RC Circuits

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Response Terminology

<u>Source dependence</u>

Natural response – response in absence of sources

Forced response – response due to external source

Complete response = Natural + Forced

<u>Time dependence</u>

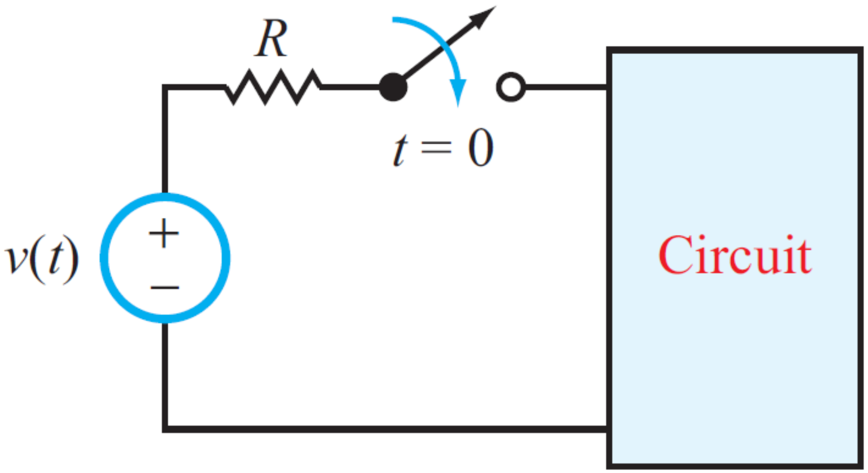
Transient response – time-varying response (temporary)

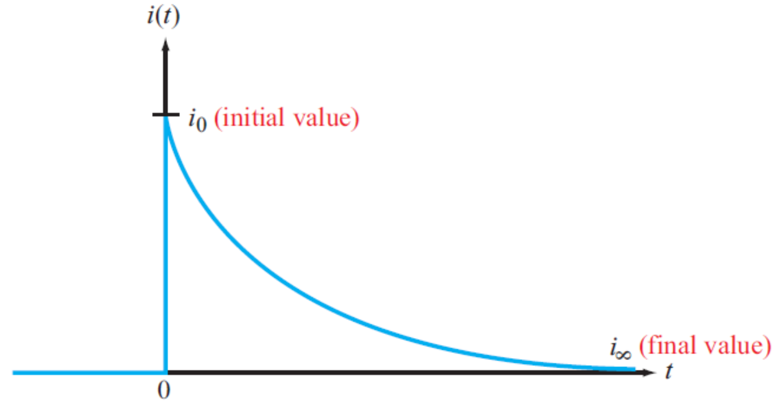
Steady state response – time-independent or periodic (permanent)

Complete response = Transient + Steady State

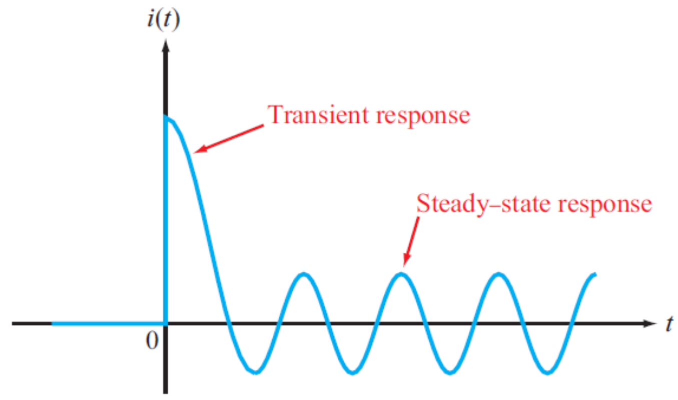
Transient Response

The transient response represents the initial reaction immediately after a sudden change, such as closing or opening a switch to connect a source to the circuit.





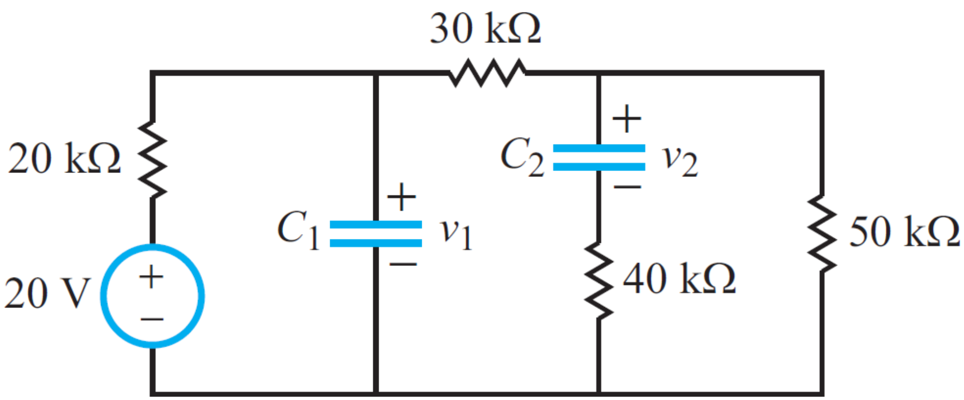
(a) dc transient response



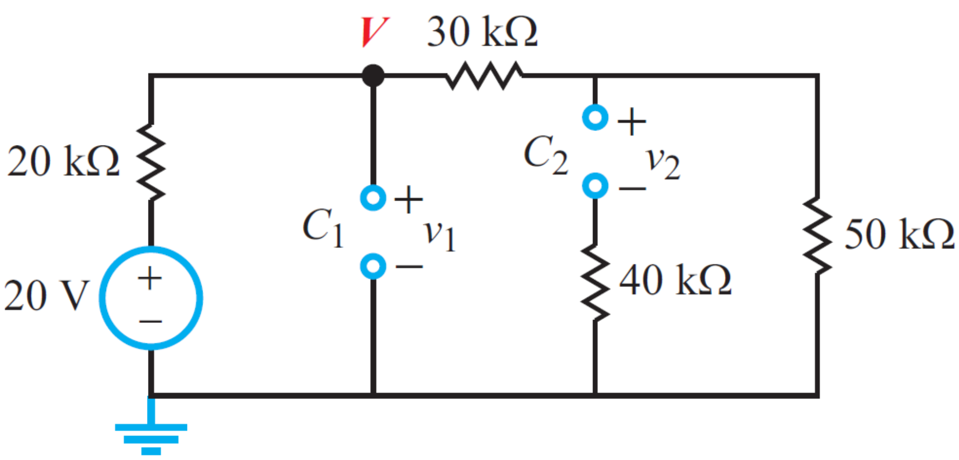
(b) Combined response to ac excitation

RC Circuits at DC

- At dc no currents flow through capacitors: open circuits



$$\frac{V-20}{20\times 10^3}+\frac{V}{(30+50)\times 10^3}=0,$$



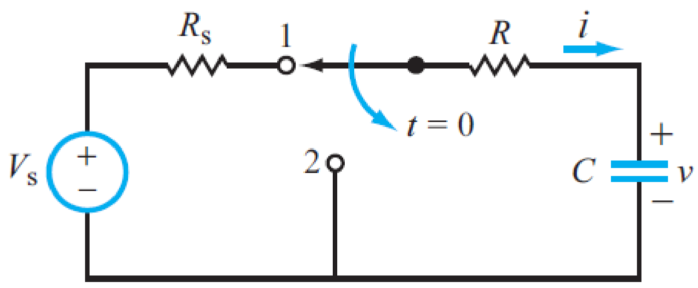
which gives $V = 16$ V. Hence,

$$v_1 = V = 16\text{ V}.$$

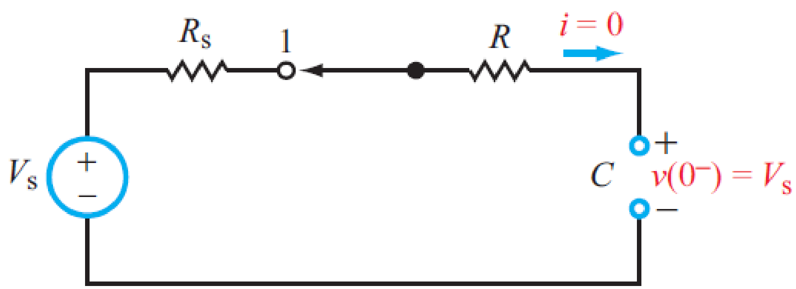
Through voltage division, v_2 across the 50-kΩ resistor is given by

$$v_2 = \frac{V\times 50\text{k}}{(30+50)\text{k}} = \frac{16\times 50}{80} = 10\text{ V}.$$

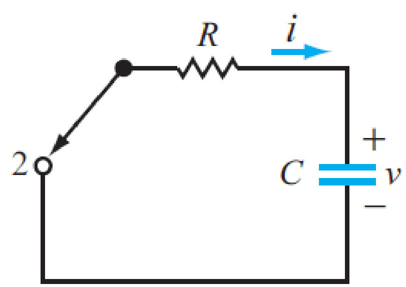
Natural Response of Charged Capacitor



(a) RC circuit



(b) At $t = 0^-$



(c) At $t \geq 0$

(a) $t = 0^-$ is the instant just before the switch is moved from terminal 1 to terminal 2

(b) $t = 0$ is the instant just after it was moved; $t = 0$ is synonymous with $t = 0^+$

since the voltage across the capacitor cannot change instantaneously, it follows that

$$v(0) = v(0^-) = V_s.$$

$$Ri + v = 0 \qquad (\text{for } t \geq 0),$$

where i is the current through and v is the voltage across the capacitor. Since $i = C\, dv/dt$,

$$RC\,\frac{dv}{dt} + v = 0.$$

Upon dividing both terms by RC, Eq. (5.69) takes the form

$$\frac{dv}{dt} + av = 0 \qquad (\text{source-free}),$$

where

$$a = \frac{1}{RC}.$$

Solution to the differential equation

The standard procedure for solving Eq. (5.70) starts by multiplying both sides by e^{at},

$$\frac{dv}{dt}\, e^{at} + ave^{at} = 0. \qquad (5.72)$$

Next, we recognize that the sum of the two terms on the left-hand side is equal to the expansion of the differential of (ve^{at}),

$$\frac{d}{dt}(ve^{at}) = \frac{dv}{dt}\, e^{at} + ave^{at}. \qquad (5.73)$$

Hence, Eq. (5.72) becomes

$$\frac{d}{dt}(ve^{at}) = 0. \qquad (5.74)$$

Integrating both sides, we have

$$\int_0^t \frac{d}{dt}(ve^{at})\, dt = 0, \qquad (5.75)$$

Solution to the differential equation

. Performing the integration gives

$$v e^{at}\Big|_0^t = 0$$

or

$$v(t)\, e^{at} - v(0) = 0. \qquad (5.76)$$

Solving for $v(t)$, we have

$$v(t) = v(0)\, e^{-at},$$
$$= v(0)\, e^{-t/RC} \qquad \text{(for } t \geq 0\text{)}, \qquad (5.77)$$

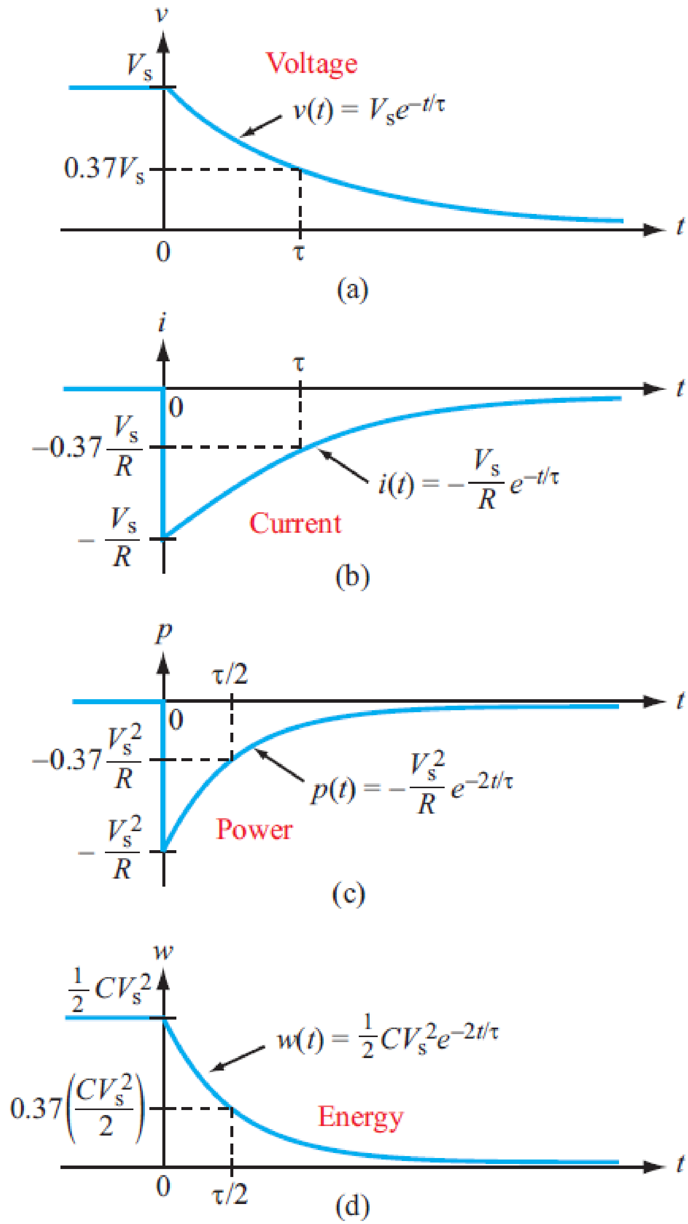
$$v(t) = v(0)\, e^{-t/\tau} \qquad \text{(natural response)},$$

with

$$\tau = RC \qquad \text{(s)},$$

τ is called the ***time constant of the circuit.***

Natural Response of Charged Capacitor



$$i(t) = C\frac{dv}{dt} = C\frac{d}{dt}(V_s e^{-t/\tau})$$

$$= -C\frac{V_s}{\tau}e^{-t/\tau} \qquad (\text{for } t \geq 0),$$

which simplifies to

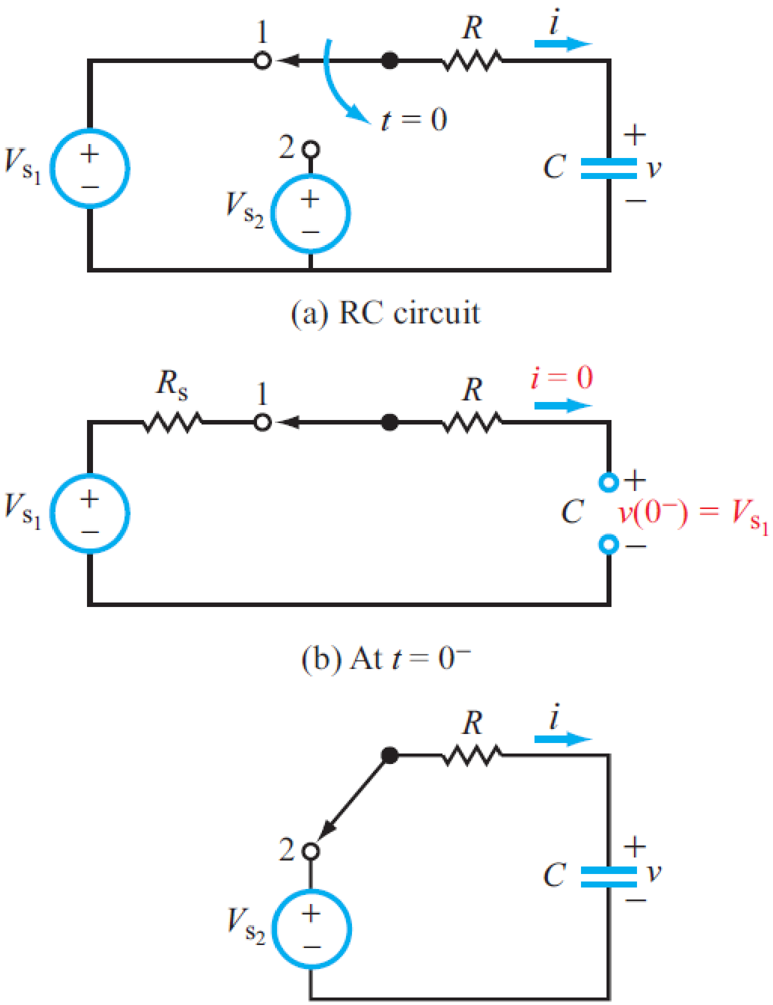
$$i(t) = -\frac{V_s}{R}e^{-t/\tau}\, u(t) \qquad (\text{for } t \geq 0)$$

(natural response).

$$p(t) = iv = -\frac{V_s}{R}e^{-t/\tau} \times V_s e^{-t/\tau}$$

$$= -\frac{V_s^2}{R}e^{-2t/\tau} \qquad (\text{for } t \geq 0).$$

General Response of RC Circuit



(a) RC circuit

(b) At $t = 0^-$

(c) At $t \geq 0$

$$v(0) = v(0^-) = V_{s_1}. \tag{5.86}$$

For $t \geq 0$, the voltage equation for the loop in Fig. 5-30(c) is

$$-V_{s_2} + iR + v = 0. \tag{5.87}$$

Upon using $i = C\,dv/dt$ and rearranging its terms, Eq. (5.87) can be written in the differential-equation form

$$\frac{dv}{dt} + av = b, \tag{5.88}$$

where

$$a = \frac{1}{RC} \quad \text{and} \quad b = \frac{V_{s_2}}{RC}. \tag{5.89}$$

Solution of $\frac{dv}{dt} + av = b,$

$$\frac{d}{dt}(ve^{at}) = be^{at}.$$

Integrating both sides,

$$\int_0^t \frac{d}{dt}(ve^{at})\, dt = \int_0^t be^{at}$$

gives

$$ve^{at}|_0^t = \frac{b}{a}\, e^{at}\Big|_0^t.$$

Upon evaluating the functions at the two limits, we have

$$v(t)\, e^{at} - v(0) = \frac{b}{a}\, e^{at} - \frac{b}{a},$$

and then solving for $v(t)$, we have

$$v(t) = v(0)\, e^{-at} + \frac{b}{a}\,(1 - e^{-at}).$$

As $t \to \infty$, $v(t)$ reduces to

$$v(\infty) = \frac{b}{a} = V_{S2}.$$

Solution of $\frac{dv}{dt} + av = b,$

We have:

$$v(t) = v(0)\, e^{-at} + \frac{b}{a}\,(1 - e^{-at}). \qquad v(\infty) = \frac{b}{a} = V_{s_2}.$$

By reintroducing the time constant $\tau = RC = 1/a$ and replacing b/a with $v(\infty)$, we can rewrite Eq. (5.94) in the general form:

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} \qquad (\text{for } t \geq 0)$$

(switch action at $t = 0$).

If the switch action causing the change in voltage across the capacitor occurs at time T_0 instead of at $t = 0$, Eq. (5.96) assumes the form

$$v(t) = v(\infty) + [v(T_0) - v(\infty)]e^{-(t-T_0)/\tau} \qquad (\text{for } t \geq T_0)$$

(switch action at $t = T_0$),