Bode Plots

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Frequency Response

Transfer function of a circuit or system describes the output response to an input excitation as a function of the angular frequency ω.

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_{\text{out}}(\omega)}{\mathbf{V}_{\text{in}}(\omega)}$$

$$\mathbf{H}(\omega) = M(\omega)\, e^{j\phi(\omega)},$$

where by definition,

$$M(\omega) = |\mathbf{H}(\omega)|, \qquad \phi(\omega) = \tan^{-1}\left\{\frac{\mathfrak{Im}[\mathbf{H}(\omega)]}{\mathfrak{Re}[\mathbf{H}(\omega)]}\right\}$$

Magnitude　　Phase

dB Scale

If G is defined as the power gain,

$$G = \frac{P}{P_0},$$

then the corresponding gain in dB is defined as

$$G\ [\text{dB}] = 10 \log G = 10 \log \left(\frac{P}{P_0} \right) \qquad (\text{dB}).$$

$$G\ [\text{dB}] = 10 \log \left(\frac{\frac{1}{2}|\mathbf{V}|^2 R}{\frac{1}{2}|\mathbf{V}_0|^2 R} \right) = 20 \log \left(\frac{|\mathbf{V}|}{|\mathbf{V}_0|} \right)$$

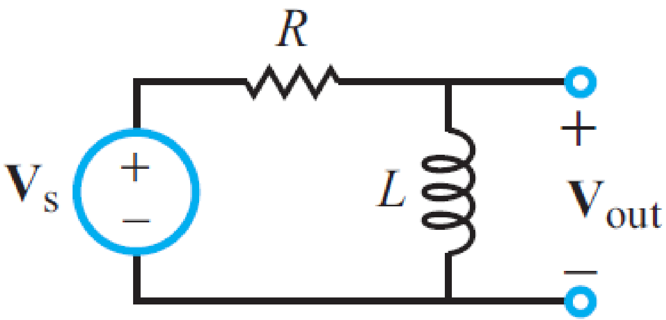
$$G = XY \rightarrow G\ [\text{dB}] = X\ [\text{dB}] + Y\ [\text{dB}].$$

$$G = \frac{X}{Y} \rightarrow G\ [\text{dB}] = X\ [\text{dB}] - Y\ [\text{dB}].$$

$\frac{P}{P_0}$	dB
10^N	$10N$ dB
10^3	30 dB
100	20 dB
10	10 dB
4	$\simeq$ 6 dB
2	$\simeq$ 3 dB
1	0 dB
0.5	$\simeq -3$ dB
0.25	$\simeq -6$ dB
0.1	-10 dB
10^{-N}	$-10N$ dB

$\left\lvert \frac{\mathbf{V}}{\mathbf{V}_0} \right\rvert$ or $\left\lvert \frac{\mathbf{I}}{\mathbf{I}_0} \right\rvert$	dB
10^N	$20N$ dB
10^3	60 dB
100	40 dB
10	20 dB
4	$\simeq$ 12 dB
2	$\simeq$ 6 dB
1	0 dB
0.5	$\simeq -6$ dB
0.25	$\simeq -12$ dB
0.1	-20 dB
10^{-N}	$-20N$ dB

Example: RL filter, Magnitude Plot



$$\mathbf{V}_{\text{out}} = \frac{j\omega L \mathbf{V}_s}{R + j\omega L},$$

which leads to

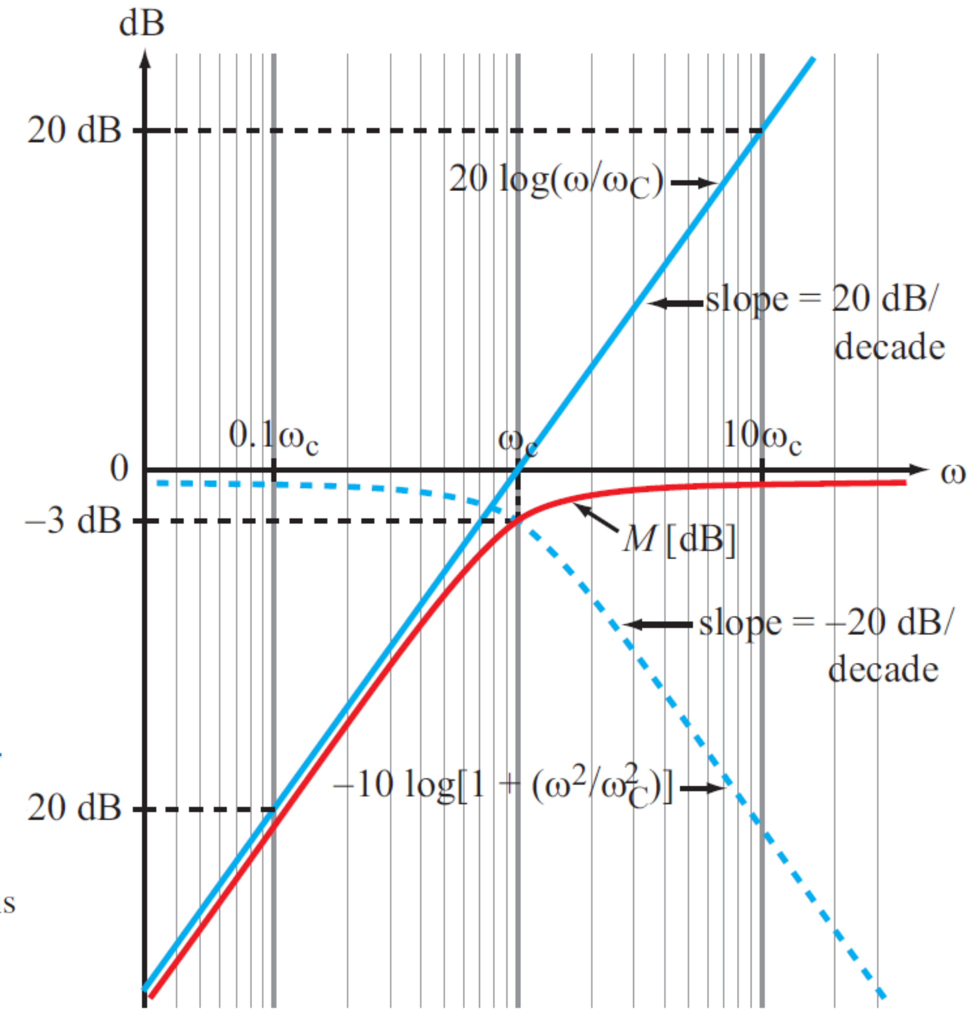
$$\mathbf{H} = \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_s} = \frac{j\omega L}{R + j\omega L} = \frac{j(\omega/\omega_c)}{1 + j(\omega/\omega_c)},$$

with $\omega_c = R/L$.

$$M = |\mathbf{H}| = \frac{(\omega/\omega_c)}{|1 + j(\omega/\omega_c)|} = \frac{(\omega/\omega_c)}{\sqrt{1 + (\omega/\omega_c)^2}}.$$

Since H is a voltage ratio, the appropriate dB scaling factor is 20, so

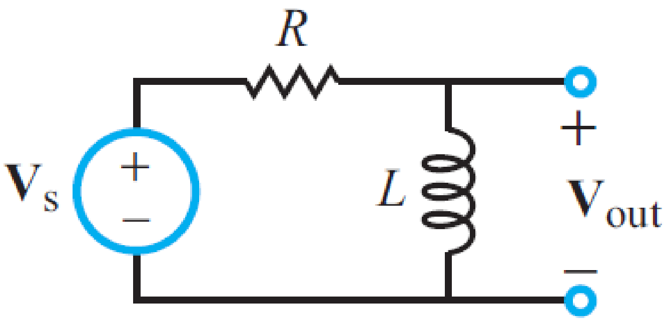
$$\begin{aligned} M\,[\text{dB}] &= 20 \log M \\ &= 20 \log(\omega/\omega_c) - 20 \log[1 + (\omega/\omega_c)^2]^{1/2} \\ &= 20 \log(\omega/\omega_c) - 10 \log[1 + (\omega/\omega_c)^2]. \qquad (9.35) \end{aligned}$$



(b) Magnitude plot

Log scale for ω and dB scale for M

Example: RL filter, Phase Plot



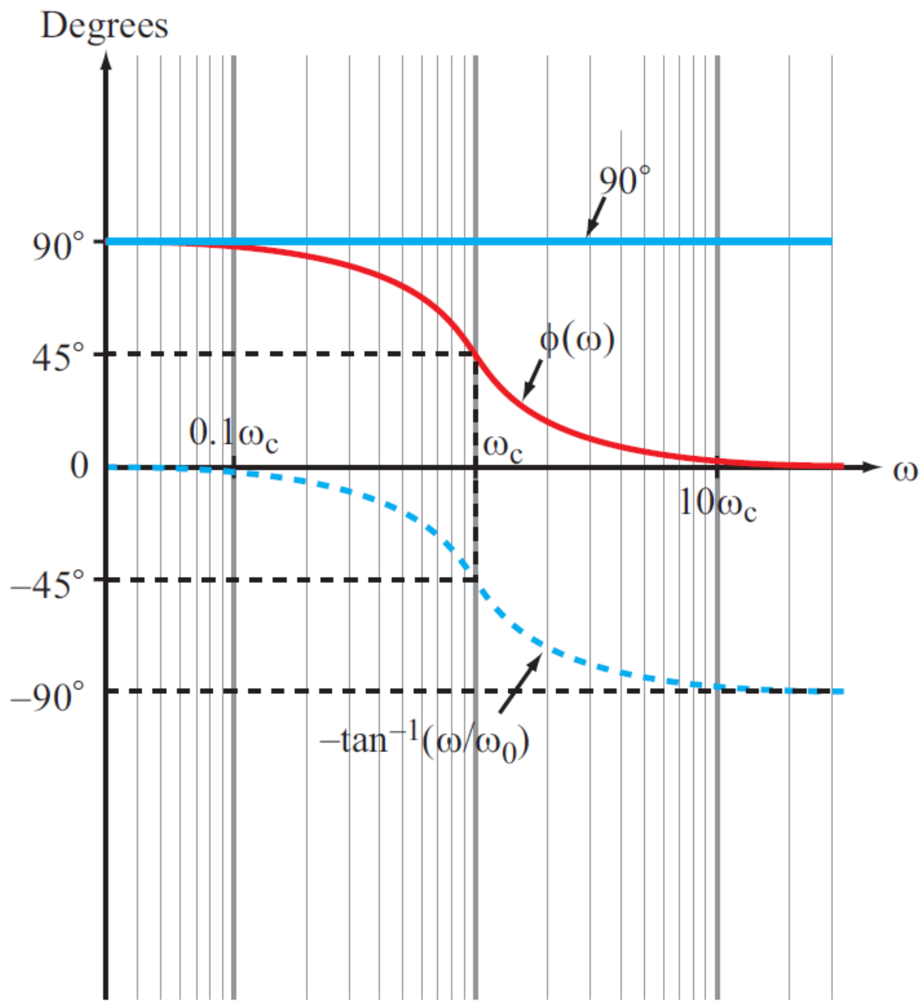
$$\mathbf{V}_{out} = \frac{j\omega L \mathbf{V}_s}{R + j\omega L},$$

which leads to

$$\mathbf{H} = \frac{\mathbf{V}_{out}}{\mathbf{V}_s} = \frac{j\omega L}{R + j\omega L} = \frac{j(\omega/\omega_c)}{1 + j(\omega/\omega_c)},$$

with $\omega_c = R/L$.

$$\phi(\omega) = 90° - \tan^{-1}\left(\frac{\omega}{\omega_c}\right)$$



(c) Phase plot

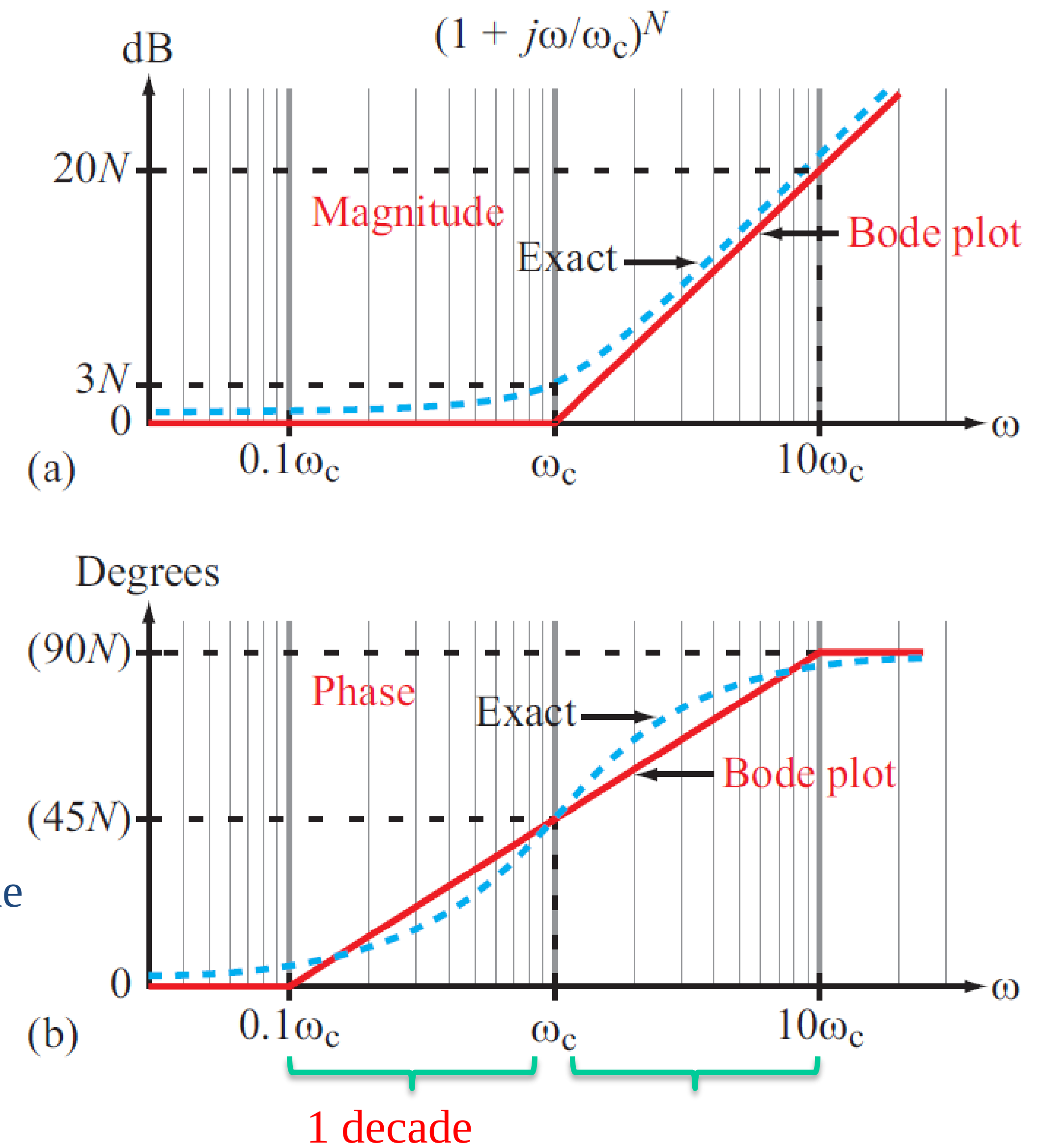
Log scale for ω and linear scale for $\varphi(\omega)$

Bode Plots: Straight line Approximations

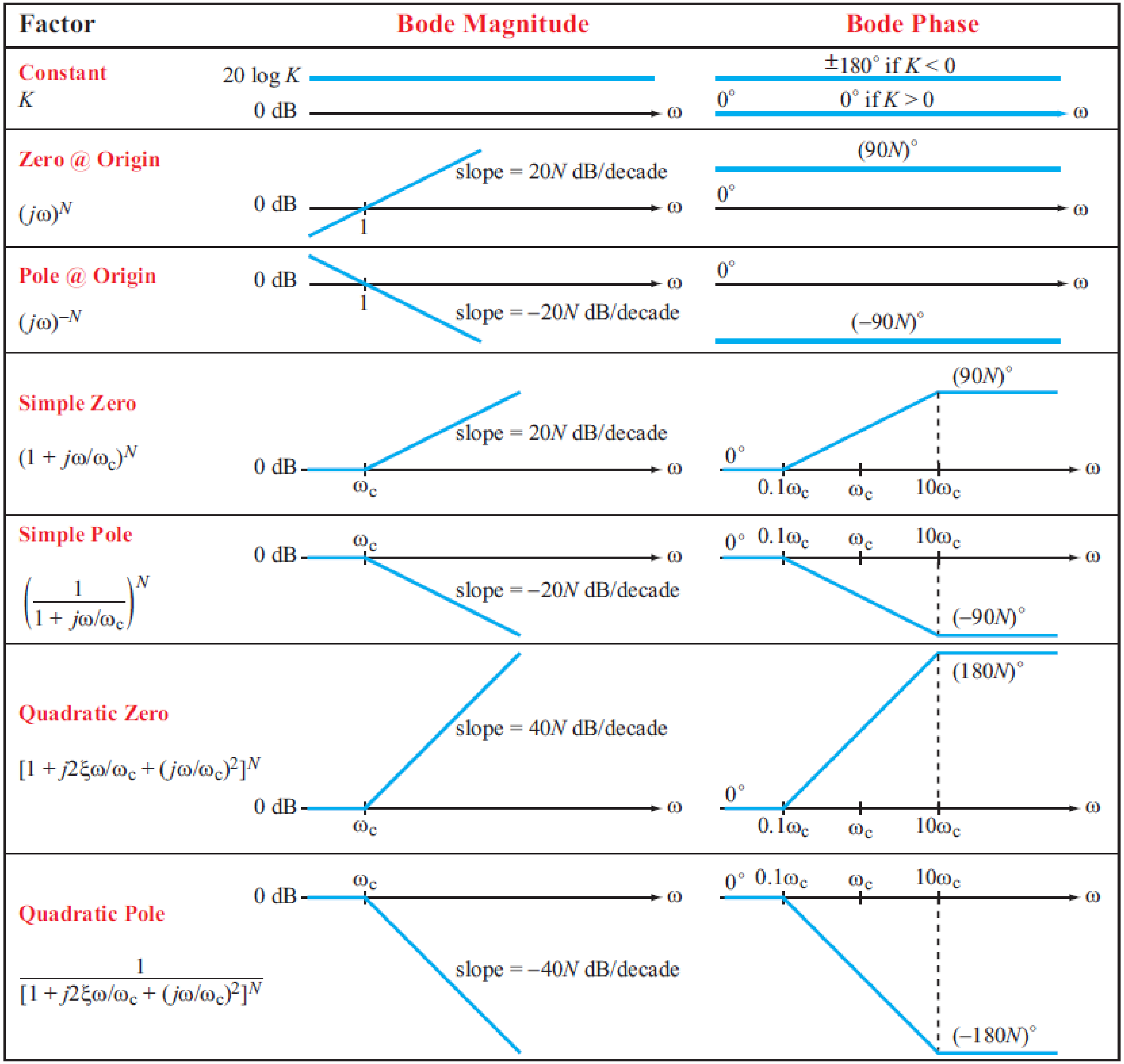
Simple zero: $\mathbf{H} = (1 + j\omega/\omega_c)^N$

Bode Magnitude Slope= 20N dB per decade

Bode Phase Slope= 45N degrees per decade



Bode Plots


Factor
Bode Magnitude
Bode Phase
Constant
K
20 log K
0 dB
ω
±180° if K < 0
0°
0° if K > 0
Zero @ Origin
(jω)^N
slope = 20N dB/decade
1
(90N)°
Pole @ Origin
(jω)^−N
slope = −20N dB/decade
(−90N)°
Simple Zero
(1 + jω/ωc)^N
ωc
0.1ωc
10ωc
(90N)°
Simple Pole
(1/(1 + jω/ωc))^N
slope = −20N dB/decade
(−90N)°
Quadratic Zero
[1 + j2ξω/ωc + (jω/ωc)²]^N
slope = 40N dB/decade
(180N)°
Quadratic Pole
1/[1 + j2ξω/ωc + (jω/ωc)²]^N
slope = −40N dB/decade
(−180N)°