filter shown below, how much is the magnitude of any potential interference at 60Hz ( $v_i$ ) attenuated by? How much is the lowest frequency component of the desired signal (90Hz) attenuated by?



- (b) Noticing the limitations of the simple high-pass filter from part (a), one of your colleagues suggests trying the notch filter described in lecture. However, after looking at some data sheets for the inductors, you both quickly realize that any real inductor has some parasitic series resistance associated with it, resulting in the overall filter circuit shown below.



Derive  $H(\omega)$  for this circuit.

- (c) Assuming that  $R_{par} = 10\Omega$  and  $R_{src} = 1k\Omega$ , select values for  $L$  and  $C$  to maximize the attenuation provided by this filter at 60Hz. For the values you have selected, how much attenuation do you get at 90Hz?
- (d) Another one of your colleagues suggests that perhaps instead of filtering the 60Hz interference in the analog domain we can instead implement a discrete-time (digital) filter to get rid of it. In particular, your colleague suggests using a filter of the form  $y[n] = x[n] + x[n - M]$  where  $M$  is some positive integer. Given the sample rate of 1.2kHz, what value of  $M$  would you select in order to make the filter's response be zero at the discrete-time frequency corresponding to an interference input of 60Hz?
- (e) Given the potential frequency content of the actual and desired sensor signal, explain why it isn't a good idea to use the discrete-time filter proposed in (d). (Hint: Try evaluating the magnitude response

of the filter for frequencies other than those that correspond to 60Hz.)

It is worth noting that one can build discrete-time notch filters that do not suffer from the specific issue that plagues the filter described in part (d), so you shouldn't consider the conclusion about discrete-time vs. continuous-time filters to be a general one.

### **3. Robots are Cool!**

Robotics is a great topic to wrap up the EE16 series with because building a robot integrates a wide variety of the topics we have discussed/learned about. For this problem, you should research some commercially-available robot (whether it be for manufacturing, a toy, a self-driving car, etc.) and describe in as detailed of a manner as possible how the robot achieves its operational goals. In your description, you should point out the specific sub-systems and/or components of the robot that draw upon material we've learned about in the EE16 series.