

# Lecture 2. Intensity Transformation and Spatial Filtering



# Spatial Domain vs. Transform Domain

## ► Spatial domain

image plane itself, directly process the intensity values of the image plane

## ► Transform domain

process the transform coefficients, not directly process the intensity values of the image plane

# Spatial Domain Process

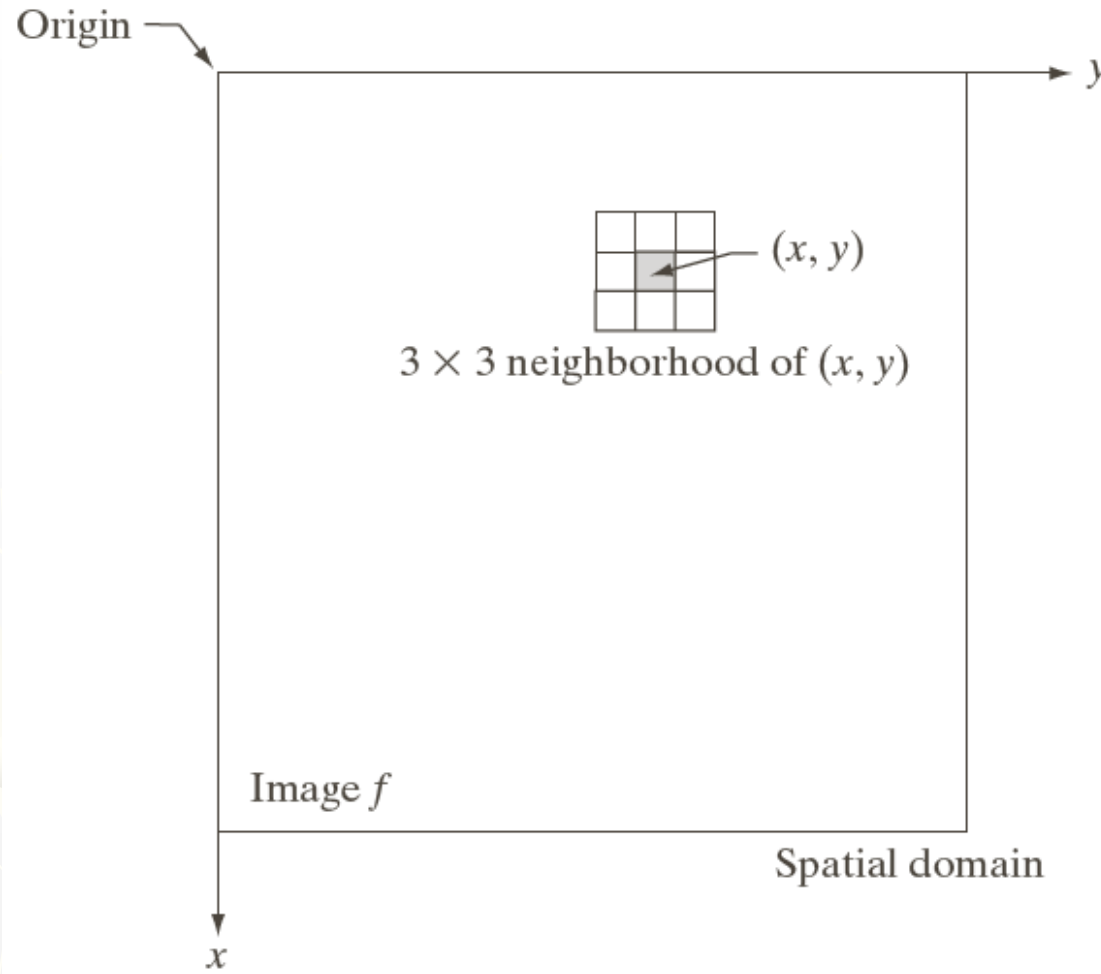
$$g(x, y) = T[f(x, y)]$$

$f(x, y)$  : input image

$g(x, y)$  : output image

$T$  : an operator on  $f$  defined over  
a neighborhood of point  $(x, y)$

# Spatial Domain Process



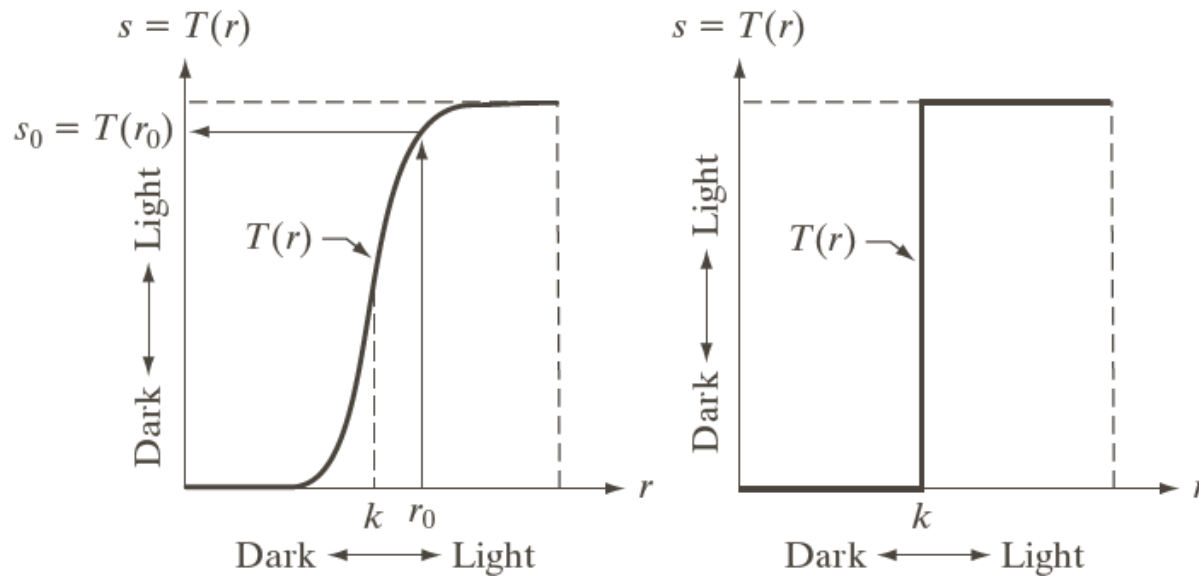
**FIGURE 3.1**

A  $3 \times 3$  neighborhood about a point  $(x, y)$  in an image in the spatial domain. The neighborhood is moved from pixel to pixel in the image to generate an output image.

# Spatial Domain Process

## Intensity transformation function

$$s = T(r)$$

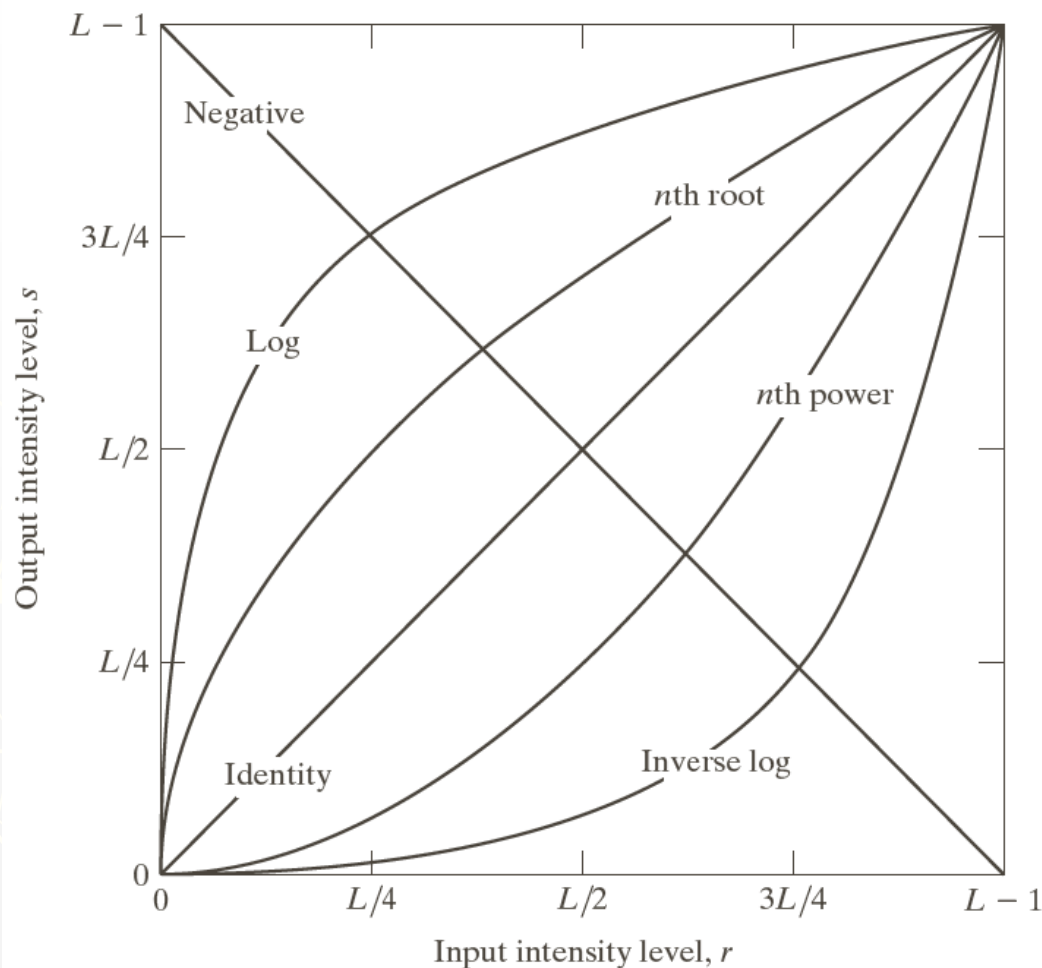


a b

**FIGURE 3.2**

Intensity transformation functions.  
(a) Contrast-stretching function.  
(b) Thresholding function.

# Some Basic Intensity Transformation Functions



**FIGURE 3.3** Some basic intensity transformation functions. All curves were scaled to fit in the range shown.

# Image Negatives

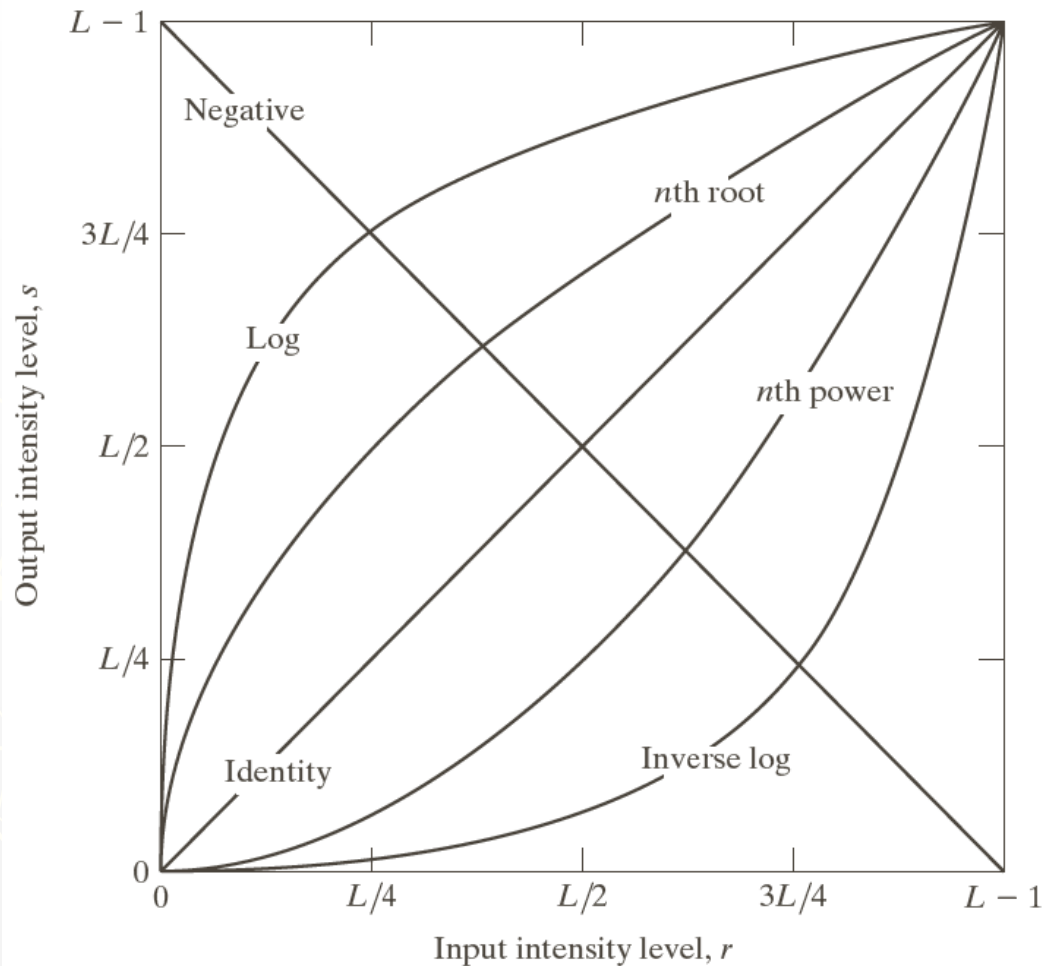


Image negatives

$$s = L - 1 - r$$

# Example: Image Negatives



a b

## FIGURE 3.4

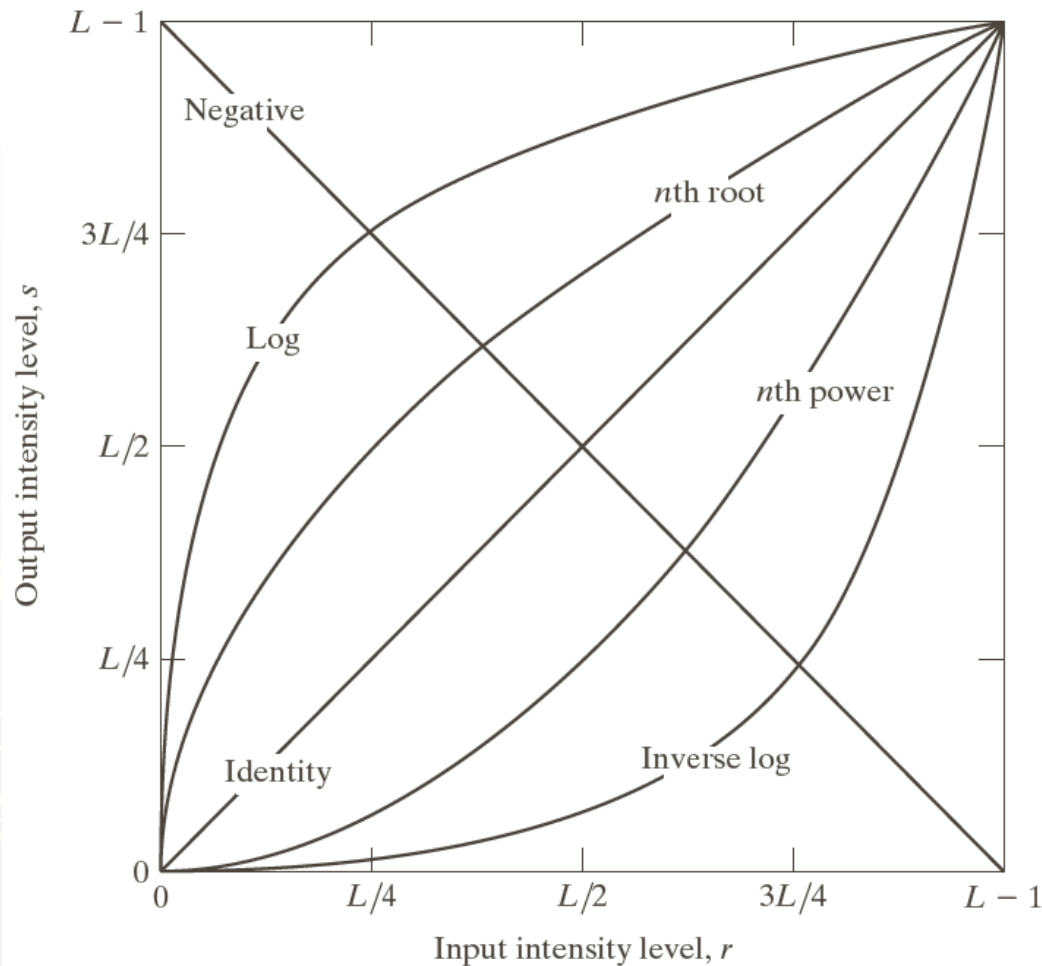
(a) Original digital mammogram.

(b) Negative image obtained using the negative transformation in Eq. (3.2-1).  
(Courtesy of G.E. Medical Systems.)

Small lesion



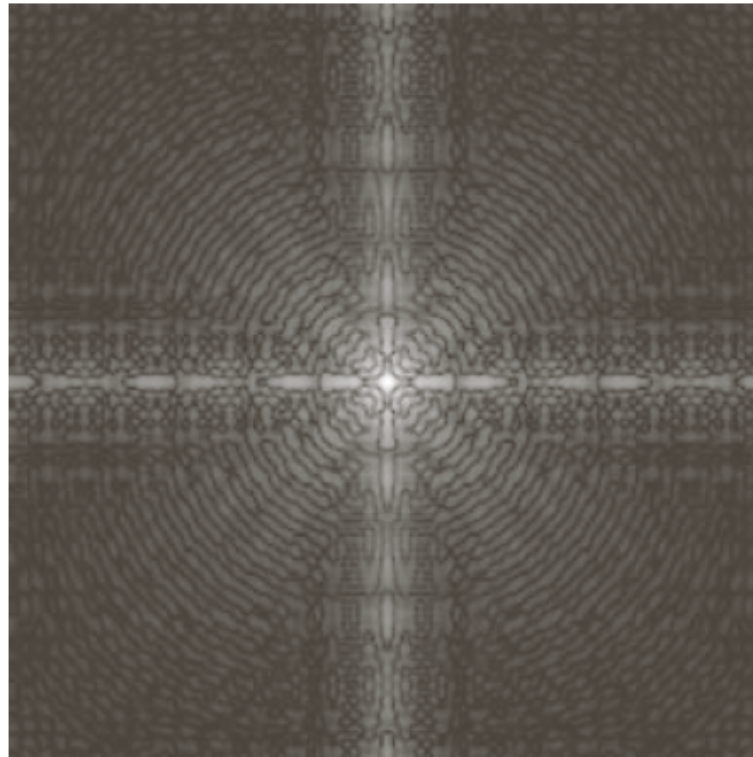
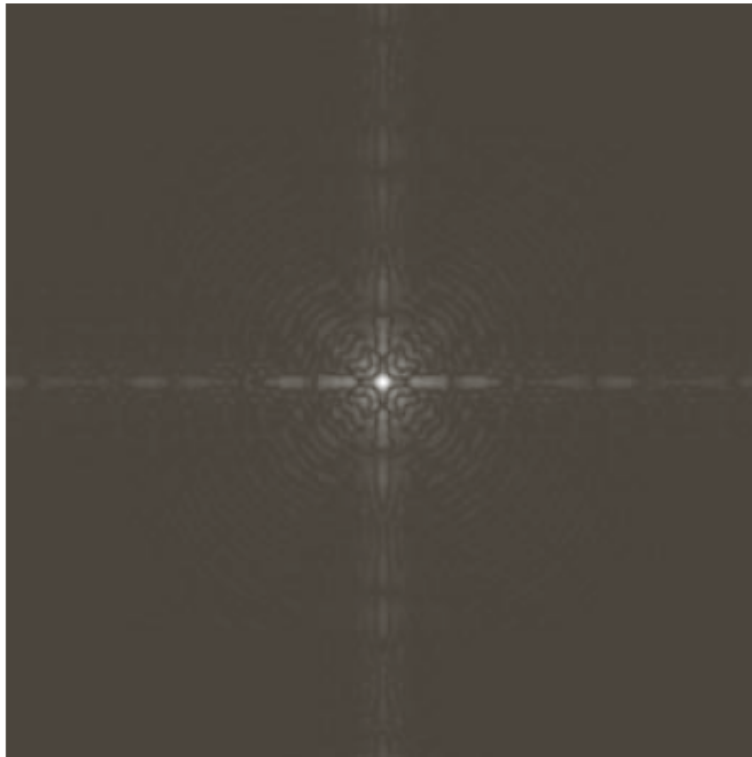
# Log Transformations



## Log Transformations

$$s = c \log(1 + r)$$

# Example: Log Transformations



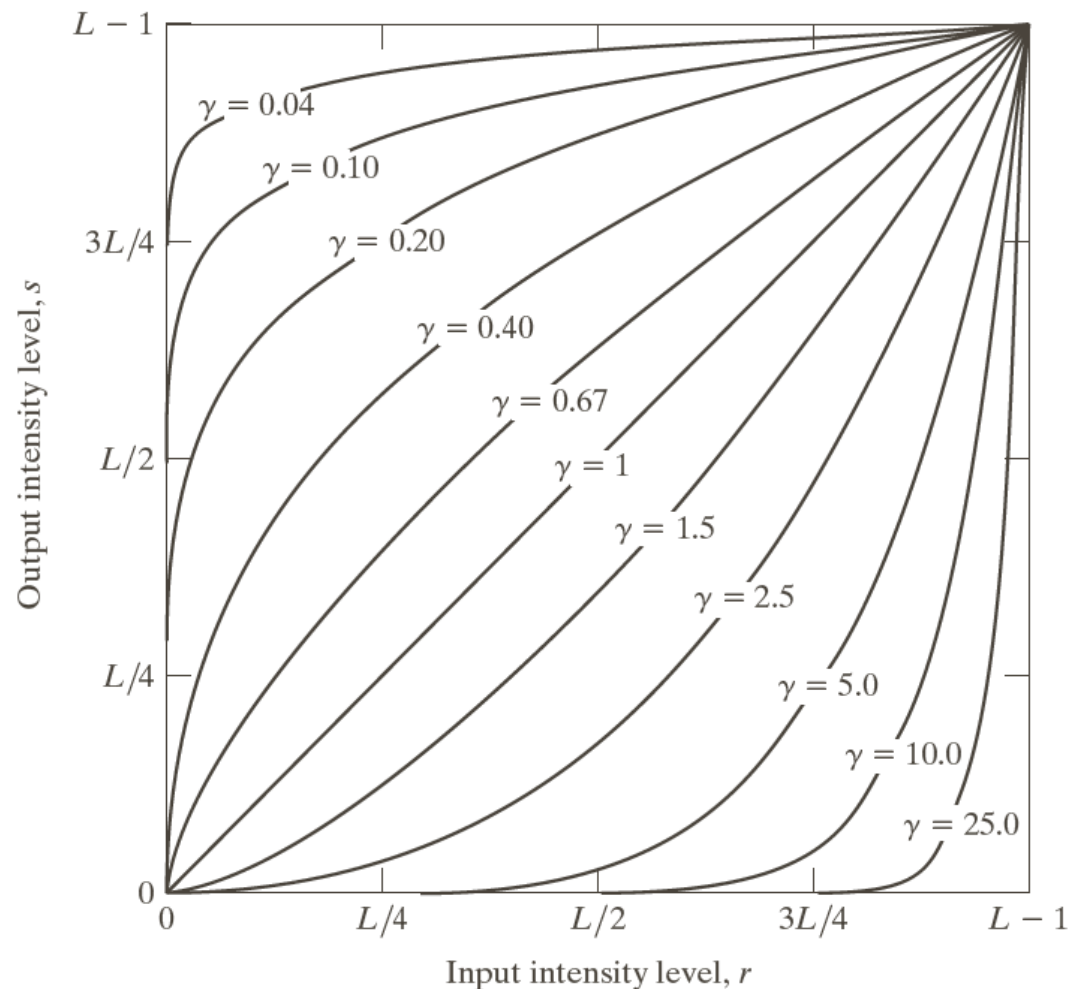
a b

**FIGURE 3.5**

(a) Fourier spectrum.

(b) Result of applying the log transformation in Eq. (3.2-2) with  $c = 1$ .

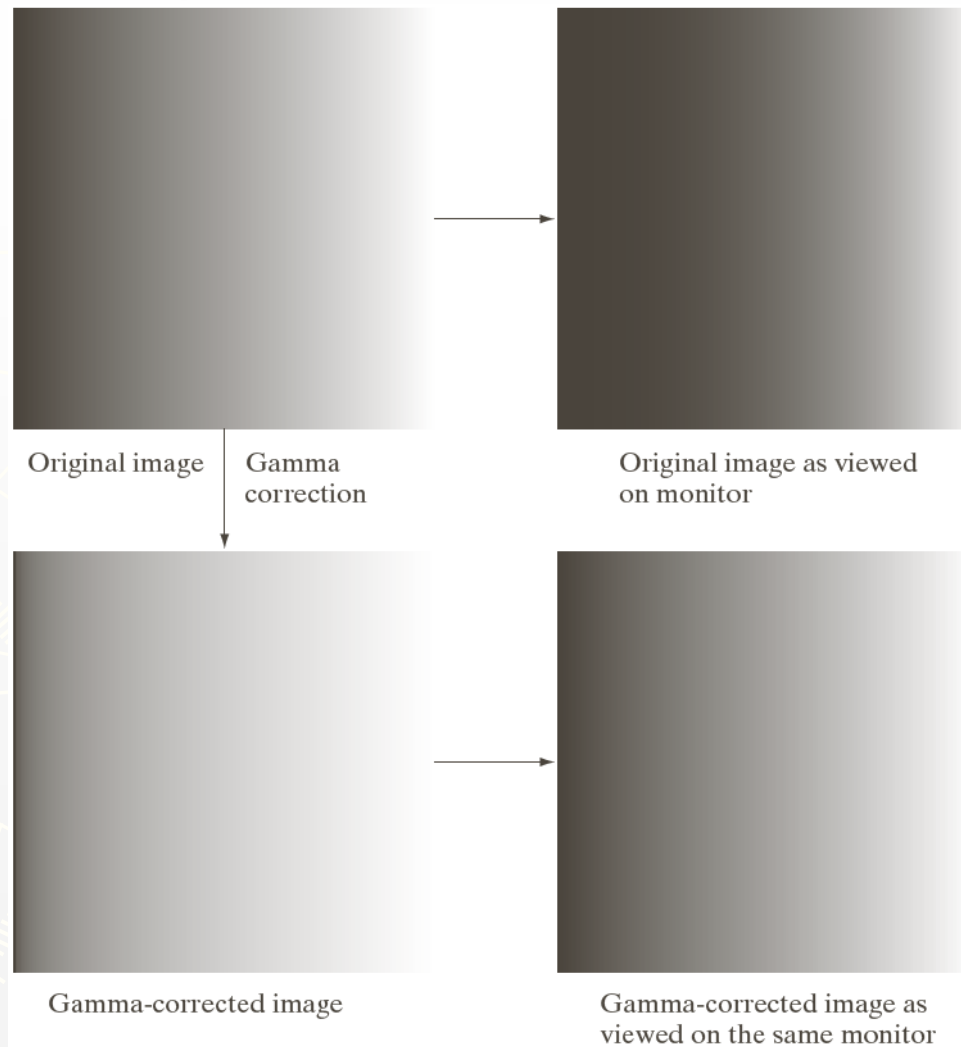
# Power-Law (Gamma) Transformations



$$s = cr^\gamma$$

**FIGURE 3.6** Plots of the equation  $s = cr^\gamma$  for various values of  $\gamma$  ( $c = 1$  in all cases). All curves were scaled to fit in the range shown.

# Example: Gamma Transformations

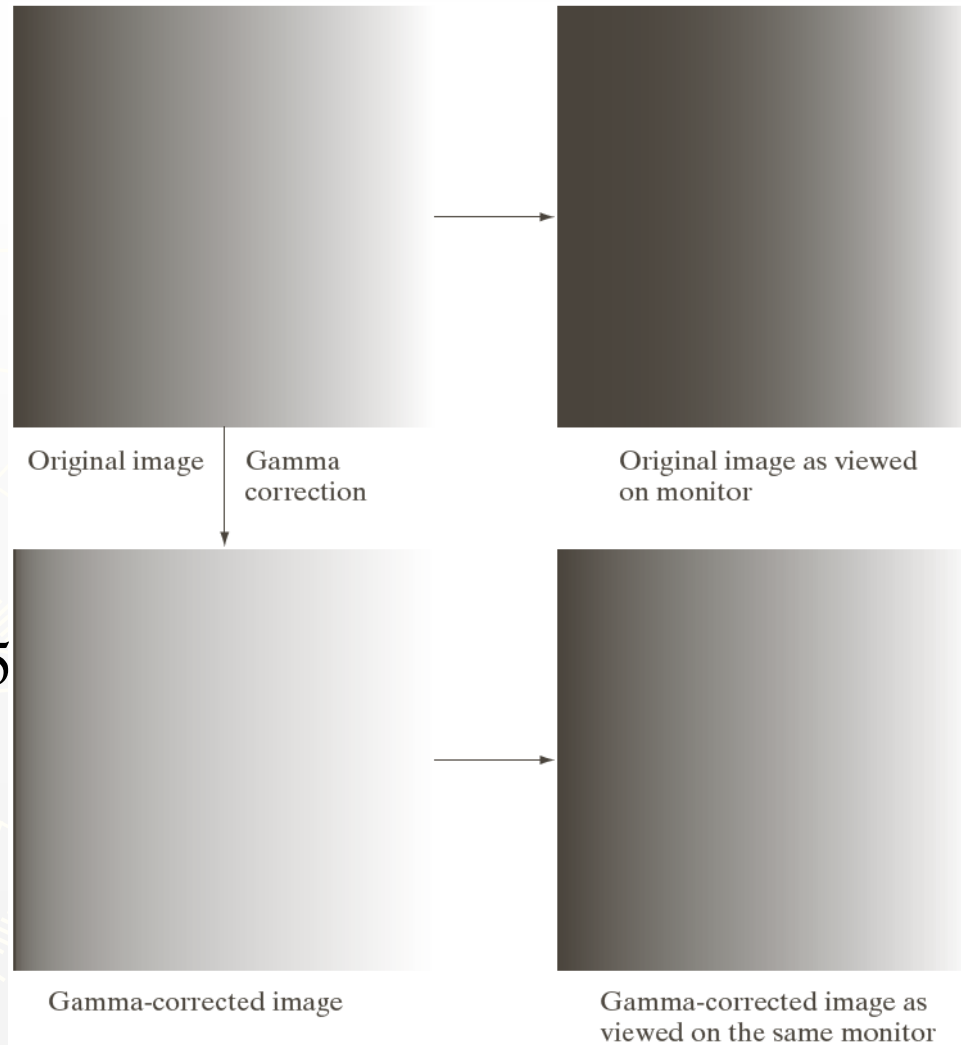


a	b
c	d

**FIGURE 3.7**

(a) Intensity ramp image. (b) Image as viewed on a simulated monitor with a gamma of 2.5. (c) Gamma-corrected image. (d) Corrected image as viewed on the same monitor. Compare (d) and (a).

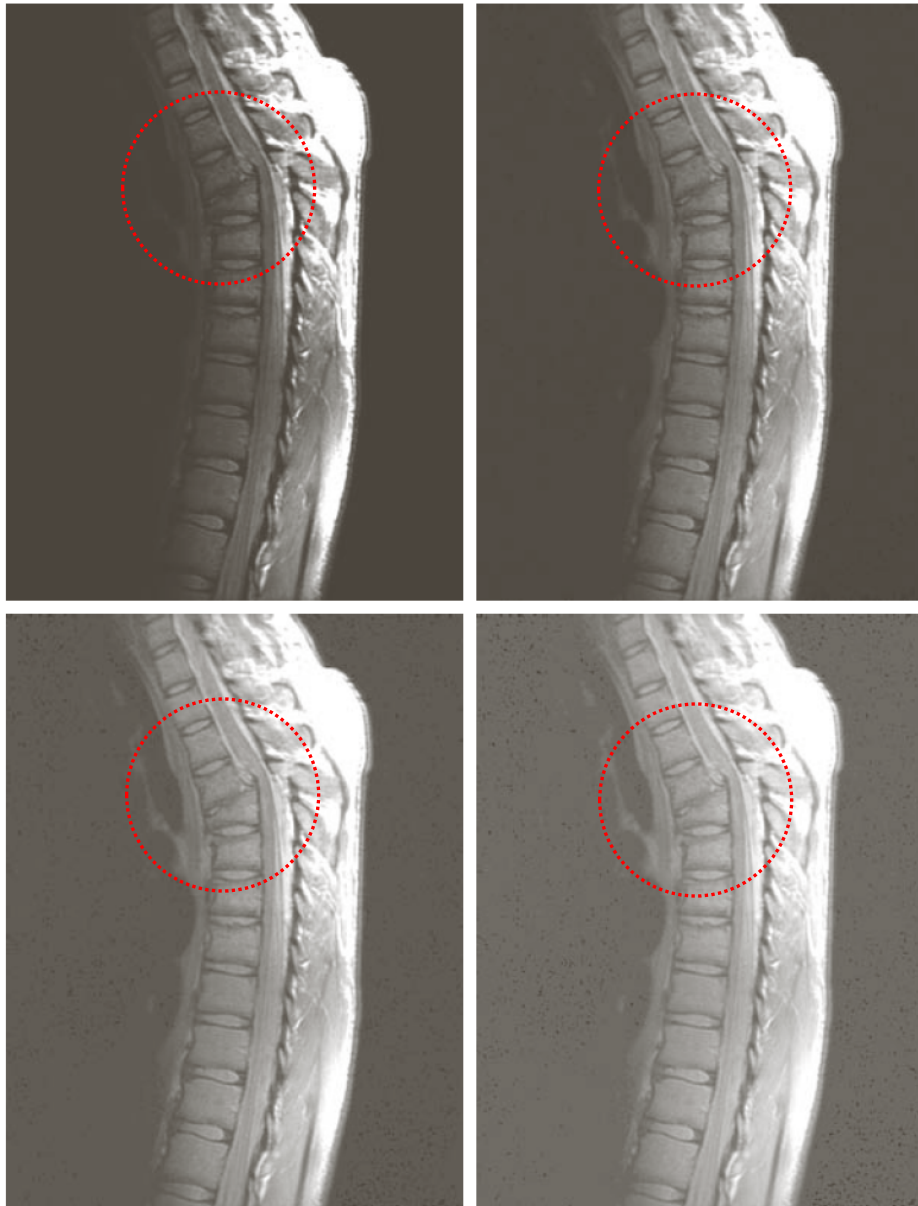
# Example: Gamma Transformations



Cathode ray tube (CRT) devices have an intensity-to-voltage response that is a power function, with exponents varying from approximately 1.8 to 2.5

$$s = r^{1/2.5}$$

# Example: Gamma Transformations



a	b
c	d

**FIGURE 3.8**

(a) Magnetic resonance image (MRI) of a fractured human spine.

(b)–(d) Results of applying the transformation in Eq. (3.2-3) with  $c = 1$  and  $\gamma = 0.6, 0.4$ , and  $0.3$ , respectively. (Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)



# Example: Gamma Transformations



a	b
c	d

**FIGURE 3.9**

(a) Aerial image.  
(b)–(d) Results of applying the transformation in Eq. (3.2-3) with  $c = 1$  and  $\gamma = 3.0, 4.0,$  and  $5.0$ , respectively. (Original image for this example courtesy of NASA.)

# Piecewise-Linear Transformations

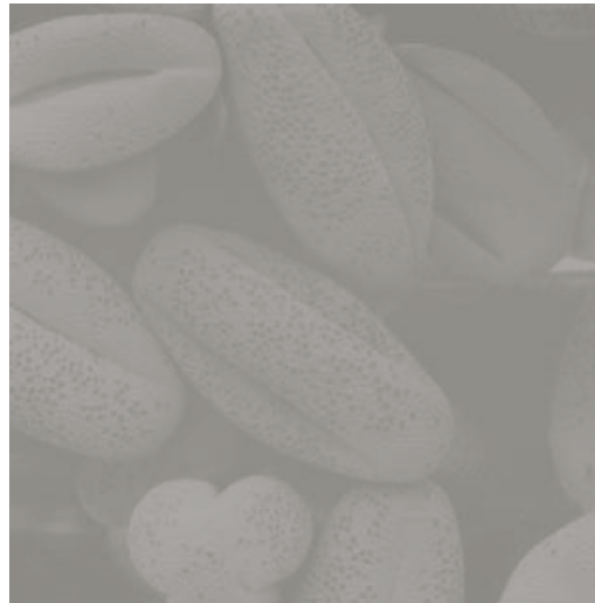
## ► Contrast Stretching

- Expands the range of intensity levels in an image so that it spans the full intensity range of the recording medium or display device.

## ► Intensity-level Slicing

- Highlighting a specific range of intensities in an image often is of interest.





a	b
c	d

### **FIGURE 3.10**

Contrast stretching.

(a) Form of transformation function. (b) A low-contrast image.

(c) Result of contrast stretching.

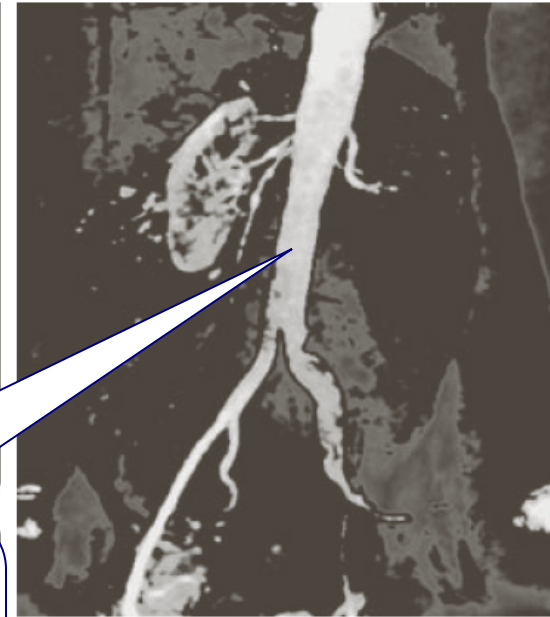
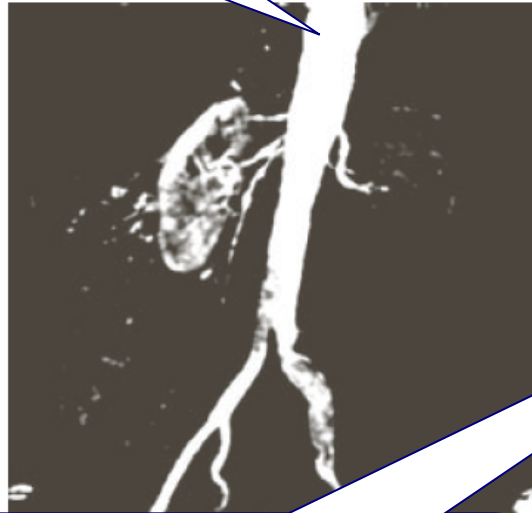
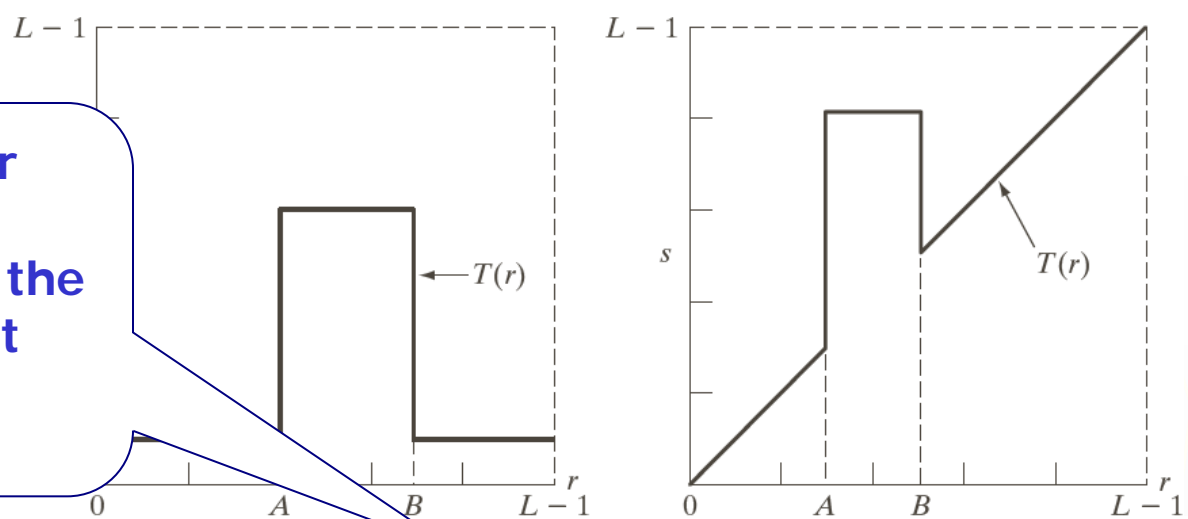
(d) Result of thresholding.

(Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

a b

FIGURE 3.11 (a) This

Highlight the major blood vessels and study the shape of the flow of the contrast medium (to detect blockages, etc.)



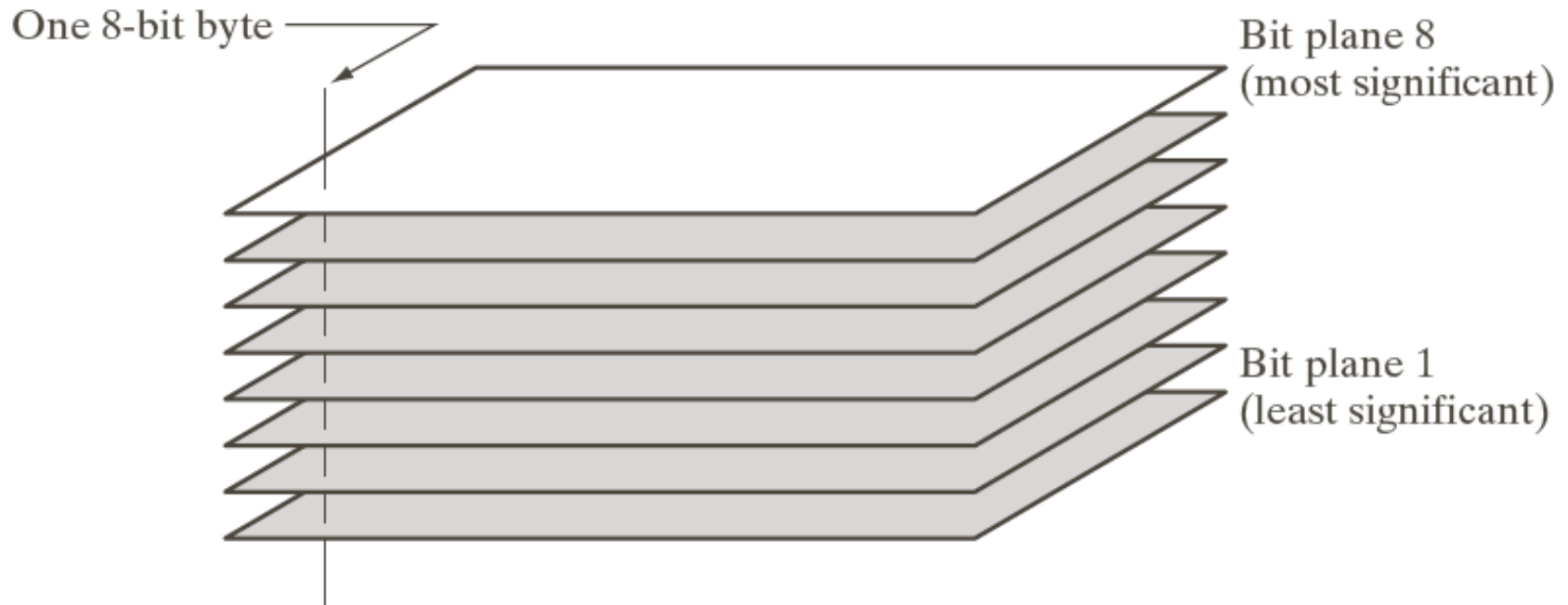
a b c

**FIGURE 3.12** (a) Aortic angiogram, (b) result of the transformation in Fig. 3.11(a), with the range of intensities of the contrast medium flow set to the full range of the gray scale. (c) Result of the transformation in Fig. 3.11(b), where the contrast medium flow is set to black, so that grays in the area of the kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)

Measuring the actual flow of the contrast medium as a function of time in a series of images

information of the type illustrated in Fig. 3.11(b). (c) Result of the transformation in Fig. 3.11(b), where the contrast medium flow is set to black, so that grays in the area of the kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)

# Bit-plane Slicing



**FIGURE 3.13**  
Bit-plane  
representation of  
an 8-bit image.

# Bit-plane Slicing



a	b	c
d	e	f
g	h	i

**FIGURE 3.14** (a) An 8-bit gray-scale image of size  $500 \times 1192$  pixels. (b) through (i) Bit planes 1 through 8, with bit plane 1 corresponding to the least significant bit. Each bit plane is a binary image.

# Bit-plane Slicing



a b c

**FIGURE 3.15** Images reconstructed using (a) bit planes 8 and 7; (b) bit planes 8, 7, and 6; and (c) bit planes 8, 7, 6, and 5. Compare (c) with Fig. 3.14(a).

# Histogram Processing

- ▶ Histogram Equalization
- ▶ Histogram Matching
- ▶ Local Histogram Processing
- ▶ Using Histogram Statistics for Image Enhancement

# Histogram Processing

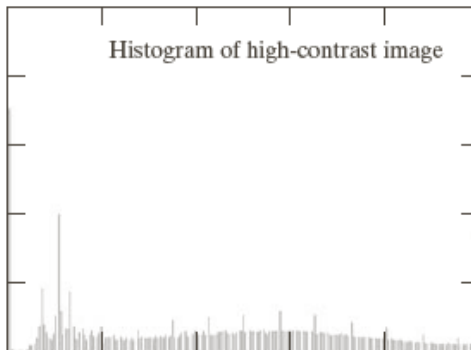
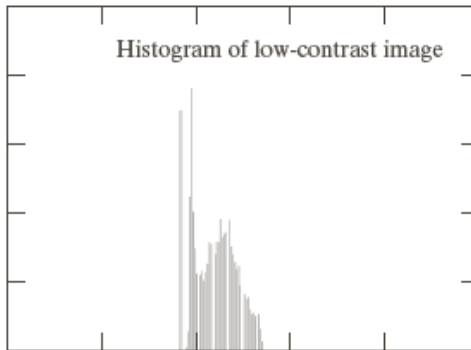
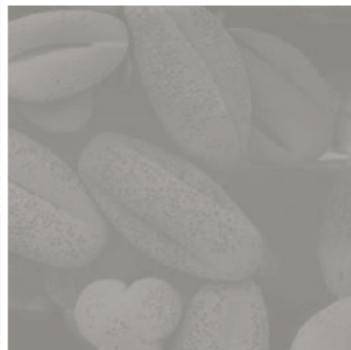
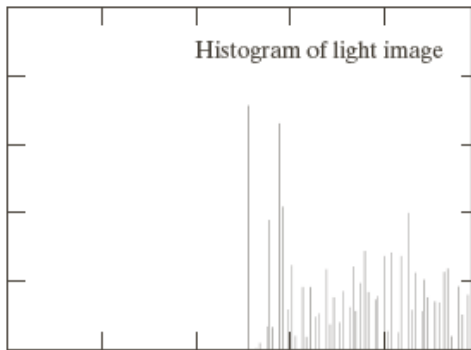
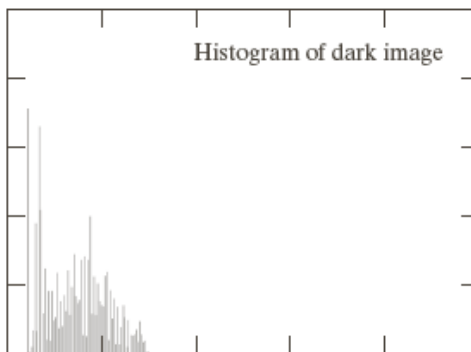
Histogram  $h(r_k) = n_k$

$r_k$  is the  $k^{th}$  intensity value

$n_k$  is the number of pixels in the image with intensity  $r_k$

Normalized histogram  $p(r_k) = \frac{n_k}{MN}$

$n_k$ : the number of pixels in the image of size  $M \times N$  with intensity  $r_k$

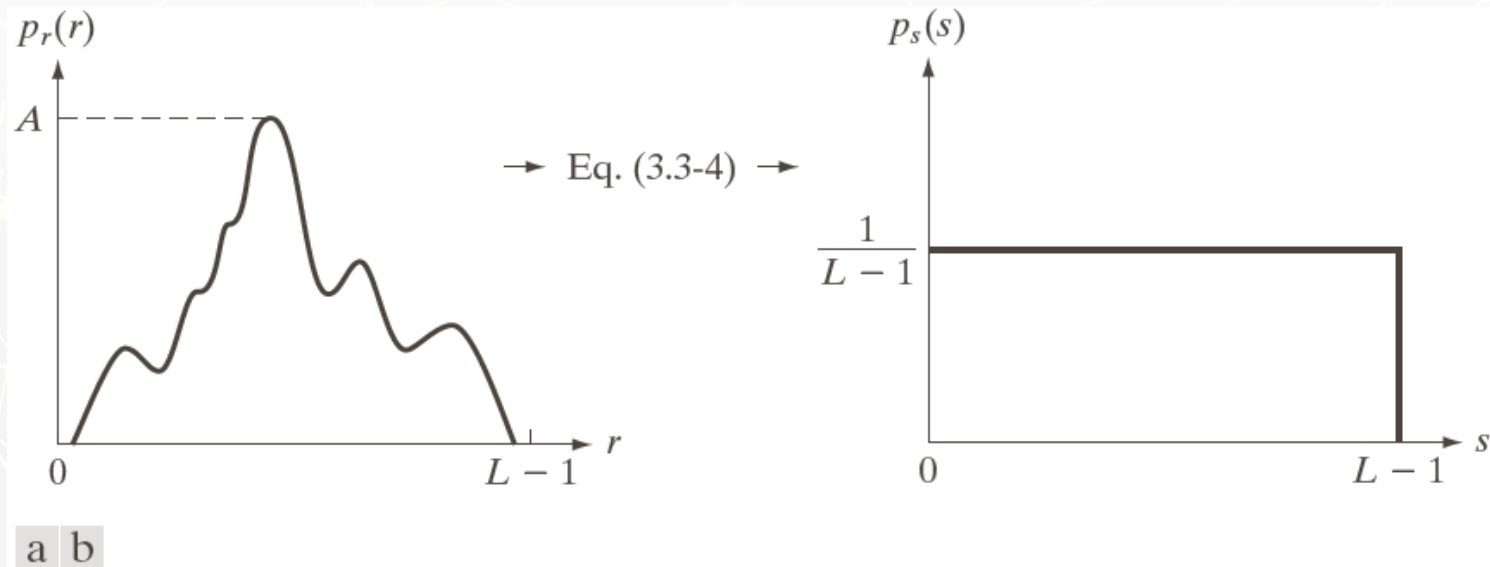




# Histogram Equalization

The intensity levels in an image may be viewed as random variables in the interval  $[0, L-1]$ .

Let  $p_r(r)$  and  $p_s(s)$  denote the probability density function (PDF) of random variables  $r$  and  $s$ .

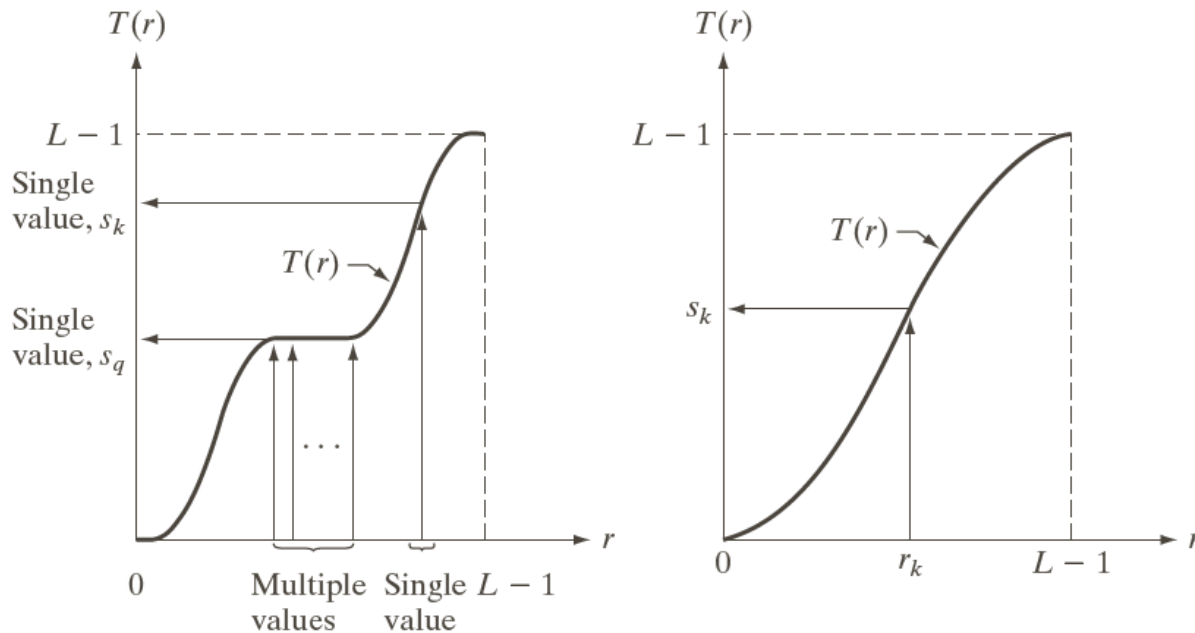


**FIGURE 3.18** (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels,  $r$ . The resulting intensities,  $s$ , have a uniform PDF, independently of the form of the PDF of the  $r$ 's.

# Histogram Equalization

$$s = T(r) \quad 0 \leq r \leq L-1$$

- a.  $T(r)$  is a strictly monotonically increasing function in the interval  $0 \leq r \leq L-1$ ;
- b.  $0 \leq T(r) \leq L-1$  for  $0 \leq r \leq L-1$ .



a b

**FIGURE 3.17**

(a) Monotonically increasing function, showing how multiple values can map to a single value. (b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.

# Histogram Equalization

$$s = T(r) \quad 0 \leq r \leq L-1$$

- a.*  $T(r)$  is a strictly monotonically increasing function in the interval  $0 \leq r \leq L-1$ ;
- b.*  $0 \leq T(r) \leq L-1$  for  $0 \leq r \leq L-1$ .

$T(r)$  is continuous and differentiable.

$$p_s(s)ds = p_r(r)dr$$

# Histogram Equalization

$$s = T(r) = (L-1) \int_0^r p_r(w) dw$$

$$\begin{aligned} \frac{ds}{dr} &= \frac{dT(r)}{dr} = (L-1) \frac{d}{dr} \left[ \int_0^r p_r(w) dw \right] \\ &= (L-1) p_r(r) \end{aligned}$$

$$p_s(s) = \frac{p_r(r) dr}{ds} = p_r(r) \bigg/ \left( \frac{ds}{dr} \right) = p_r(r) \bigg/ ((L-1) p_r(r)) = \frac{1}{L-1}$$

# Example

Suppose that the (continuous) intensity values in an image have the PDF

$$p_r(r) = \begin{cases} \frac{2r}{(L-1)^2}, & \text{for } 0 \leq r \leq L-1 \\ 0, & \text{otherwise} \end{cases}$$

Find the transformation function for equalizing the image histogram.

# Example

$$s = T(r) = (L-1) \int_0^r p_r(w) dw$$

# Histogram Equalization

Continuous case:

$$s = T(r) = (L-1) \int_0^r p_r(w) dw$$

Discrete values:

$$\begin{aligned} s_k = T(r_k) &= (L-1) \sum_{j=0}^k p_r(r_j) \\ &= (L-1) \sum_{j=0}^k \frac{n_j}{MN} = \frac{L-1}{MN} \sum_{j=0}^k n_j \quad k=0,1,\dots, L-1 \end{aligned}$$

# Example: Histogram Equalization

Suppose that a 3-bit image ( $L=8$ ) of size  $64 \times 64$  pixels ( $MN = 4096$ ) has the intensity distribution shown in following table.

Get the histogram equalization transformation function and give the  $p_s(s_k)$  for each  $s_k$ .

$r_k$	$n_k$	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02



# Example: Histogram Equalization

$r_k$	$n_k$	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

$$s_0 = T(r_0) = 7 \sum_{j=0}^0 p_r(r_j) = 7 \times 0.19 = 1.33 \rightarrow 1$$

$$s_1 = T(r_1) = 7 \sum_{j=0}^1 p_r(r_j) = 7 \times (0.19 + 0.25) = 3.08 \rightarrow 3$$

$$s_2 = 4.55 \rightarrow 5$$

$$s_3 = 5.67 \rightarrow 6$$

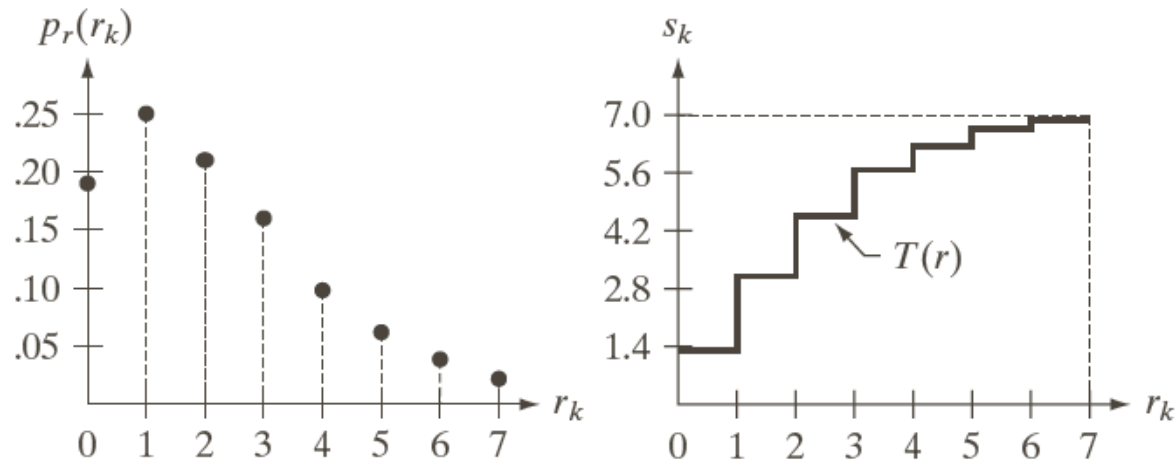
$$s_4 = 6.23 \rightarrow 6$$

$$s_5 = 6.65 \rightarrow 7$$

$$s_6 = 6.86 \rightarrow 7$$

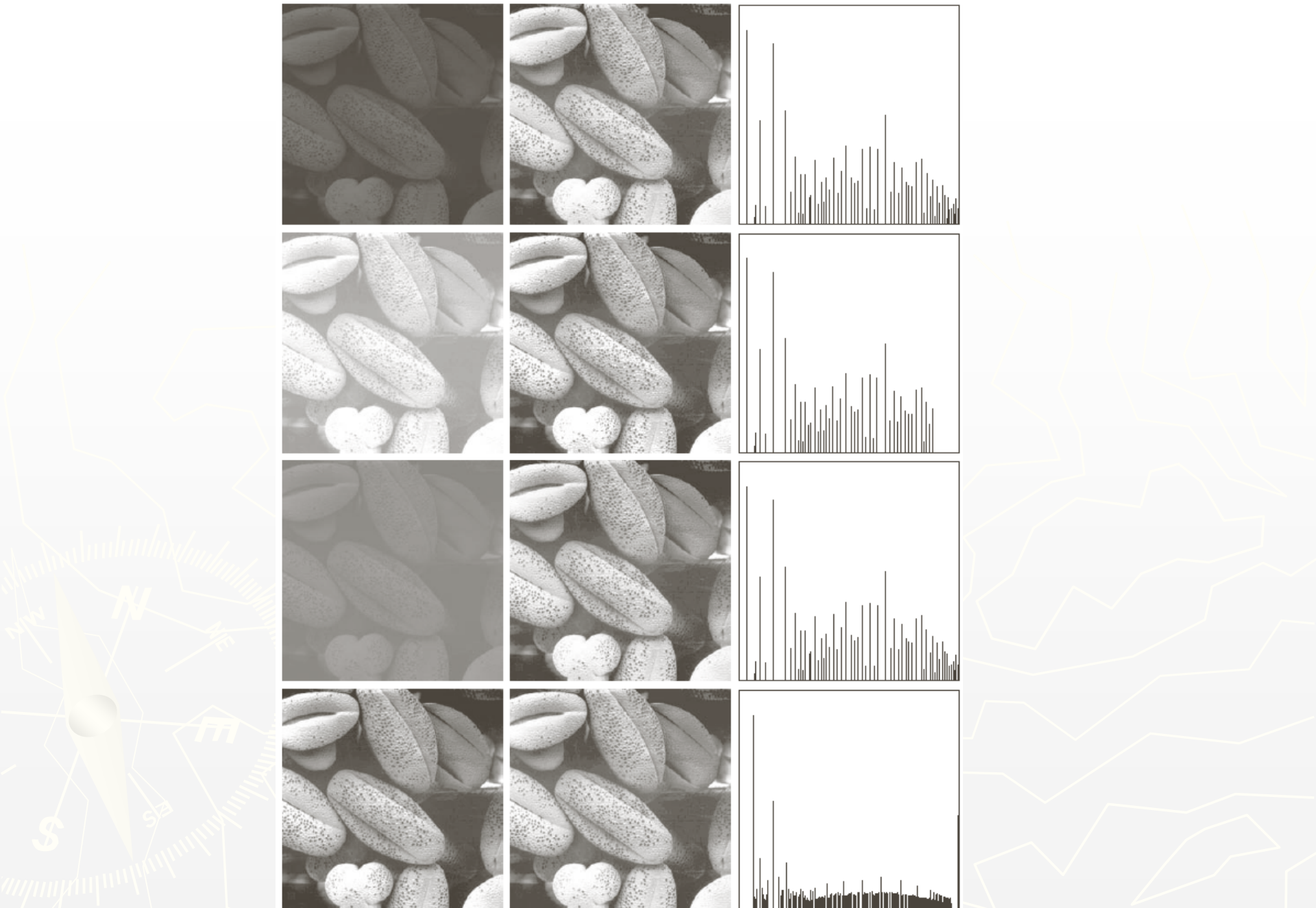
$$s_7 = 7.00 \rightarrow 7$$

# Example: Histogram Equalization

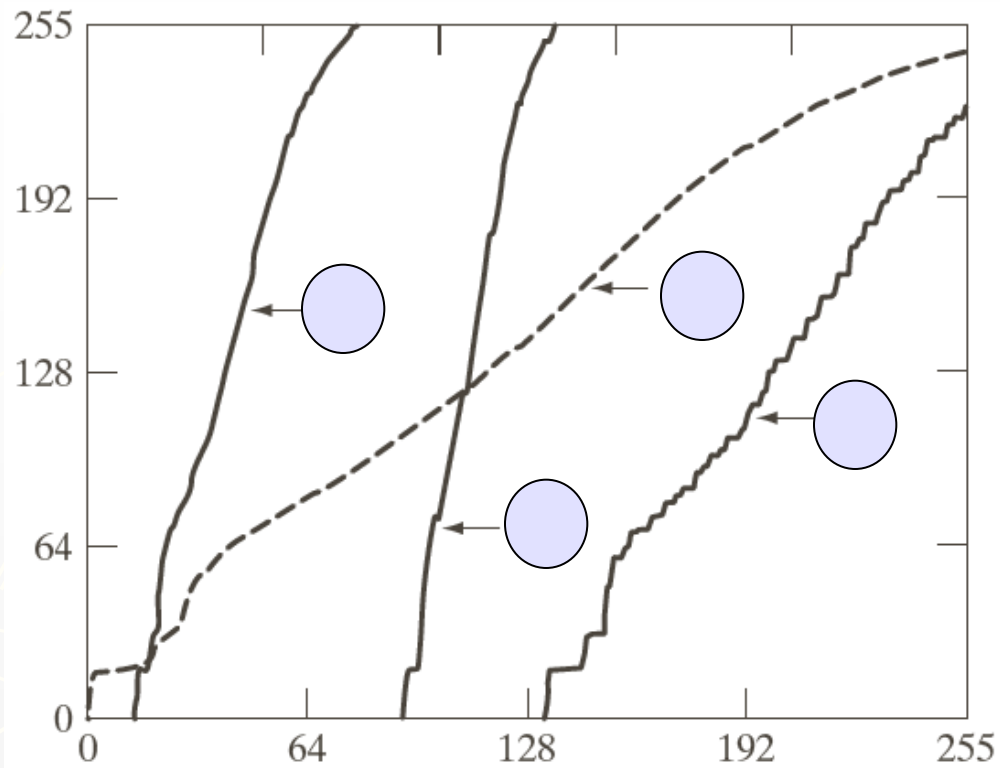


a b c

**FIGURE 3.19** Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.



**FIGURE 3.20** Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.



**FIGURE 3.21** Transformation functions for histogram equalization. Transformations (1) through (4) were obtained from the histograms of the images (from top to bottom) in the left column of Fig. 3.20 using Eq. (3.3-8).

# Question

Is histogram equalization always good?

No

# Histogram Matching

## Histogram matching (histogram specification)

— generate a processed image that has a specified histogram

Let  $p_r(r)$  and  $p_z(z)$  denote the continuous probability density functions of the variables  $r$  and  $z$ .  $p_z(z)$  is the specified probability density function.

Let  $s$  be the random variable with the probability

$$s = T(r) = (L-1) \int_0^r p_r(w) dw$$

Define a random variable  $z$  with the probability

$$G(z) = (L-1) \int_0^z p_z(t) dt = s$$

# Histogram Matching

$$s = T(r) = (L-1) \int_0^r p_r(w) dw$$

$$G(z) = (L-1) \int_0^z p_z(t) dt = s$$

$$z = G^{-1}(s) = G^{-1}[T(r)]$$

# Histogram Matching: Procedure

- ▶ Obtain  $p_r(r)$  from the input image and then obtain the values of  $s$

$$s = (L-1) \int_0^r p_r(w) dw$$

- ▶ Use the specified PDF and obtain the transformation function  $G(z)$

$$G(z) = (L-1) \int_0^z p_z(t) dt = s$$

- ▶ Mapping from  $s$  to  $z$

$$z = G^{-1}(s)$$



# Histogram Matching: Example

Assuming continuous intensity values, suppose that an image has the intensity PDF

$$p_r(r) = \begin{cases} \frac{2r}{(L-1)^2}, & \text{for } 0 \leq r \leq L-1 \\ 0, & \text{otherwise} \end{cases}$$

Find the transformation function that will produce an image whose intensity PDF is

$$p_z(z) = \begin{cases} \frac{3z^2}{(L-1)^3}, & \text{for } 0 \leq z \leq (L-1) \\ 0, & \text{otherwise} \end{cases}$$

# Histogram Matching: Example

Find the histogram equalization transformation for the input image

$$s = T(r) = (L-1) \int_0^r p_r(w) dw = (L-1) \int_0^r \frac{2w}{(L-1)^2} dw = \frac{r^2}{L-1}$$

Find the histogram equalization transformation for the specified histogram

$$G(z) = (L-1) \int_0^z p_z(t) dt = (L-1) \int_0^z \frac{3t^2}{(L-1)^3} dt = \frac{z^3}{(L-1)^2} = s$$

The transformation function

$$z = \left[ (L-1)^2 s \right]^{1/3} = \left[ (L-1)^2 \frac{r^2}{L-1} \right]^{1/3} = \left[ (L-1) r^2 \right]^{1/3}$$

# Histogram Matching: Discrete Cases

- ▶ Obtain  $p_r(r_j)$  from the input image and then obtain the values of  $s_k$ , round the value to the integer range  $[0, L-1]$ .

$$s_k = T(r_k) = (L-1) \sum_{j=0}^k p_r(r_j) = \frac{(L-1)}{MN} \sum_{j=0}^k n_j$$

- ▶ Use the specified PDF and obtain the transformation function  $G(z_q)$ , round the value to the integer range  $[0, L-1]$ .

$$G(z_q) = (L-1) \sum_{i=0}^q p_z(z_i) = s_k$$

- ▶ Mapping from  $s_k$  to  $z_q$

$$z_q = G^{-1}(s_k)$$

# Example: Histogram Matching

Suppose that a 3-bit image ( $L=8$ ) of size  $64 \times 64$  pixels ( $MN = 4096$ ) has the intensity distribution shown in the following table (on the left). Get the histogram transformation function and make the output image with the specified histogram, listed in the table on the right.

$r_k$	$n_k$	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

$z_q$	<b>Specified</b> $p_z(z_q)$
$z_0 = 0$	0.00
$z_1 = 1$	0.00
$z_2 = 2$	0.00
$z_3 = 3$	0.15
$z_4 = 4$	0.20
$z_5 = 5$	0.30
$z_6 = 6$	0.20
$z_7 = 7$	0.15

# Example: Histogram Matching

Obtain the scaled histogram-equalized values,

$$s_0 = 1, s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 7, \\ s_5 = 7, s_6 = 7, s_7 = 7.$$

Compute all the values of the transformation function  $G$ ,

$$G(z_0) = 7 \sum_{j=0}^0 p_z(z_j) = 0.00 \rightarrow 0$$

$$G(z_1) = 0.00 \rightarrow 0$$

$$G(z_2) = 0.00 \rightarrow 0$$

$$G(z_3) = 1.05 \rightarrow 1$$

$$G(z_4) = 2.45 \rightarrow 2$$

$$G(z_5) = 4.55 \rightarrow 5$$

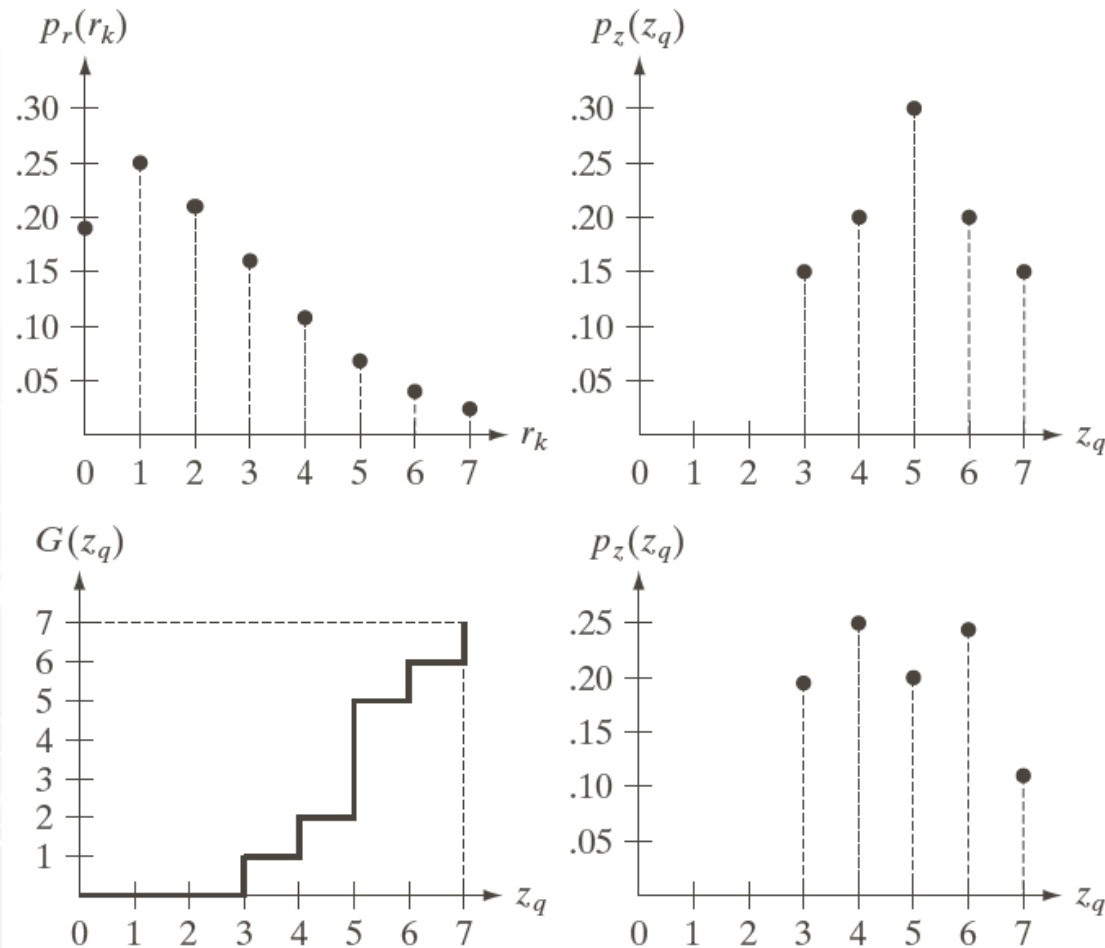
$$G(z_6) = 5.95 \rightarrow 6$$

$$G(z_7) = 7.00 \rightarrow 7$$

$z_q$	Specified $p_z(z_q)$
$z_0 = 0$	0.00
$z_1 = 1$	0.00
$z_2 = 2$	0.00
$z_3 = 3$	0.15
$z_4 = 4$	0.20
$z_5 = 5$	0.30
$z_6 = 6$	0.20
$z_7 = 7$	0.15

$r_k$	$n_k$	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

# Example: Histogram Matching



a b  
c d

**FIGURE 3.22**

(a) Histogram of a 3-bit image. (b) Specified histogram. (c) Transformation function obtained from the specified histogram. (d) Result of performing histogram specification. Compare (b) and (d).

# Example: Histogram Matching

Obtain the scaled histogram-equalized values,

$$s_0 = 1, s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 7, \\ s_5 = 7, s_6 = 7, s_7 = 7.$$

Compute all the values of the transformation function  $G$ ,

$$G(z_0) = 7 \sum_{j=0}^0 p_z(z_j) = 0.00 \rightarrow 0$$

$$G(z_1) = 0.00 \rightarrow 0$$

$$G(z_2) = 0.00 \rightarrow 0$$

$$G(z_3) = 1.05 \rightarrow 1 \quad \mathbf{s_0}$$

$$G(z_4) = 2.45 \rightarrow 2 \quad \mathbf{s_1}$$

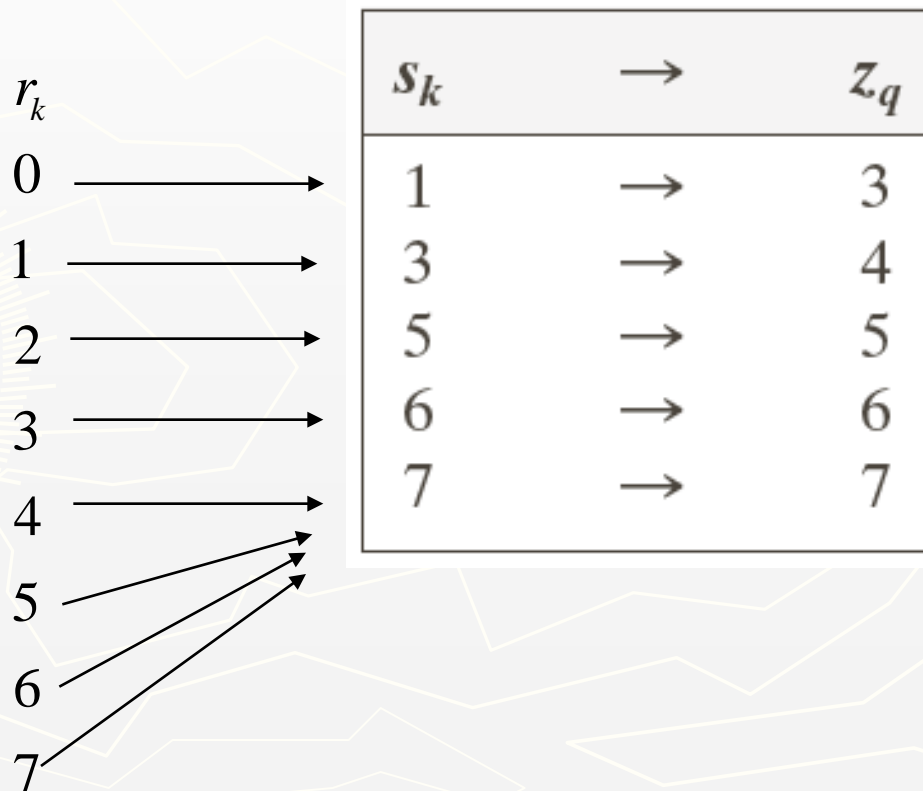
$$G(z_5) = 4.55 \rightarrow 5 \quad \mathbf{s_2}$$

$$G(z_6) = 5.95 \rightarrow 6 \quad \mathbf{s_3}$$

$$G(z_7) = 7.00 \rightarrow 7 \quad \mathbf{s_4 \quad s_5 \quad s_6 \quad s_7}$$

# Example: Histogram Matching

$$s_0 = 1, s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 7,$$
$$s_5 = 7, s_6 = 7, s_7 = 7.$$



$r_k$	$s_k$	$\rightarrow$	$z_q$
0	1	$\rightarrow$	3
1	3	$\rightarrow$	4
2	5	$\rightarrow$	5
3	6	$\rightarrow$	6
4	7	$\rightarrow$	7
5	7	$\rightarrow$	7
6	7	$\rightarrow$	7
7	7	$\rightarrow$	7



# Example: Histogram Matching

$$r_k \rightarrow z_q$$

$$0 \rightarrow 3$$

$$1 \rightarrow 4$$

$$2 \rightarrow 5$$

$$3 \rightarrow 6$$

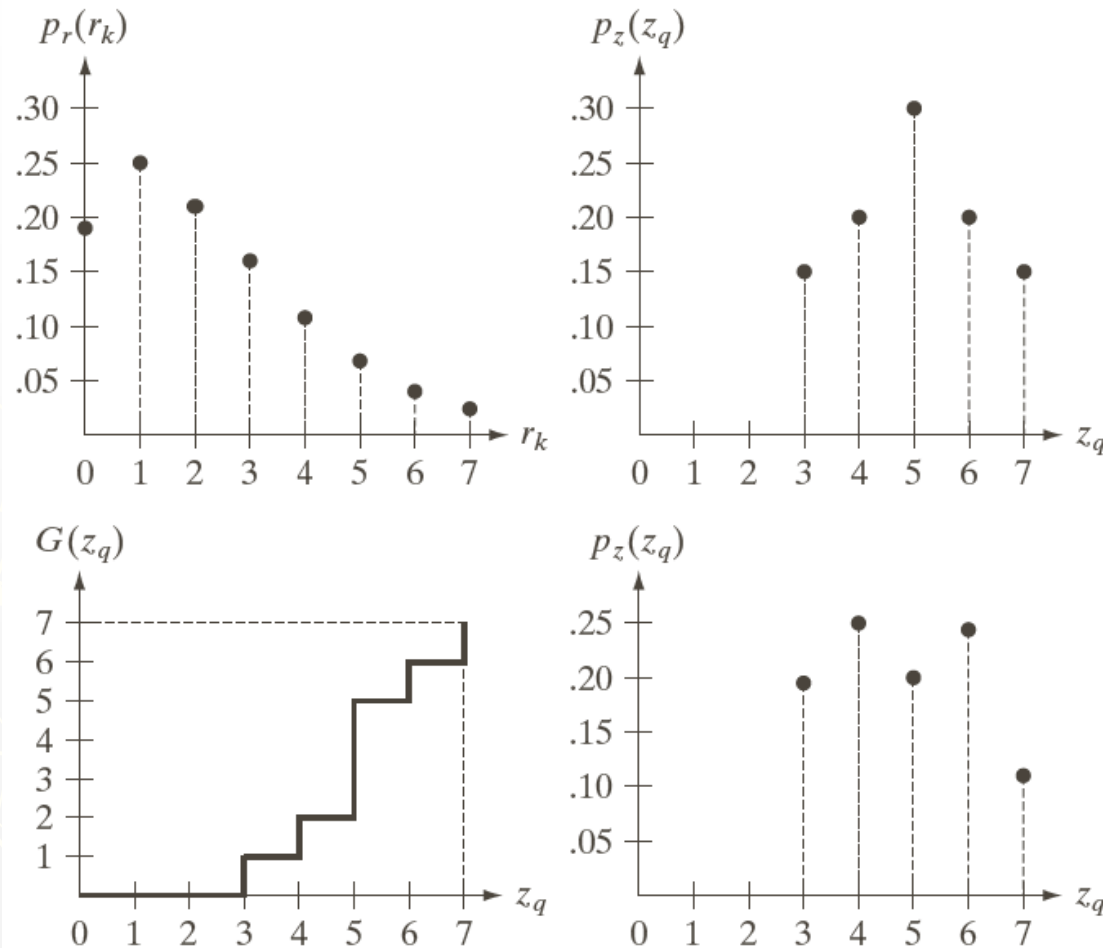
$$4 \rightarrow 7$$

$$5 \rightarrow 7$$

$$6 \rightarrow 7$$

$$7 \rightarrow 7$$

# Example: Histogram Matching

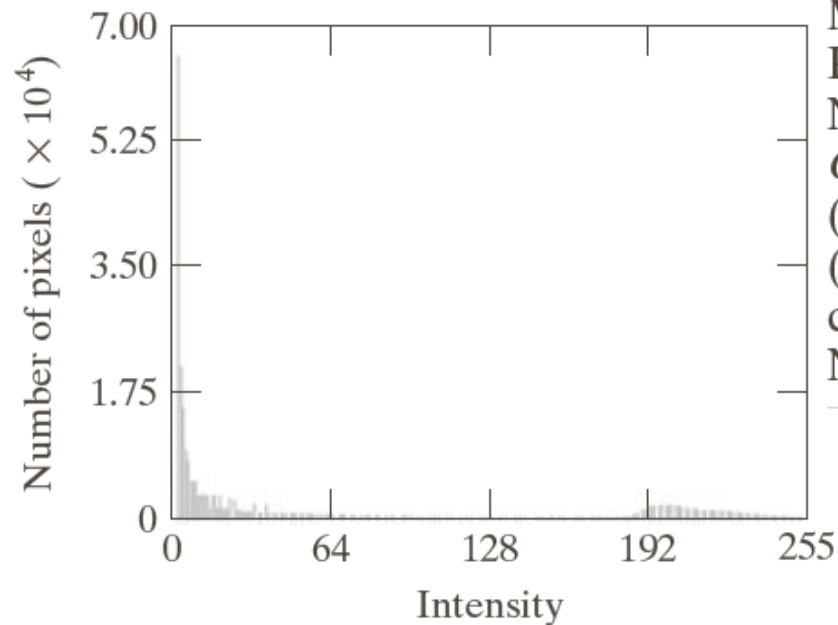


a b  
c d

**FIGURE 3.22**

(a) Histogram of a 3-bit image. (b) Specified histogram. (c) Transformation function obtained from the specified histogram. (d) Result of performing histogram specification. Compare (b) and (d).

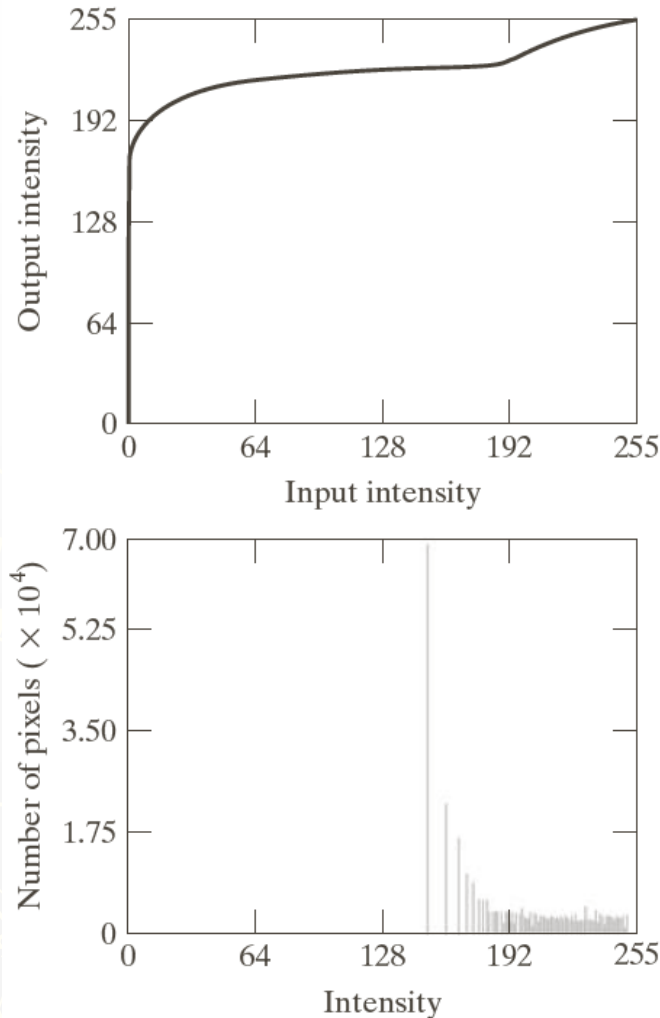
# Example: Histogram Matching



a b

**FIGURE 3.23**  
(a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor*. (b) Histogram. (Original image courtesy of NASA.)

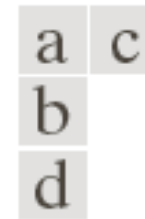
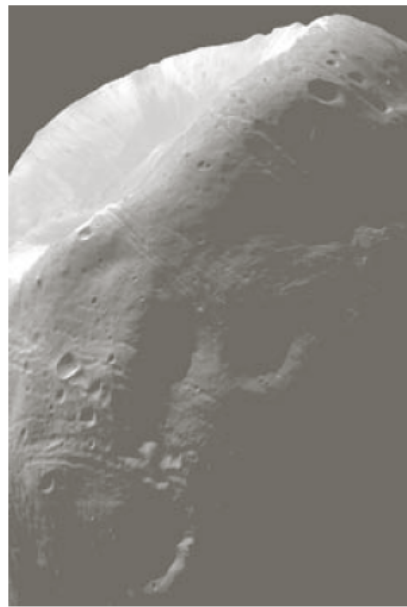
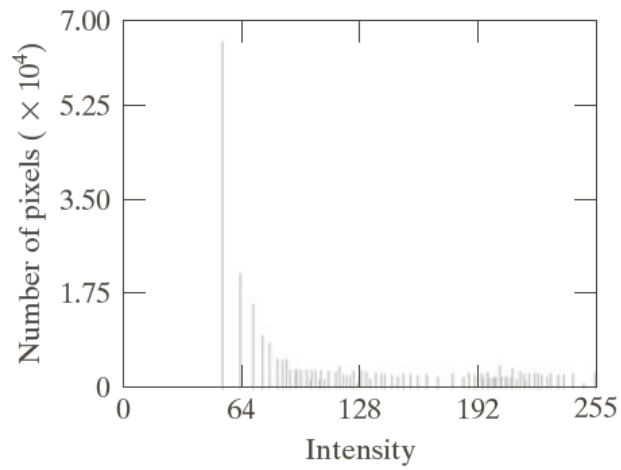
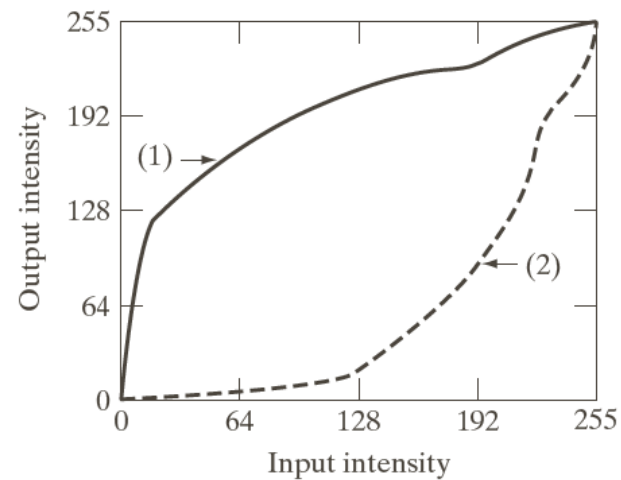
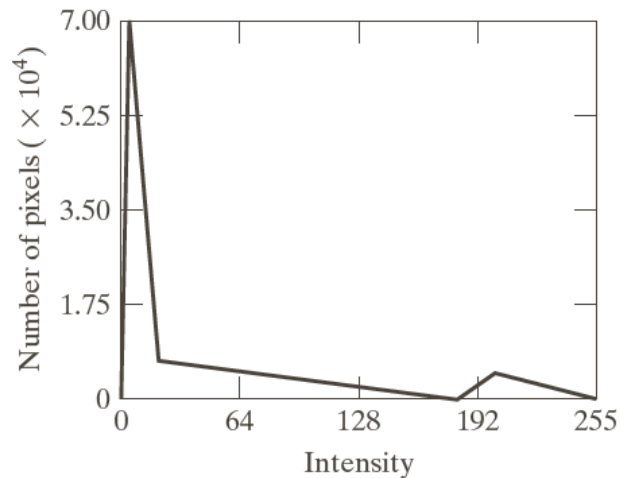
# Example: Histogram Matching



a b  
c

**FIGURE 3.24**

(a) Transformation function for histogram equalization.  
(b) Histogram-equalized image (note the washed-out appearance).  
(c) Histogram of (b).



**FIGURE 3.25**

(a) Specified histogram.

(b) Transformations.

(c) Enhanced image using mappings from curve (2).

(d) Histogram of (c).

# Local Histogram Processing

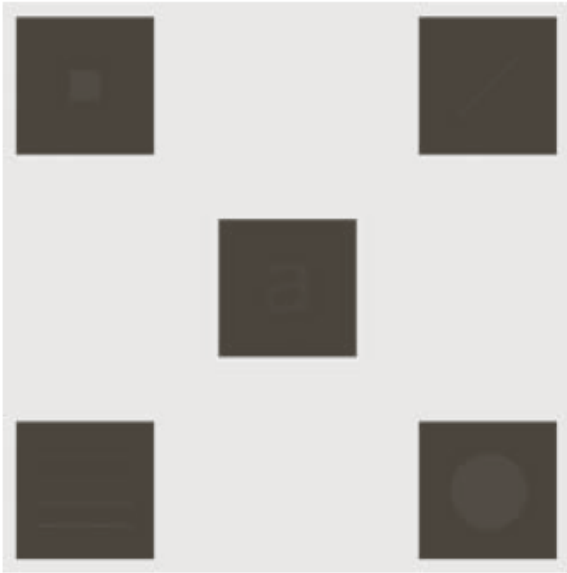
Define a neighborhood and move its center from pixel to pixel

At each location, the histogram of the points in the neighborhood is computed. Either histogram equalization or histogram specification transformation function is obtained

Map the intensity of the pixel centered in the neighborhood

Move to the next location and repeat the procedure

# Local Histogram Processing: Example



a b c

**FIGURE 3.26** (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size  $3 \times 3$ .

# Using Histogram Statistics for Image Enhancement

Average Intensity

$$m = \sum_{i=0}^{L-1} r_i p(r_i) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)$$

$$u_n(r) = \sum_{i=0}^{L-1} (r_i - m)^n p(r_i)$$

Variance

$$\sigma^2 = u_2(r) = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [f(x, y) - m]^2$$



# Using Histogram Statistics for Image Enhancement

Local average intensity

$$m_{s_{xy}} = \sum_{i=0}^{L-1} r_i p_{s_{xy}}(r_i)$$

$s_{xy}$  denotes a neighborhood

Local variance

$$\sigma_{s_{xy}}^2 = \sum_{i=0}^{L-1} (r_i - m_{s_{xy}})^2 p_{s_{xy}}(r_i)$$

# Using Histogram Statistics for Image Enhancement: Example

$$g(x, y) = \begin{cases} E[f(x, y)], & \text{if } m_{s_{xy}} \leq k_0 m_G \text{ and } k_1 \sigma_G \leq \sigma_{s_{xy}} \leq k_2 \sigma_G \\ f(x, y), & \text{otherwise} \end{cases}$$

$m_G$  : global mean;     $\sigma_G$  : global standard deviation

$k_0 = 0.4$ ;     $k_1 = 0.02$ ;  $k_2 = 0.4$ ;  $E = 4$



a b c

**FIGURE 3.27** (a) SEM image of a tungsten filament magnified approximately 130 $\times$ . (b) Result of global histogram equalization. (c) Image enhanced using local histogram statistics. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

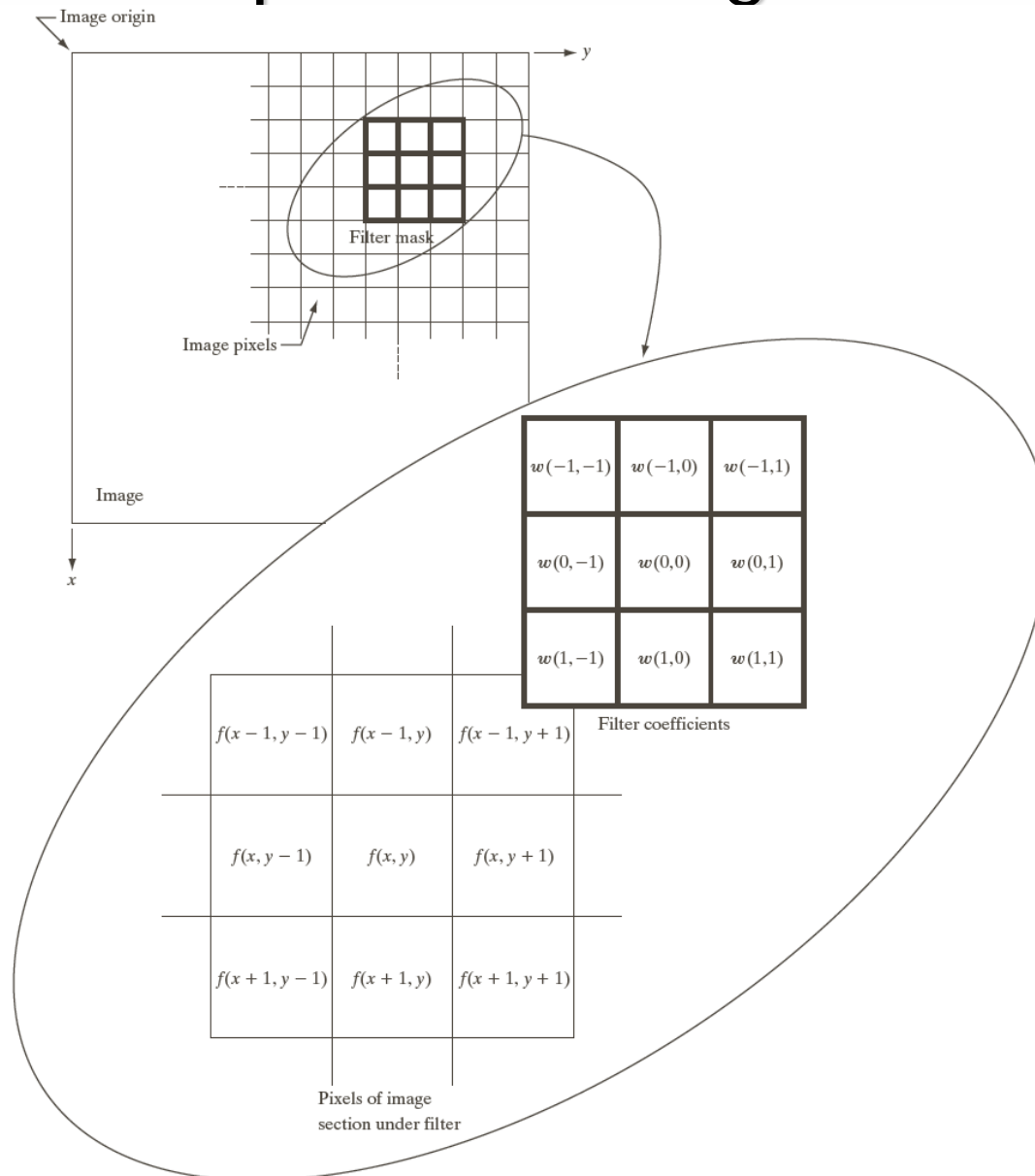
# Spatial Filtering

A spatial filter consists of (a) **a neighborhood**, and (b) **a predefined operation**

Linear spatial filtering of an image of size  $M \times N$  with a filter of size  $m \times n$  is given by the expression

$$g(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)$$

# Spatial Filtering



# Spatial Correlation

The correlation of a filter  $w(x, y)$  of size  $m \times n$  with an image  $f(x, y)$ , denoted as  $w(x, y) \star f(x, y)$

$$w(x, y) \star f(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)$$

# Spatial Convolution

The convolution of a filter  $w(x, y)$  of size  $m \times n$  with an image  $f(x, y)$ , denoted as  $w(x, y) \star f(x, y)$

$$w(x, y) \star f(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x-s, y-t)$$

↙ Origin	$f(x, y)$					
	0	0	0	0	0	
	0	0	0	0	0	$w(x, y)$
	0	0	1	0	0	1 2 3
	0	0	0	0	0	4 5 6
	0	0	0	0	0	7 8 9

(a)

**FIGURE 3.30**  
Correlation (middle row) and convolution (last row) of a 2-D filter with a 2-D discrete, unit impulse. The 0s are shown in gray to simplify visual analysis.

# Smoothing Spatial Filters

Smoothing filters are used for blurring and for noise reduction

Blurring is used in removal of small details and bridging of small gaps in lines or curves

Smoothing spatial filters include linear filters and nonlinear filters.



# Spatial Smoothing Linear Filters

The general implementation for filtering an  $M \times N$  image with a weighted averaging filter of size  $m \times n$  is given

$$g(x, y) = \frac{\sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)}{\sum_{s=-a}^a \sum_{t=-b}^b w(s, t)}$$

where  $m = 2a + 1$ ,  $n = 2b + 1$ .

# Two Smoothing Averaging Filter Masks

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \quad \frac{1}{16} \times \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

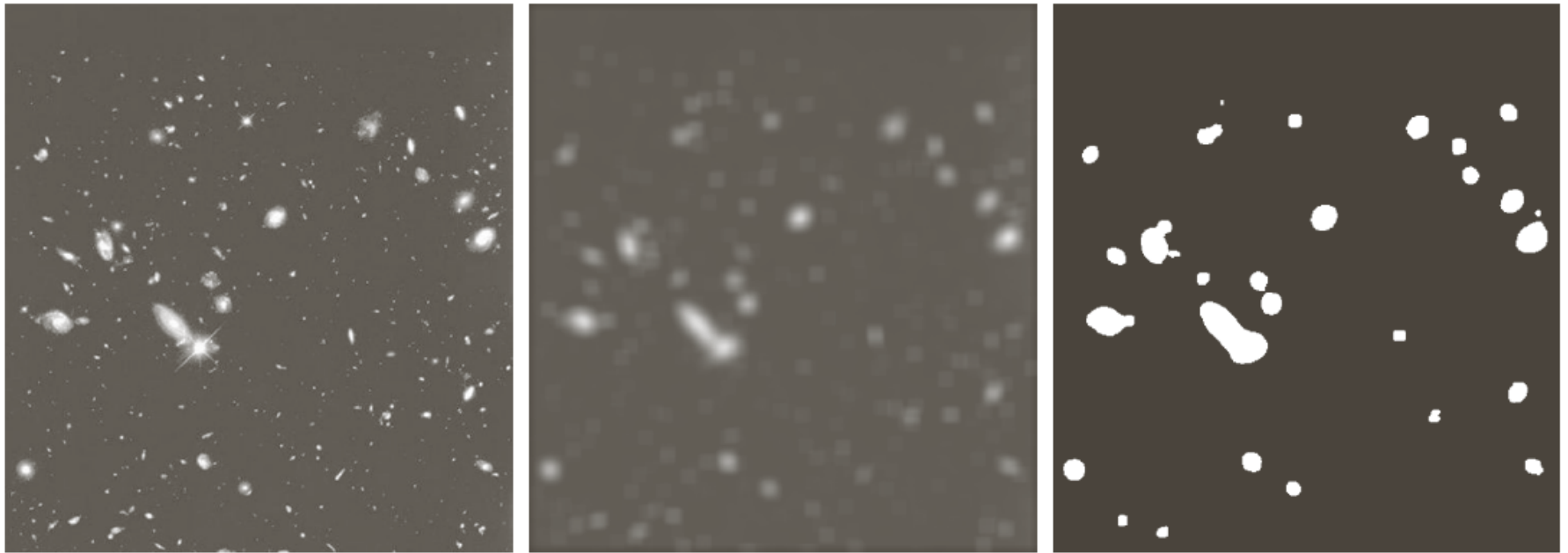
a b

**FIGURE 3.32** Two  $3 \times 3$  smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to 1 divided by the sum of the values of its coefficients, as is required to compute an average.

**FIGURE 3.33** (a) Original image, of size  $500 \times 500$  pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes  $m = 3, 5, 9, 15$ , and  $35$ , respectively. The black squares at the top are of sizes  $3, 5, 9, 15, 25, 35, 45$ , and  $55$  pixels, respectively; their borders are  $25$  pixels apart. The letters at the bottom range in size from  $10$  to  $24$  points, in increments of  $2$  points; the large letter at the top is  $60$  points. The vertical bars are  $5$  pixels wide and  $100$  pixels high; their separation is  $20$  pixels. The diameter of the circles is  $25$  pixels, and their borders are  $15$  pixels apart; their intensity levels range from  $0\%$  to  $100\%$  black in increments of  $20\%$ . The background of the image is  $10\%$  black. The noisy rectangles are of size  $50 \times 120$  pixels.



# Example: Gross Representation of Objects



a b c

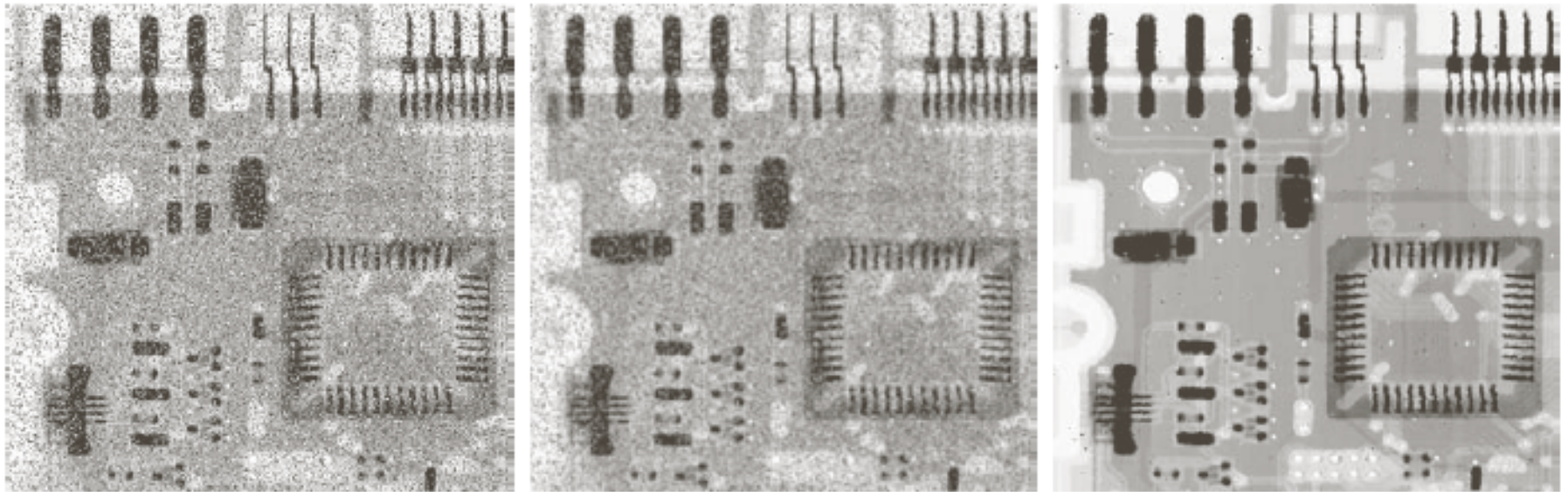
**FIGURE 3.34** (a) Image of size  $528 \times 485$  pixels from the Hubble Space Telescope. (b) Image filtered with a  $15 \times 15$  averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

# Order-statistic (Nonlinear) Filters

- Nonlinear
- Based on ordering (ranking) the pixels contained in the filter mask
- Replacing the value of the center pixel with the value determined by the ranking result

E.g., median filter, max filter, min filter

# Example: Use of Median Filtering for Noise Reduction



a b c

**FIGURE 3.35** (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a  $3 \times 3$  averaging mask. (c) Noise reduction with a  $3 \times 3$  median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

# Sharpening Spatial Filters

- ▶ Foundation
- ▶ Laplacian Operator
- ▶ Unsharp Masking and Highboost Filtering
- ▶ Using First-Order Derivatives for Nonlinear Image Sharpening — The Gradient



# Sharpening Spatial Filters: Foundation

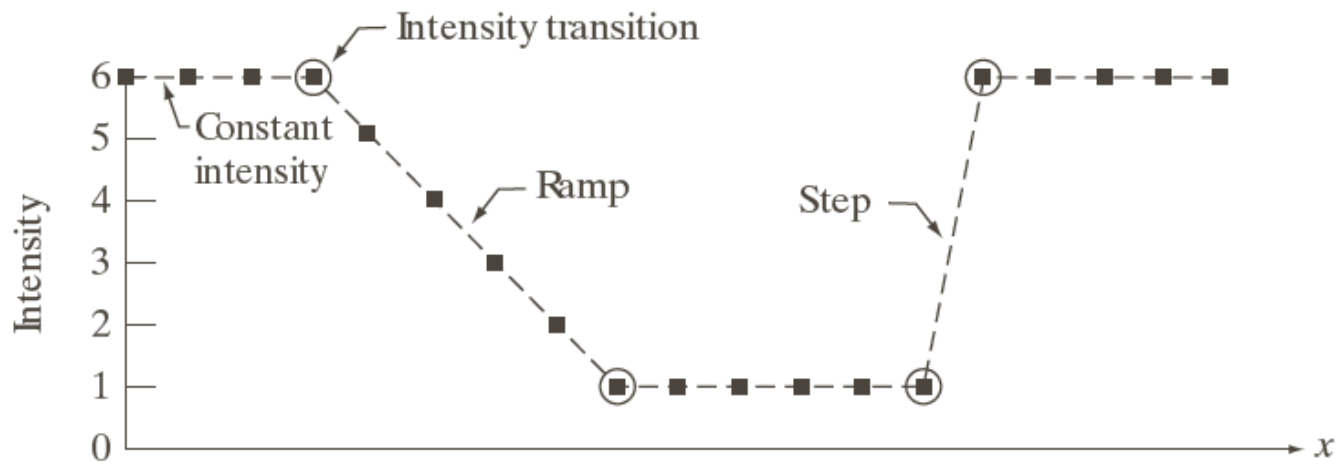
- ▶ The first-order derivative of a one-dimensional function  $f(x)$  is the difference

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

- ▶ The second-order derivative of  $f(x)$  as the difference

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$





a  
b  
c

**FIGURE 3.36** Illustration of the first and second derivatives of a 1-D digital function representing a section of a horizontal intensity profile from an image. In (a) and (c) data points are joined by dashed lines as a visualization aid.

# Sharpening Spatial Filters: Laplace Operator

The second-order isotropic derivative operator is the Laplacian for a function (image)  $f(x,y)$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

$$\nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

# Sharpening Spatial Filters: Laplace Operator

0	1	0	1	1	1
1	-4	1	1	-8	1
0	1	0	1	1	1

0	-1	0	-1	-1	-1
-1	4	-1	-1	8	-1
0	-1	0	-1	-1	-1

a b  
c d

**FIGURE 3.37**

(a) Filter mask used to implement Eq. (3.6-6).

(b) Mask used to implement an extension of this equation that includes the diagonal terms.

(c) and (d) Two other implementations of the Laplacian found frequently in practice.

# Sharpening Spatial Filters: Laplace Operator

Image sharpening in the way of using the Laplacian:

$$g(x, y) = f(x, y) + c \left[ \nabla^2 f(x, y) \right]$$

where,

$f(x, y)$  is input image,

$g(x, y)$  is sharpened images,

$c = -1$  if  $\nabla^2 f(x, y)$  corresponding to Fig. 3.37(a) or (b)

and  $c = 1$  if either of the other two filters is used.



a	
b	c
d	e

### FIGURE 3.38

(a) Blurred image of the North Pole of the moon.

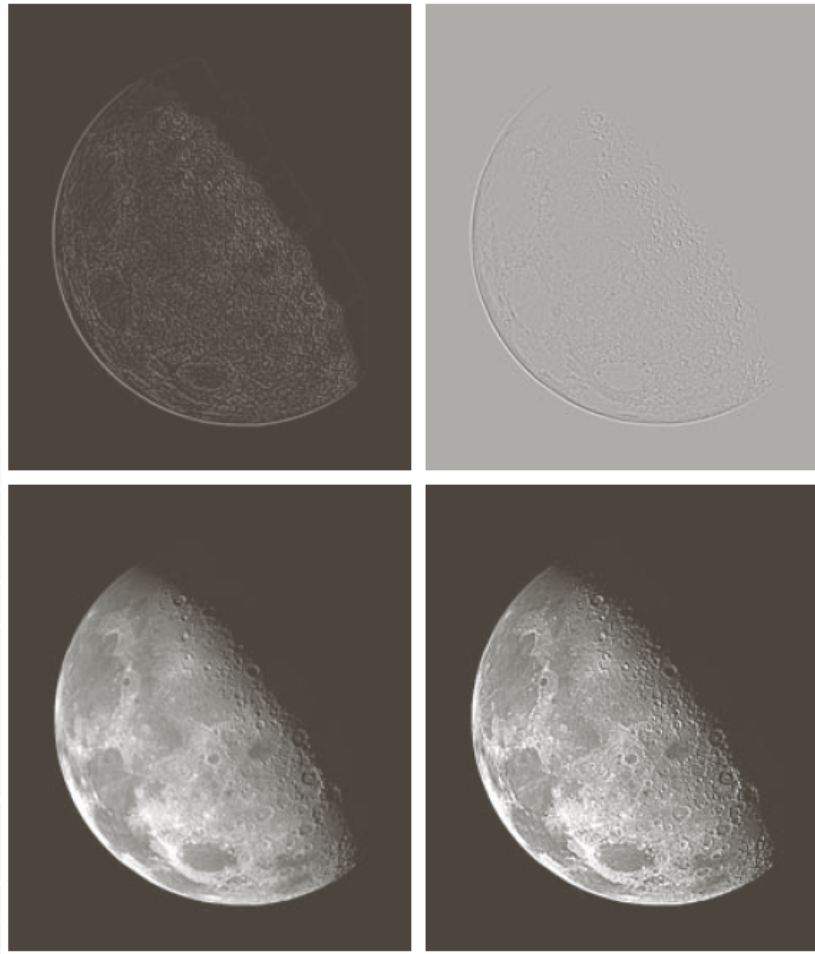
(b) Laplacian without scaling.

(c) Laplacian with scaling.

(d) Image sharpened using the mask in Fig. 3.37(a).

(e) Result of using the mask in Fig. 3.37(b).

(Original image courtesy of NASA.)



# Unsharp Masking and Highboost Filtering

## ► Unsharp masking

Sharpen images consists of subtracting an unsharp (smoothed) version of an image from the original image

e.g., printing and publishing industry

## ► Steps

1. Blur the original image
2. Subtract the blurred image from the original
3. Add the mask to the original

# Unsharp Masking and Highboost Filtering

Let  $\bar{f}(x, y)$  denote the blurred image, unsharp masking is

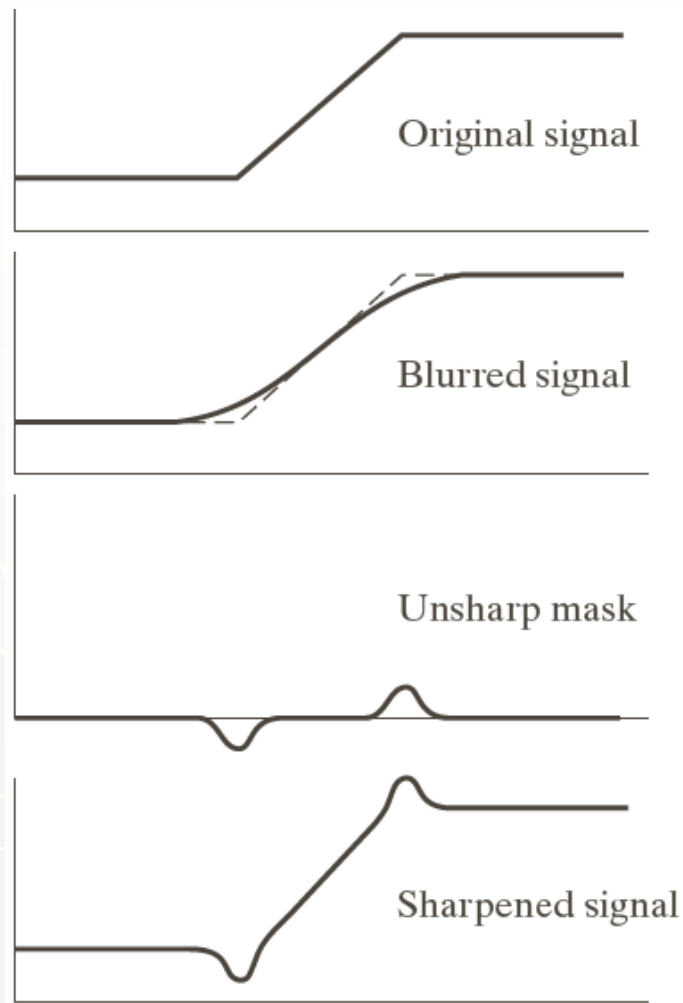
$$g_{mask}(x, y) = f(x, y) - \bar{f}(x, y)$$

Then add a weighted portion of the mask back to the original

$$g(x, y) = f(x, y) + k * g_{mask}(x, y) \quad k \geq 0$$

when  $k > 1$ , the process is referred to as highboost filtering.

# Unsharp Masking: Demo



a  
b  
c  
d

**FIGURE 3.39** 1-D illustration of the mechanics of unsharp masking. (a) Original signal. (b) Blurred signal with original shown dashed for reference. (c) Unsharp mask. (d) Sharpened signal, obtained by adding (c) to (a).



# Unsharp Masking and Highboost Filtering: Example



a  
b  
c  
d  
e

**FIGURE 3.40**

(a) Original image.  
(b) Result of blurring with a Gaussian filter.  
(c) Unsharp mask. (d) Result of using unsharp masking.  
(e) Result of using highboost filtering.

# Image Sharpening based on First-Order Derivatives

For function  $f(x, y)$ , the gradient of  $f$  at coordinates  $(x, y)$  is defined as

$$\nabla f \equiv \text{grad}(f) \equiv \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

The *magnitude* of vector  $\nabla f$ , denoted as  $M(x, y)$

Gradient Image  $M(x, y) = \text{mag}(\nabla f) = \sqrt{g_x^2 + g_y^2}$

# Image Sharpening based on First-Order Derivatives

The *magnitude* of vector  $\nabla f$ , denoted as  $M(x, y)$

$$M(x, y) = \text{mag}(\nabla f) = \sqrt{g_x^2 + g_y^2}$$

$$M(x, y) \approx |g_x| + |g_y|$$

$z_1$	$z_2$	$z_3$
$z_4$	$z_5$	$z_6$
$z_7$	$z_8$	$z_9$

$$M(x, y) = |z_8 - z_5| + |z_6 - z_5|$$

# Image Sharpening based on First-Order Derivatives

## Roberts Cross-gradient Operators

$$M(x, y) \approx |z_9 - z_5| + |z_8 - z_6|$$

## Sobel Operators

$$M(x, y) \approx |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)| \\ + |(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)|$$

$z_1$	$z_2$	$z_3$
$z_4$	$z_5$	$z_6$
$z_7$	$z_8$	$z_9$

# Image Sharpening based on First-Order Derivatives

$z_1$	$z_2$	$z_3$
$z_4$	$z_5$	$z_6$
$z_7$	$z_8$	$z_9$

-1	0	0	-1
0	1	1	0

-1	-2	-1	-1	0	1
0	0	0	-2	0	2
1	2	1	-1	0	1

a
b c
d e

**FIGURE 3.41**

A  $3 \times 3$  region of an image (the  $z$ s are intensity values).

(b)–(c) Roberts cross gradient operators.

(d)–(e) Sobel operators. All the mask coefficients sum to zero, as expected of a derivative operator.

# Example

a b

**FIGURE 3.42**

(a) Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock).

(b) Sobel gradient. (Original image courtesy of Pete Sites, Perceptics Corporation.)

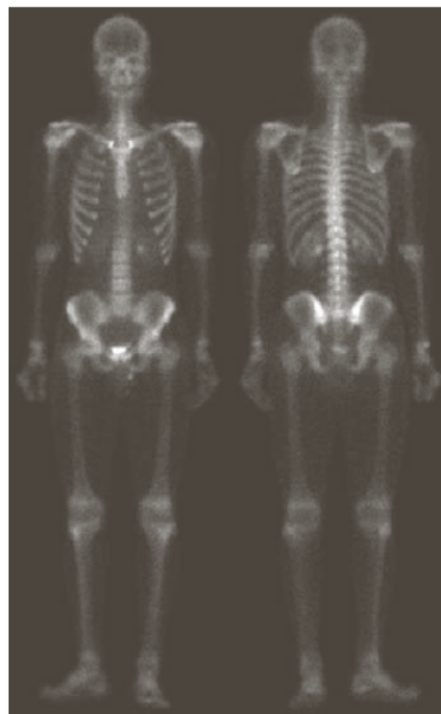


Example:

## Combining Spatial Enhancement Methods

Goal:

Enhance the  
image by  
sharpening it  
and by bringing  
out more of the  
skeletal detail



a	b
c	d

**FIGURE 3.43**

(a) Image of whole body bone scan.

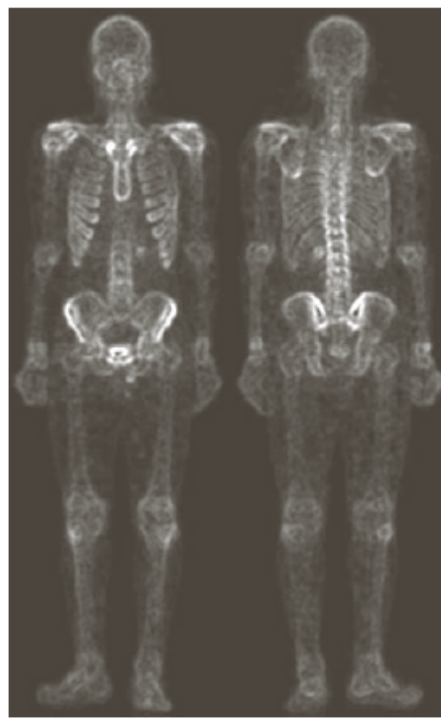
(b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel gradient of (a).

Example:

## Combining Spatial Enhancement Methods

Goal:

Enhance the  
image by  
sharpening it  
and by bringing  
out more of the  
skeletal detail



e f  
g h

**FIGURE 3.43**

*(Continued)*

(e) Sobel image smoothed with a  $5 \times 5$  averaging filter. (f) Mask image formed by the product of (c) and (e).

(g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)