Phasors

Michel M. Maharbiz
Vivek Subramanian

Phasor Domain

A ***domain transformation*** is a mathematical process that converts a set of variables from their domain into a corresponding set of variables defined in another domain.

1. The phasor-analysis technique transforms equations from the time domain to the phasor domain.

2. Integro-differential equations get converted into linear equations with no sinusoidal functions.

3. After solving for the desired variable in the phasor domain, conversion back to the time domain provides the same solution that would have been obtained had the original integro-differential equations been solved entirely in the time domain.

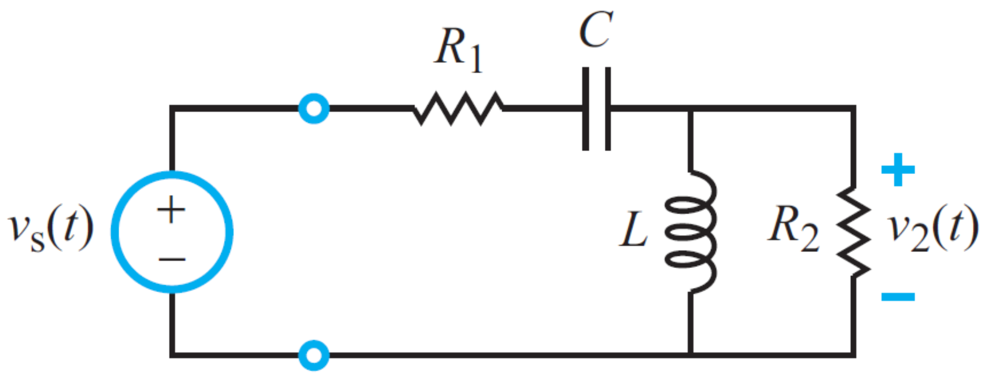
Why Phasors?

Objective: To determine the steady state response of a linear circuit to ac signals

$$v_s(t) = V_0 \cos(\omega t + \phi)$$

angular frequency ω

ϕ is called its ***phase angle***



- Sinusoidal input is common in electronic circuits
- Any time-varying periodic signal can be represented by a series of sinusoids (Fourier Series)
- Time-domain solution method can be cumbersome

Complex Numbers

We will find it is useful to represent sinusoids as complex numbers

$$j = \sqrt{-1}$$

$$z = x + jy$$

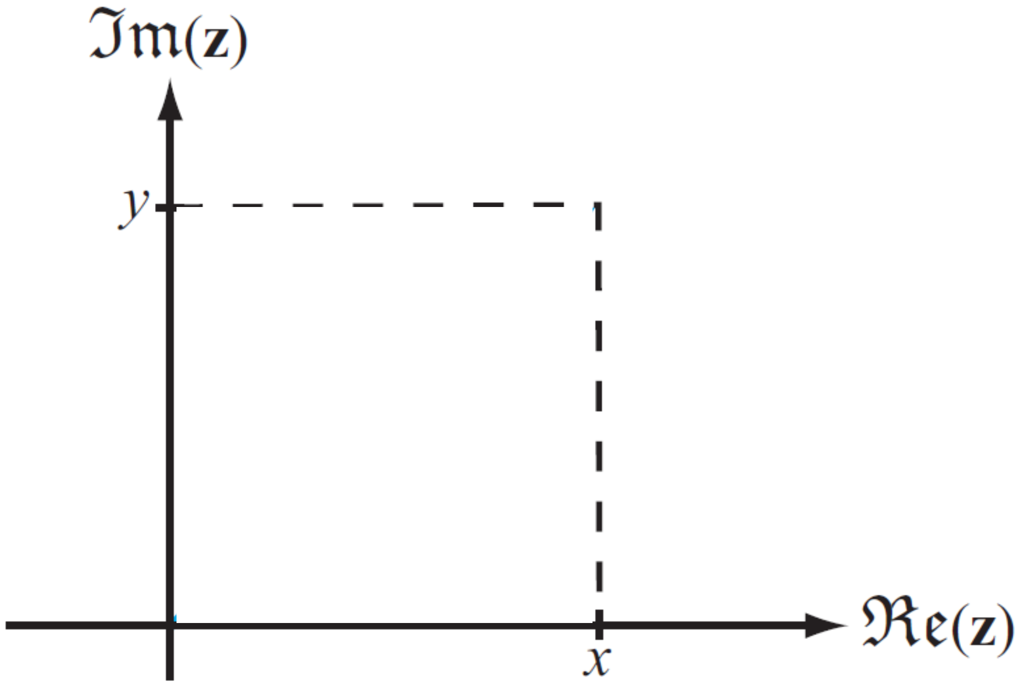
Rectangular coordinates

$$z = |z|\angle\theta = |z|e^{j\theta}$$

Polar coordinates

$$\mathrm{Re}(z) = x$$

$$\mathrm{Im}(z) = y$$



Relations based on Euler's Identity

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

Phasor Domain

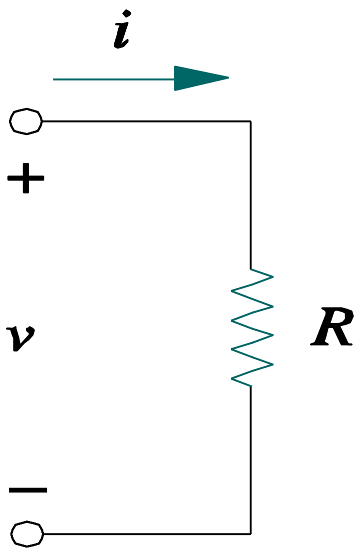
$$\upsilon(t) = V_0 \cos(\omega t + \phi)$$

Time Domain		Phasor Domain
$\upsilon(t) = V_0 \cos \omega t$	$\leftrightarrow$	$\mathbf{V} = V_0$
$\upsilon(t) = V_0 \cos(\omega t + \phi)$	$\leftrightarrow$	$\mathbf{V} = V_0 e^{j\phi}$ $\quad V_0\angle\phi$

Phasor Relation for Resistors

Current through a resistor

Time Domain



$v = iR$

Time domain

$$i = I_m \cos(\omega t + \phi)$$

$$v = iR = RI_m \cos(\omega t + \phi)$$

Phasor Relation for Inductors

Current through inductor in time domain

$$i = I_m \cos(\omega t + \phi)$$

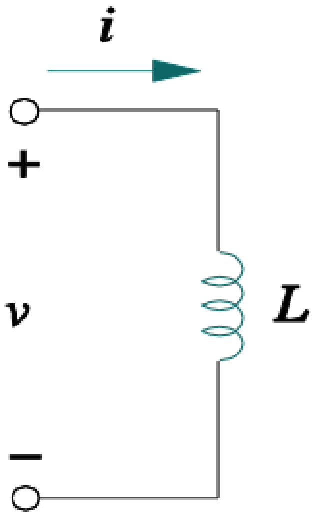
Time domain $v = L \frac{di}{dt}$

Phasor Domain

$$v_L = \Re e[\mathbf{V}_L e^{j\omega t}]$$

$$i_L = \Re e[\mathbf{I}_L e^{j\omega t}].$$

Time Domain



$$v = L \frac{di}{dt}$$

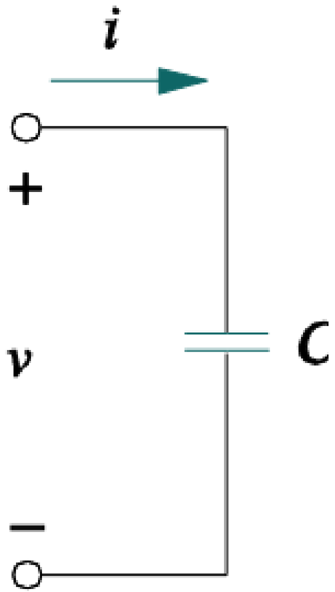
Impedance:

Phasor Relation for Capacitors

Voltage across capacitor in time domain is

$$v = V_m \cos(\omega t + \phi)$$

Time Domain



$$i = C\frac{dv}{dt}$$

Time domain

$$i = C\frac{dv}{dt}$$

Phasor Domain

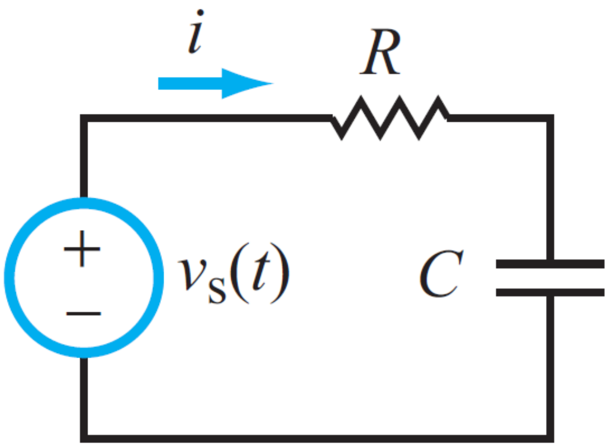
$$\mathbf{I}_C = j\omega C\mathbf{V}_C$$

$$\mathbf{Z}_C = \frac{\mathbf{V}_C}{\mathbf{I}_C} = \frac{1}{j\omega C}.$$

ac Phasor Analysis General Procedure

Step 1

Adopt Cosine Reference
(Time Domain)



$$v_S(t) = 12\sin(\omega t - 45^\circ) \text{ (V)}$$

Step 2

Transfer to Phasor Domain

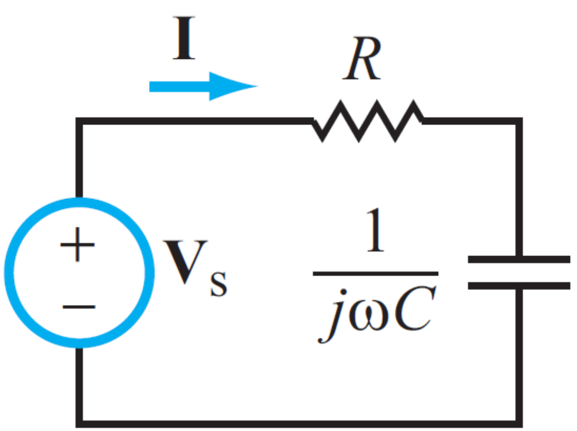
$i \rightarrow \mathbf{I}$

$v \rightarrow \mathbf{V}$

$R \rightarrow \mathbf{Z}_R = R$

$L \rightarrow \mathbf{Z}_L = j\omega L$

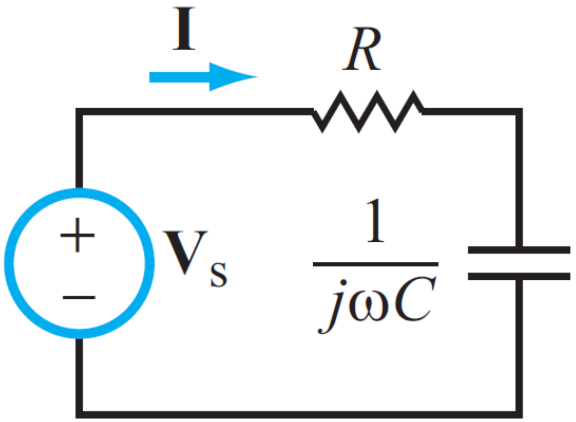
$C \rightarrow \mathbf{Z}_C = 1/j\omega C$



$$\mathbf{V}_s = 12e^{-j135^\circ} \text{ (V)}$$

ac Phasor Analysis General Procedure

Step 3 Cast Equations in Phasor Form	$\mathbf{I}\left(R + \frac{1}{j\omega C}\right) = \mathbf{V}_s$
Step 4 Solve for Unknown Variable (Phasor Domain)	$\mathbf{I} = \frac{\mathbf{V}_s}{R + \frac{1}{j\omega C}}$
Step 5 Transform Solution Back to Time Domain	$i(t) = \mathfrak{Re}[\mathbf{I}e^{j\omega t}]$ $= 6\cos(\omega t - 105°)$ (mA)

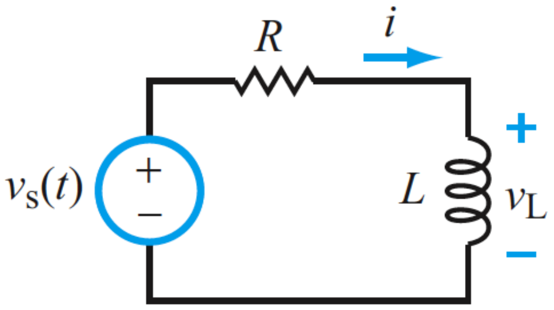


$\mathbf{V}_s = 12e^{-j135°}$ (V)

Example: RL Circuit

$$v_s(t) = 15\sin(4 \times 10^4 t - 30°) \text{ V}.$$

Also, $R = 3\ \Omega$ and $L = 0.1$ mH. Obtain an expression for t voltage across the inductor.



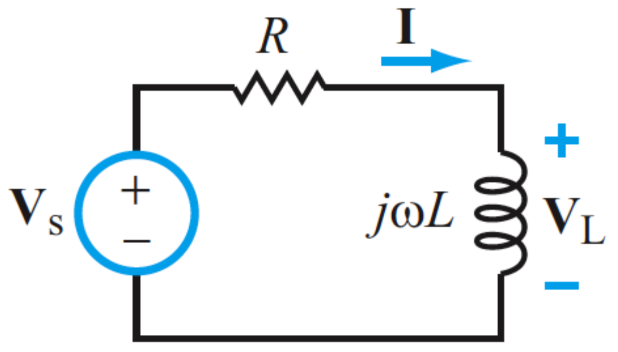
(a) Time domain

Step 2: Transform circuit to the phasor domain

Example: RL Circuit cont.

Step 3: Cast KVL in phasor domain

$$R\mathbf{I} + j\omega L\mathbf{I} = \mathbf{V}_s.$$



(b) Phasor domain

Step 4: Solve for unknown variable

$$\mathbf{I} = \frac{\mathbf{V}_s}{R + j\omega L} = \frac{15e^{-j120^\circ}}{3 + j4 \times 10^4 \times 10^{-4}}$$

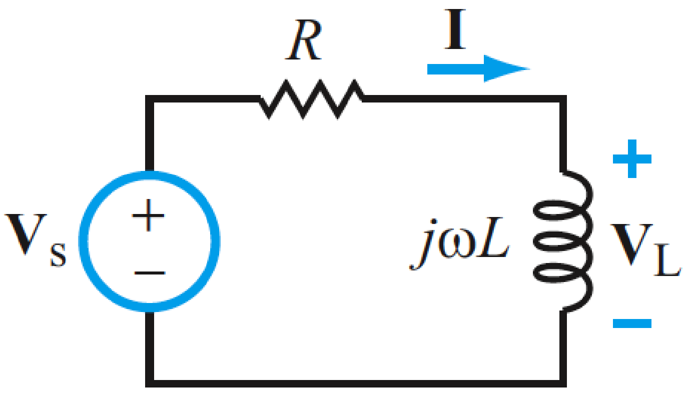
$$= \frac{15e^{-j120^\circ}}{3 + j4} = \frac{15e^{-j120^\circ}}{5e^{j53.1^\circ}} = 3e^{-j173.1^\circ} \text{ A}.$$

Example: RL Circuit cont.

The phasor voltage across the inductor is related to **I** by

$$\mathbf{V}_L = j\omega L\mathbf{I}$$

Reminder:
$\omega = 4 \times 10^4$
L=0.1mH



(b) Phasor domain

Example: RL Circuit cont.

Step 5: Transform solution to the time domain

The corresponding time-domain voltage is

$$
\begin{aligned}
v_{\mathrm{L}}(t) &= \Re\mathfrak{e}[\mathbf{V}_{\mathrm{L}} e^{j\omega t}] \\
&= \Re\mathfrak{e}[12 e^{-j83.1^\circ} e^{j4\times 10^4 t}] \\
&= 12\cos(4\times 10^4 t - 83.1^\circ)\ \mathrm{V}.
\end{aligned}
$$