Active Filters

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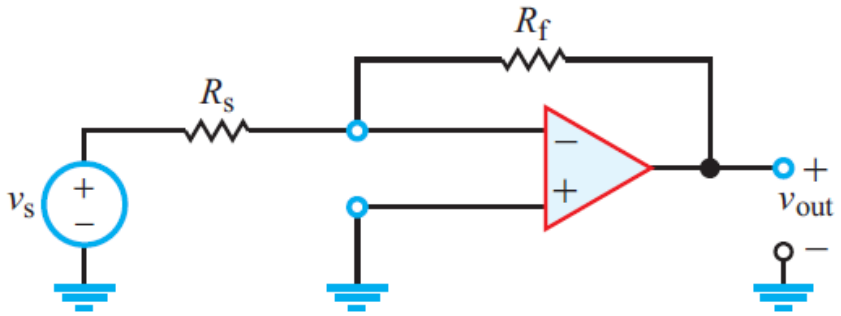
Active Lowpass Filter

$$\mathbf{V}_{\text{out}} = -\frac{\mathbf{Z}_\text{f}}{\mathbf{Z}_\text{s}}\,\mathbf{V}_\text{s}$$



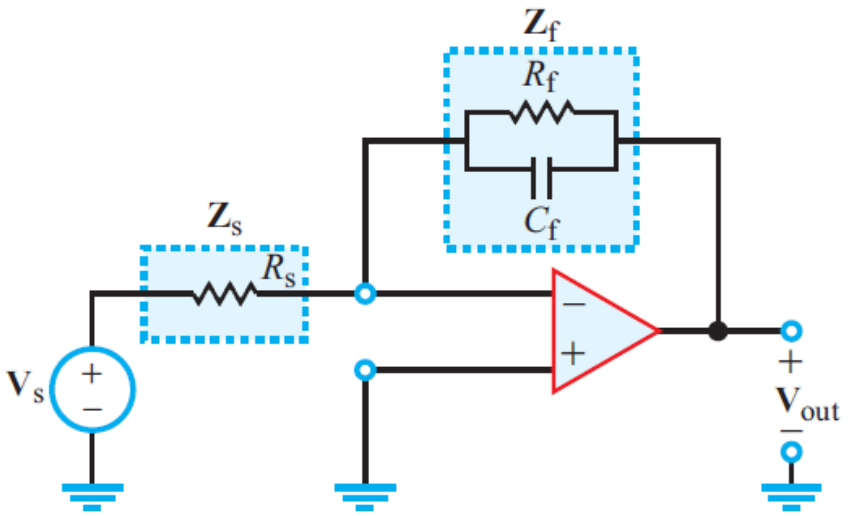
(a) Inverting amplifier

$$\mathbf{Z}_\text{f} = R_\text{f} \parallel \left(\frac{1}{j\omega C_\text{f}}\right) = \frac{R_\text{f}}{1 + j\omega R_\text{f} C_\text{f}}.$$

$$\mathbf{H}_{\text{LP}}(\omega) = \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_\text{s}} = -\frac{\mathbf{Z}_\text{f}}{\mathbf{Z}_\text{s}} = -\frac{R_\text{f}}{R_\text{s}}\left(\frac{1}{1 + j\omega R_\text{f} C_\text{f}}\right)$$
$$= G_{\text{LP}}\left(\frac{1}{1 + j\omega/\omega_{\text{LP}}}\right),$$

where

$$G_{\text{LP}} = -\frac{R_\text{f}}{R_\text{s}}, \qquad \omega_{\text{LP}} = \frac{1}{R_\text{f} C_\text{f}}.$$

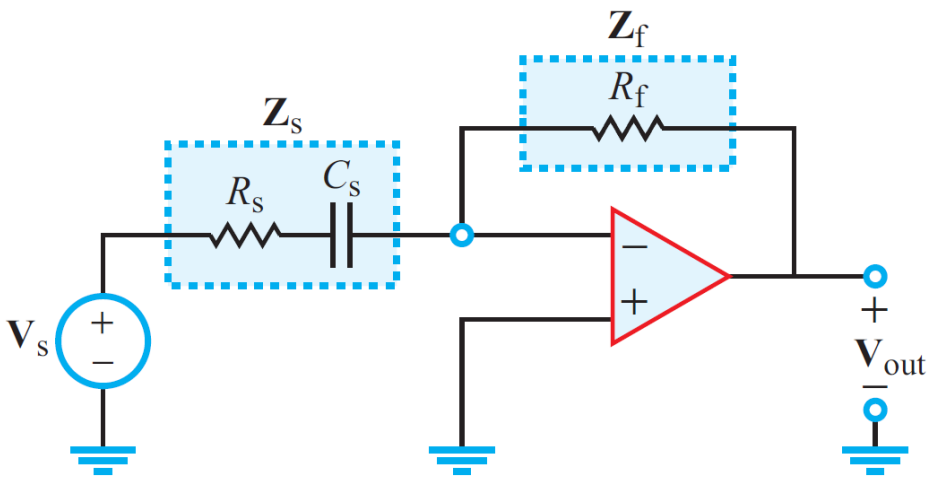


(b) Phasor domain with impedances

Active Highpass Filter

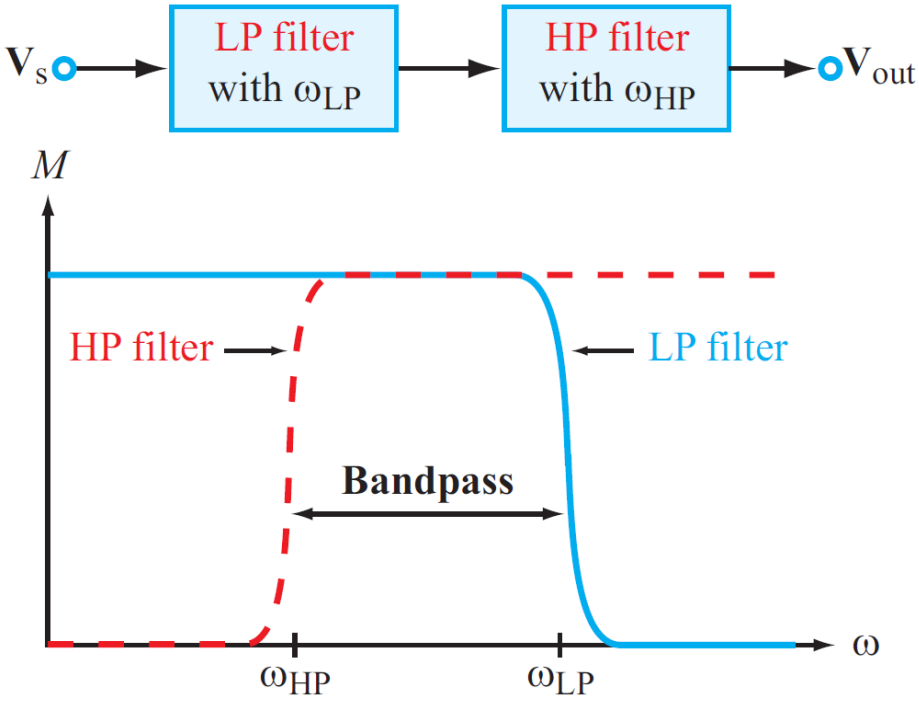
$$\mathbf{Z}_s = R_s - \frac{j}{\omega C_s} \qquad \text{and} \qquad \mathbf{Z}_f = R_f,$$

$$\mathbf{H}_{HP}(\omega) = \frac{\mathbf{V}_{out}}{\mathbf{V}_s} = -\frac{\mathbf{Z}_f}{\mathbf{Z}_s} = -\frac{R_f}{R_s - j/\omega C_s}$$
$$= G_{HP}\left[\frac{j\omega/\omega_{HP}}{1 + j\omega/\omega_{HP}}\right],$$

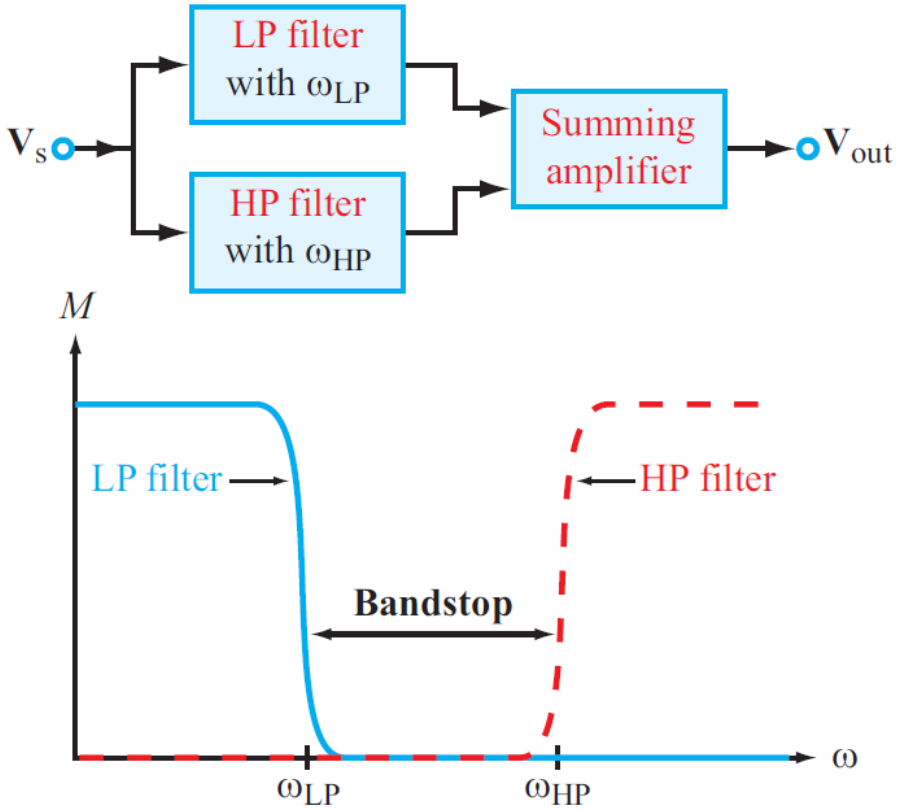


$$G_{HP} = -\frac{R_f}{R_s} \qquad \text{and} \qquad \omega_{HP} = \frac{1}{R_s C_s}.$$

Cascading Active Filters

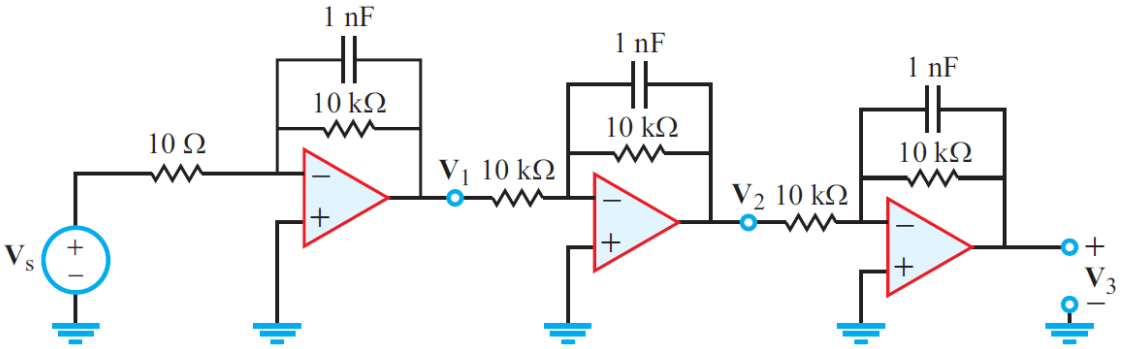


(a) Bandpass filter

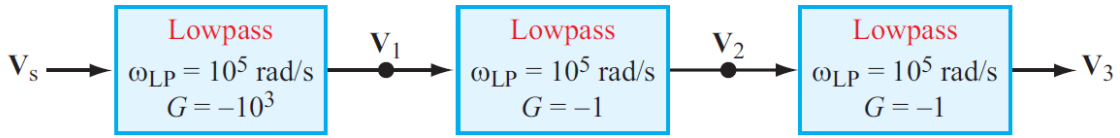


(b) Bandreject filter

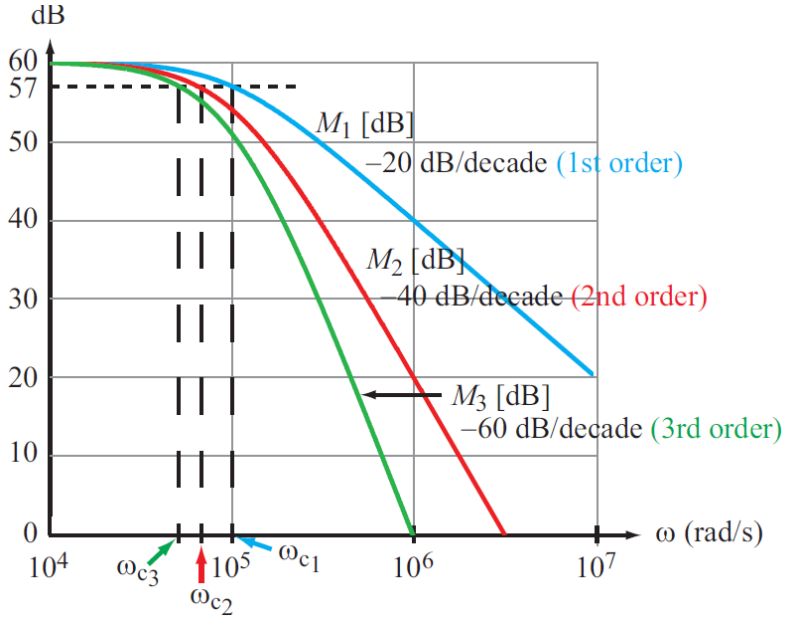
Example: Third-Order Lowpass Filter



(a) Circuit diagram



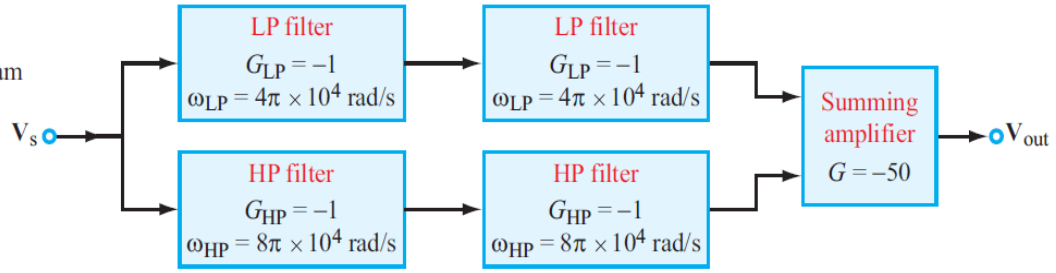
(b) Block diagram



Example: Bandreject Filter

Design a bandreject filter with the specifications: (a) Gain $= -50$, (b) bandstop extends from 20 kHz to 40 kHz, and (c) gain roll-off rate $= -40$ dB/decade along both boundaries of the bandstop.

(a) Block diagram



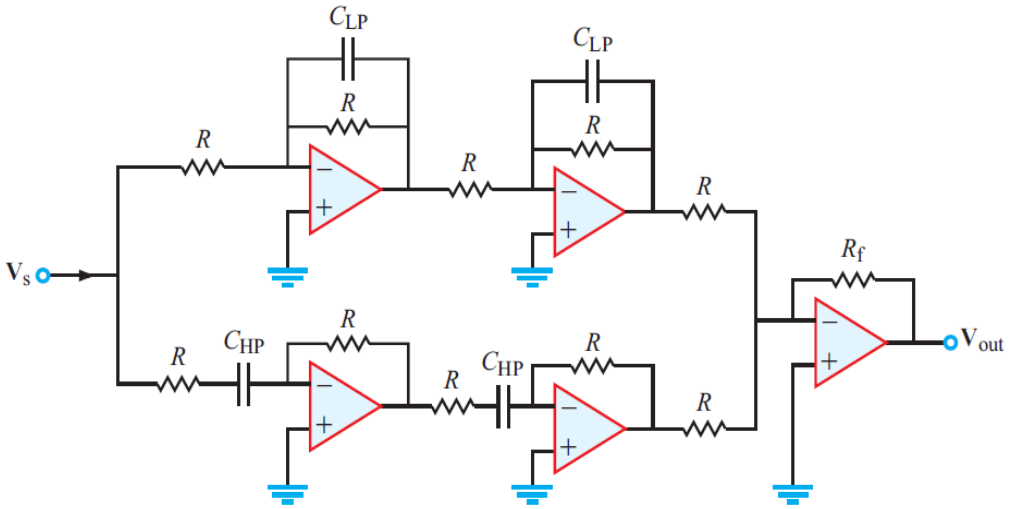
Somewhat arbitrarily, we select $R = 1\ \text{k}\Omega$. From

$$\omega_{LP} = 4\pi \times 10^4 = \frac{1}{RC_{LP}} \rightarrow C_{LP} = 7.96\ \text{nF} \simeq 8\ \text{nF},$$

$$\omega_{HP} = 8\pi \times 10^4 = \frac{1}{RC_{HP}} \rightarrow C_{HP} \simeq 4\ \text{nF}.$$

The value of R_f is specified by the gain of the summing amplifier as

$$G = -50 = -\frac{R_f}{R} \rightarrow R_f = 50\ \text{k}\Omega.$$



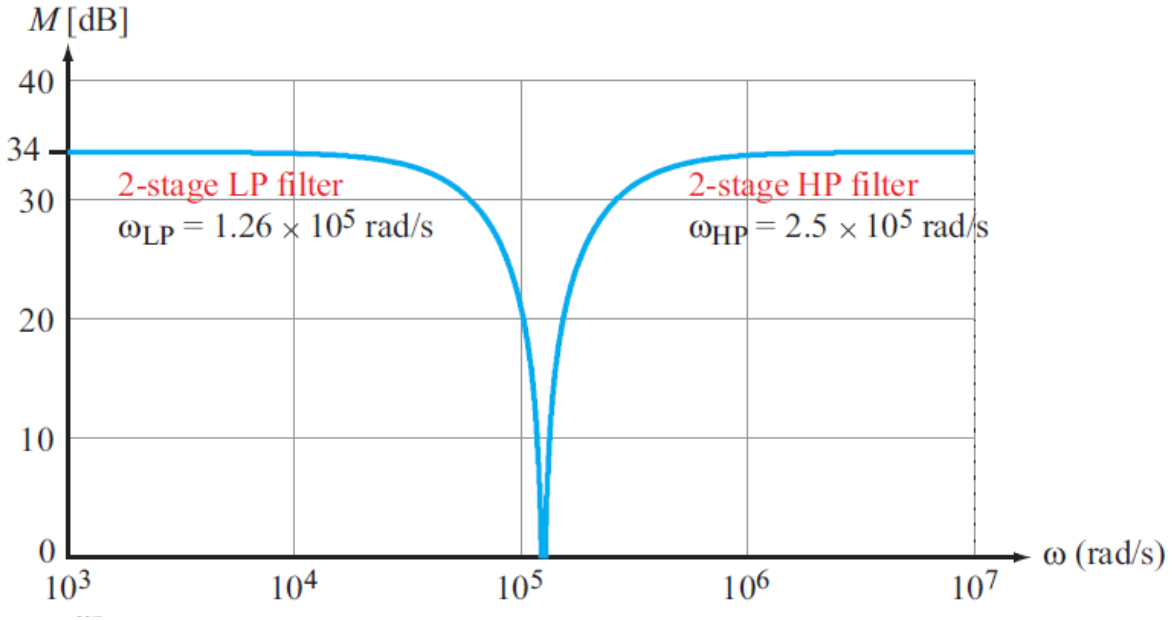
The transfer function of the bandreject filter is given by

$$\mathbf{H}(\omega) = G[\mathbf{H}_{LP}^2 + \mathbf{H}_{HP}^2]$$

$$= -50\left[\left(\frac{1}{1 + j\omega RC_{LP}}\right)^2 + \left(\frac{j\omega RC_{HP}}{1 + j\omega RC_{HP}}\right)^2\right]$$

$$= -50\left[\left(\frac{1}{1 + j\omega/4\pi \times 10^4}\right)^2 + \left(\frac{j\omega/8\pi \times 10^4}{1 + j\omega/8\pi \times 10^4}\right)^2\right].$$

Example: Bandreject Filter



$$G = -50 = -\frac{R_f}{R} \quad \Rightarrow \quad R_f = 50\ \text{k}\Omega.$$

The transfer function of the bandreject filter is given by

$$\begin{aligned}\mathbf{H}(\omega) &= G[\mathbf{H}_{LP}^2 + \mathbf{H}_{HP}^2] \\ &= -50\left[\left(\frac{1}{1 + j\omega RC_{LP}}\right)^2 + \left(\frac{j\omega RC_{HP}}{1 + j\omega RC_{HP}}\right)^2\right] \\ &= -50\left[\left(\frac{1}{1 + j\omega/4\pi \times 10^4}\right)^2 + \left(\frac{j\omega/8\pi \times 10^4}{1 + j\omega/8\pi \times 10^4}\right)^2\right].\end{aligned}$$