Bode Plot Examples

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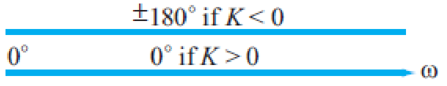
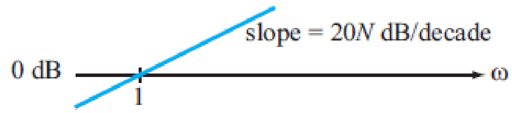
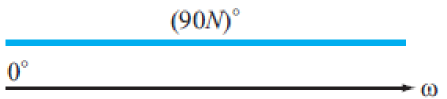
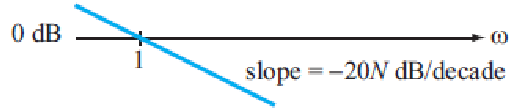
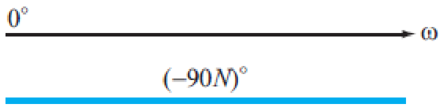
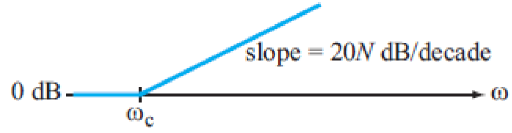
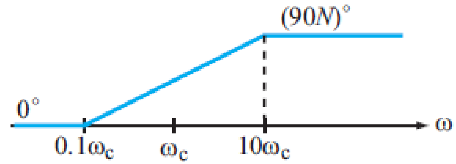
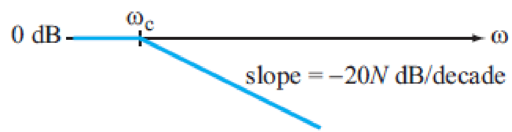
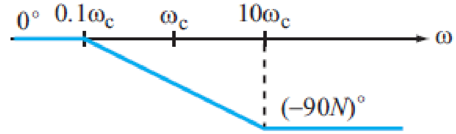
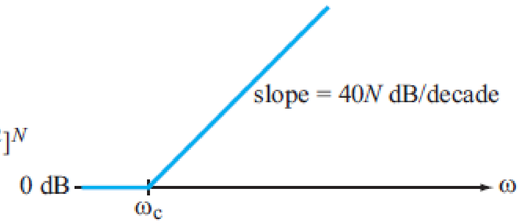
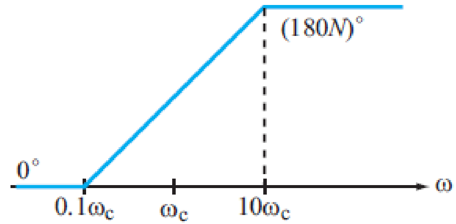
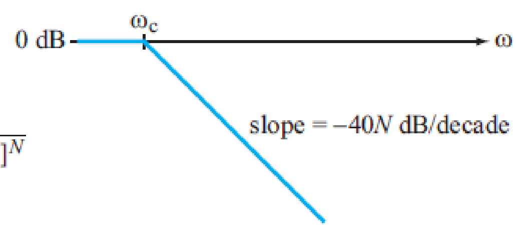
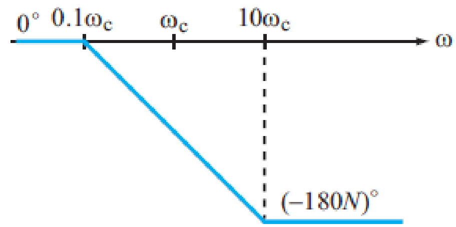
Bode Plots

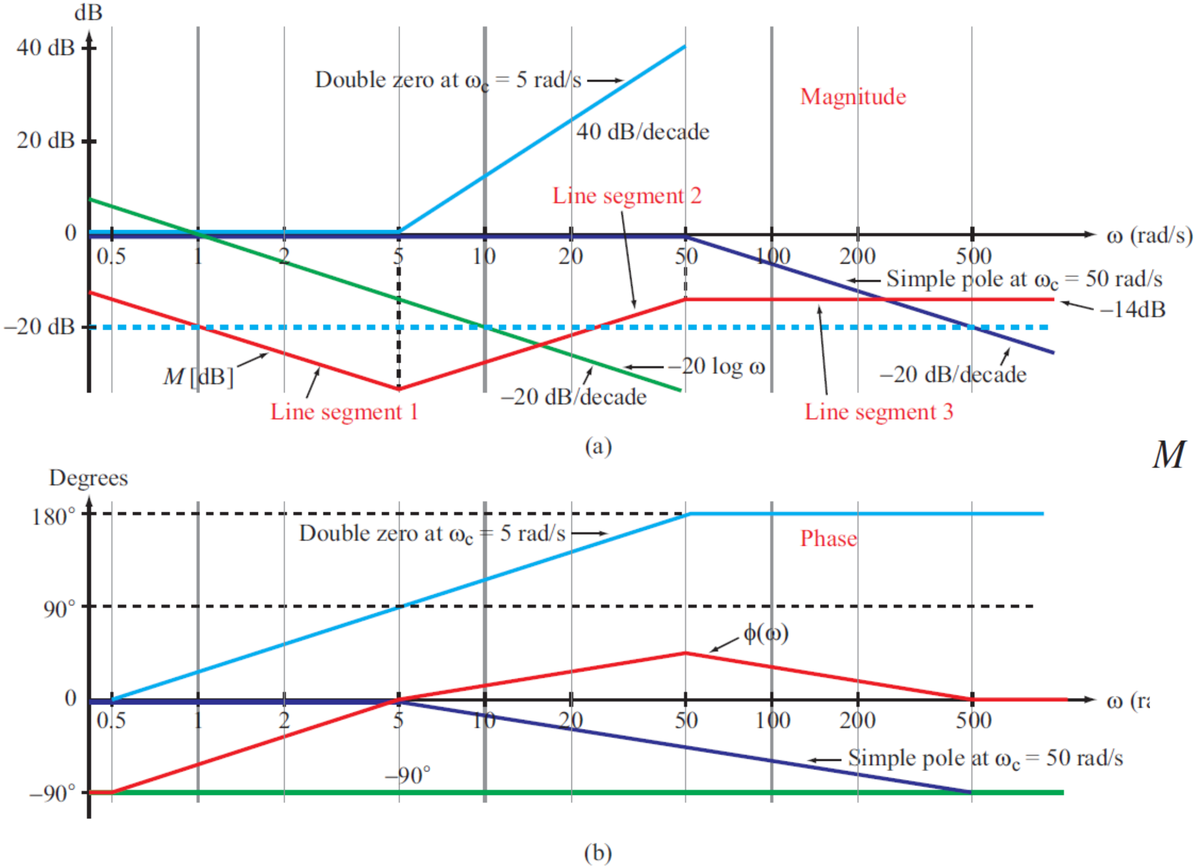
Factor	Bode Magnitude	Bode Phase
Constant K		
Zero @ Origin $(j\omega)^N$		
Pole @ Origin $(j\omega)^{-N}$		
Simple Zero $(1 + j\omega/\omega_c)^N$		
Simple Pole $\left(\frac{1}{1 + j\omega/\omega_c}\right)^N$		
Quadratic Zero $[1 + j2\xi\omega/\omega_c + (j\omega/\omega_c)^2]^N$		
Quadratic Pole $\frac{1}{[1 + j2\xi\omega/\omega_c + (j\omega/\omega_c)^2]^N}$		

Example

$$\mathbf{H}(\omega) = \frac{(20 + j4\omega)^2}{j40\omega(100 + j2\omega)}$$

Standard form

$$\mathbf{H}(\omega) = \frac{400(1 + j\omega/5)^2}{j4000\omega(1 + j\omega/50)} = \frac{-j0.1(1 + j\omega/5)^2}{\omega(1 + j\omega/50)}$$



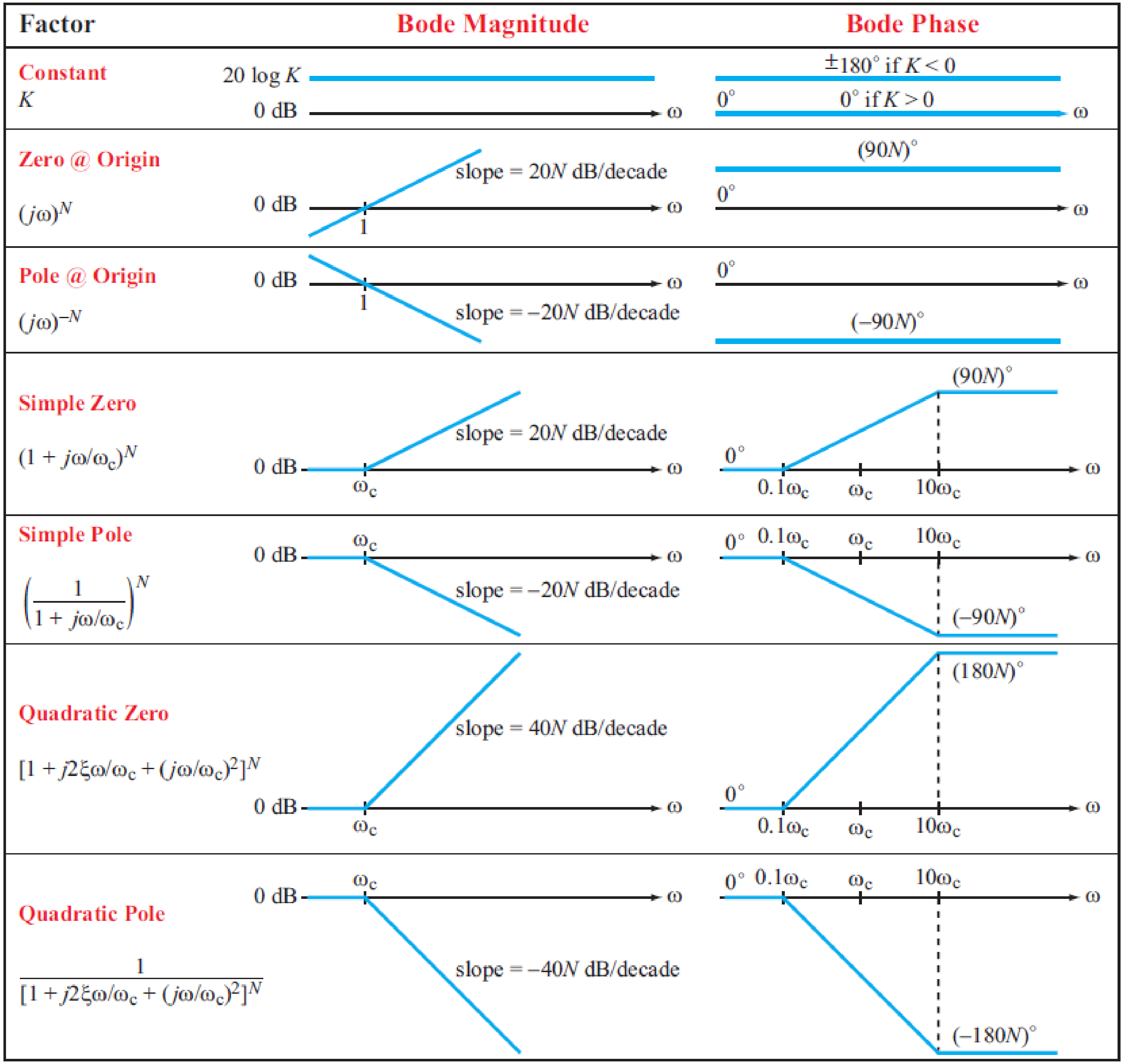
Numerator: simple zero of second order with corner frequency 5 rad/s

Denominator: pole @ origin, and simple pole with corner frequency 50 rad/s

$$\begin{aligned} M\ [\text{dB}] &= 20\log|\mathbf{H}| \\ &= 20\log 0.1 + 40\log|1 + j\omega/5| \\ &\qquad - 20\log\omega - 20\log|1 + j\omega/50| \\ &= -20\ \text{dB} + 40\log|1 + j\omega/5| \\ &\qquad - 20\log\omega - 20\log|1 + j\omega/50|. \end{aligned}$$

$$\phi = -90° + 2\tan^{-1}\frac{\omega}{5} - \tan^{-1}\frac{\omega}{50}$$

Bode Plots



Factor
Bode Magnitude
Bode Phase
Constant
K
20 log K
0 dB
ω
±180° if K < 0
0°
0° if K > 0
ω
Zero @ Origin
(jω)^N
0 dB
1
slope = 20N dB/decade
ω
(90N)°
0°
ω
Pole @ Origin
(jω)^−N
0 dB
1
slope = −20N dB/decade
ω
0°
(−90N)°
ω
Simple Zero
(1 + jω/ωc)^N
0 dB
ωc
slope = 20N dB/decade
ω
(90N)°
0°
0.1ωc
ωc
10ωc
ω
Simple Pole
(1 / (1 + jω/ωc))^N
0 dB
ωc
slope = −20N dB/decade
ω
0°
0.1ωc
ωc
10ωc
(−90N)°
ω
Quadratic Zero
[1 + j2ξω/ωc + (jω/ωc)²]^N
0 dB
ωc
slope = 40N dB/decade
ω
(180N)°
0°
0.1ωc
ωc
10ωc
ω
Quadratic Pole
1 / [1 + j2ξω/ωc + (jω/ωc)²]^N
0 dB
ωc
slope = −40N dB/decade
ω
0°
0.1ωc
ωc
10ωc
(−180N)°
ω

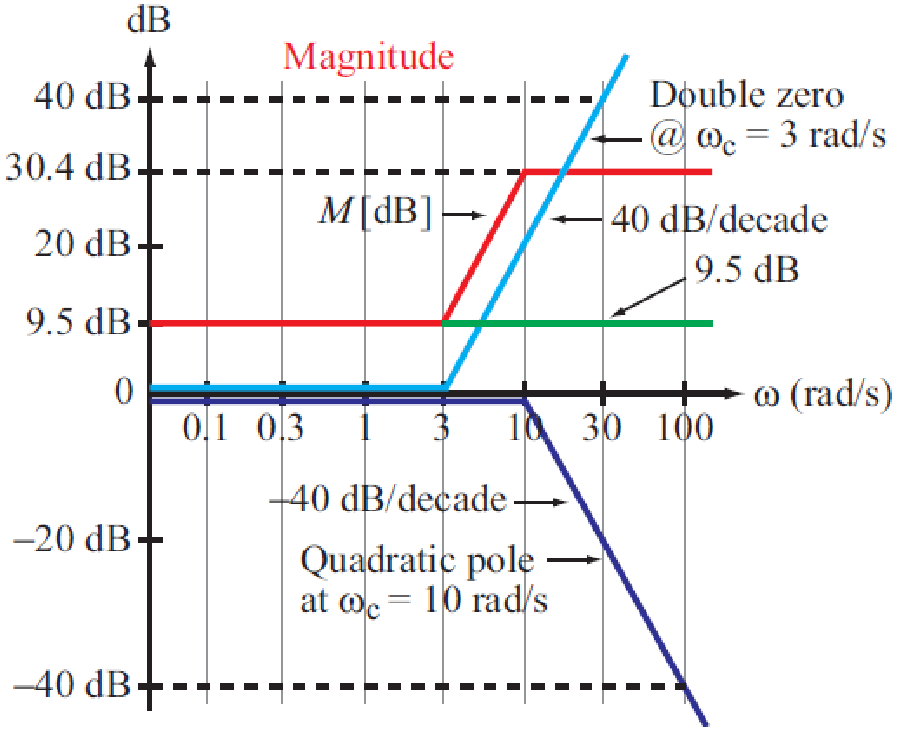
Another Example

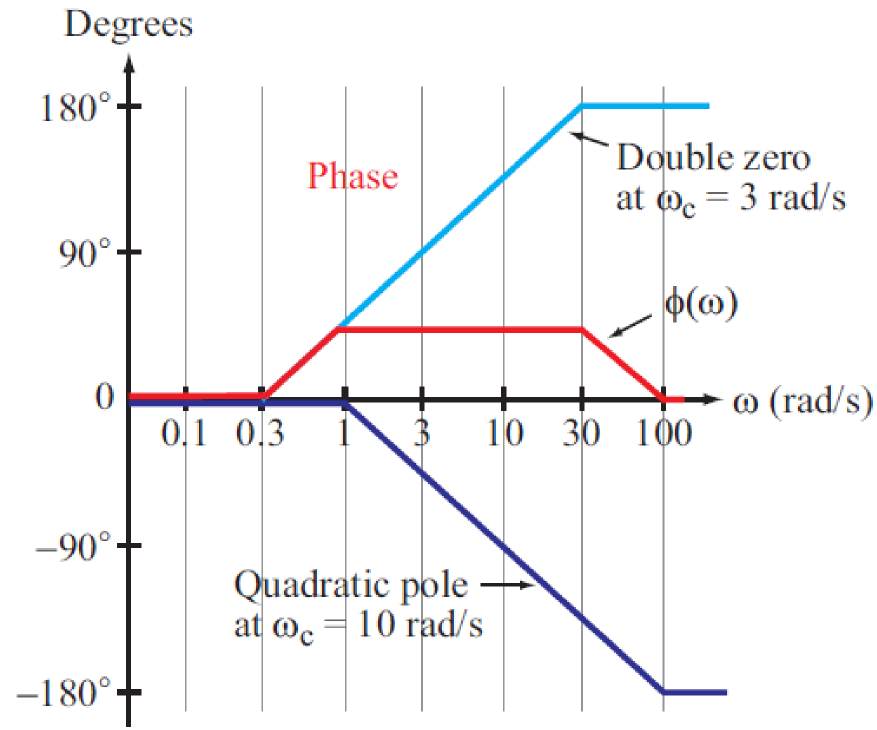
$$\mathbf{H}(\omega) = \frac{(j10\omega + 30)^2}{(300 - 3\omega^2 + j90\omega)}$$

$$\mathbf{H}(\omega) = \frac{3(1 + j\omega/3)^2}{[1 + j3\omega/10 + (j\omega/10)^2]}$$

$$\begin{aligned} M\ [\text{dB}] &= 20 \log |\mathbf{H}| \\ &= 20 \log 3 + 40 \log |1 + j\omega/3| \\ &\quad - 20 \log |1 + j3\omega/10 + (j\omega/10)^2| \\ &= 9.5\ \text{dB} + 40 \log |1 + j\omega/3| \\ &\quad - 20 \log |1 + j3\omega/10 + (j\omega/10)^2|, \end{aligned}$$

$$\phi = 2 \tan^{-1}(\omega/3) - \tan^{-1}\left(\frac{3\omega/10}{1 - \omega^2/100}\right)$$





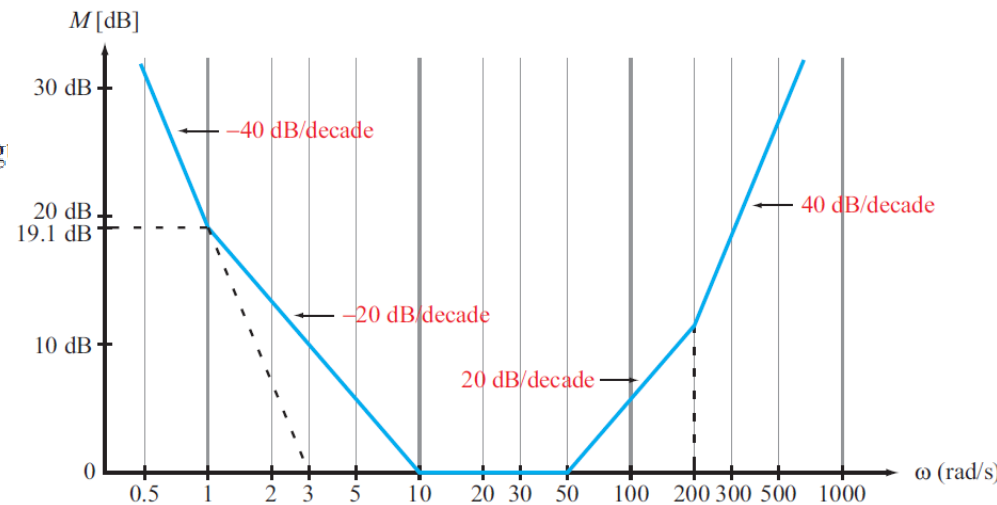
Example: given Bode Plot, obtain expression

$$\mathbf{H}_1 = \left(\frac{1}{\omega/3}\right)^2 = \frac{9}{\omega^2}.$$

To verify the validity of our expression, let us convert it to dB as

$$M_1\,[\text{dB}] = 20\log\frac{9}{\omega^2} = 20\log 9 - 40\log\omega$$
$$= 19.1\ \text{dB} - 40\log\omega.$$

At $\omega = 1$ rad/s, M_1 [dB] $= 19.1$ dB, which matches the fig



As we progress along the ω-axis, the second segment has a slope of only -20 dB/decade, which means that a simple-zero factor with a corner frequency of 1 rad/s has come into play. Hence,

$$\mathbf{H}_2 = (1 + j\omega).$$

At $\omega = 10$ rad/s, the slope becomes zero, signifying the introduction of another simple-zero factor given by

$$\mathbf{H}_3 = (1 + j\omega/10).$$

Similarly,

$$\mathbf{H}_4 = (1 + j\omega/50),$$

and

$$\mathbf{H}_5 = (1 + j\omega/200).$$

$$\mathbf{H}(\omega) = \mathbf{H}_1\mathbf{H}_2\mathbf{H}_3\mathbf{H}_4\mathbf{H}_5$$
$$= \frac{9(1 + j\omega)(1 + \frac{j\omega}{10})(1 + \frac{j\omega}{50})(1 + \frac{j\omega}{200})}{\omega^2}.$$