

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	0
13	0
13	674
14	0/4
15	65

- a b
- c d

FIGURE 9.18

(a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1s. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

Convex Hull

A set A is said to be *convex* if the straight line segment joining any two points in A lies entirely within A.

The *convex hull* H or of an arbitrary set S is the smallest convex set containing S.

Convex Hull

Let B^i , i = 1, 2, 3, 4, represent the four structuring elements. The procedure consists of implementing the equation:

 $X_k^i = (X_{k-1} \circledast B^i) \cup A$ $i = 1, 2, 3, 4 \text{ and } k = 1, 2, 3, \dots$ with $X_0^i = A$. When the procedure converges, or $X_k^i = X_{k-1}^i$, let $D^i = X_k^i$,

the convex hull of A is

$$C(A) = \bigcup_{i=1}^{4} D^i$$

$\times \times$		\times \times	$\times \times \times$
×	× ×	×	× ×
× ×	$\times \times \times$	\times \times	
B^1	B^2	B^3	B^4





 X_{8}^{3}

 X_2^4

	а	
b	с	d
e	f	g
	h	

FIGURE 9.19 (a) Structuring elements. (b) Set A. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.



 X_{2}^{2}

40



FIGURE 9.20 Result of limiting growth of the convex hull algorithm to the maximum dimensions of the original set of points along the vertical and horizontal directions.

► Thinning

The thinning of a set A by a structuring element B, defined

$$A \otimes B = A - (A^{\circledast}B)$$
$$= A \cap (A^{\circledast}B)^{c}$$

A more useful expression for thinning A symmetrically is based on a sequence of structuring elements:

$$\left\{B\right\} = \left\{B^1, B^2, B^3, ..., B^n\right\}$$

where B^i is a rotated version of B^{i-1}

The thinning of *A* by a sequence of structuring element $\{B\}$ $A \otimes \{B\} = ((...((A \otimes B^1) \otimes B^2)...) \otimes B^n)$



(c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first four elements again. (l) Result after convergence. (m) Conversion to *m*-connectivity.

2/27/2014

e f g

h i j

k 1 m

44

Thickening:

The thickening is defined by the expression $A \square B = A \cup (A^{\circledast}B)$

The thickening of *A* by a sequence of structuring element $\{B\}$ $A \square \{B\} = ((...((A \square B^1) \square B^2)...) \square B^n)$

In practice, the usual procedure is to thin the background of the set and then complement the result.



FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

Skeletons

A skeleton, S(A) of a set A has the following properties
a. if z is a point of S(A) and (D)_z is the largest disk centered at z and contained in A, one cannot find a larger disk containing (D)_z and included in A. The disk (D)_z is called a maximum disk.
b. The disk (D)_z touches the boundary of A at two or more different places.



a b c d

FIGURE 9.23 (a) Set A. (b) Various positions of maximum disks with centers on the skeleton of A. (c) Another maximum disk on a different segment of the skeleton of A. (d) Complete skeleton.

The skeleton of A can be expressed in terms of erosion and openings.

 $S(A) = \bigcup_{k=0}^{K} S_{k}(A)$ with $K = \max\{k \mid A \ominus kB \neq \phi\};$ $S_{k}(A) = (A \ominus kB) - (A \ominus kB) \circ B$ where *B* is a structuring element, and $(A \ominus kB) = ((...((A \ominus B) \ominus B) \ominus ...) \ominus B))$ *k* successive erosions of A.

k	$A \ominus kB$	$(A \ominus kB) \circ B$	$S_k(A)$	$\bigcup_{\substack{k=0}^{K}}^{K} S_{k}(A)$
0				

FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.



2/27/2014

B

k	$A \ominus kB$	$(A \ominus kB) \circ B$	$S_k(A)$	$\bigcup_{\substack{k=0}^{K}}^{K} S_k(A)$
0				
1				

FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

2/27/2014

6

B

51

A can be reconstructed from the subsets by using

$$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$$

where $S_k(A) \oplus kB$ denotes *k* successive dilations of A. $(S_k(A) \oplus kB) = ((...((S_k(A) \oplus B) \oplus B)... \oplus B))$



FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

Pruning

a. Thinning and skeletonizing tend to leave parasitic componentsb. Pruning methods are essential complement to thinning and skeletonizing procedures



FIGURE 9.25 (a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

С



(a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.



 $X_3 = (X_2 \oplus H) \cap A$ H:3×3 structuring element

2/27/2014



Morphological Reconstruction

It involves two images and a structuring element

- a. One image contains the starting points for the transformation (The image is called marker)
- b. Another image (mask) constrains the transformation
- c. The structuring element is used to define connectivity

Morphological Reconstruction: Geodesic Dilation

Let *F* denote the marker image and *G* the mask image, $F \subseteq G$. The geodesic dilation of size 1 of the marker image with respect to the mask, denoted by $D_G^{(1)}(F)$, is defined as $D_G^{(1)}(F) = (F \oplus B) \cap G$

The geodesic dilation of size *n* of the marker image *F* with respect to *G*, denoted by $D_G^{(n)}(F)$, is defined as $D_G^{(n)}(F) = D_G^{(1)}(F) \Big[D_G^{(n-1)}(F) \Big]$

with $D_G^{(0)}(F) = F$.

2/27/2014

Morphological Reconstruction: Geodesic Erosion

Let *F* denote the marker image and *G* the mask image. The geodesic erosion of size 1 of the marker image with respect to the mask, denoted by $E_G^{(1)}(F)$, is defined as $E_G^{(1)}(F) = (F \ominus B) \cup G$

The geodesic erosion of size *n* of the marker image *F* with respect to *G*, denoted by $E_G^{(n)}(F)$, is defined as $E_G^{(n)}(F) = E_G^{(1)}(F) \Big[E_G^{(n-1)}(F) \Big]$

with $E_{G}^{(0)}(F) = F$.

2/27/2014

Morphological Reconstruction by Dilation

Morphological reconstruction by dialtion of a mask image *G* from a marker image *F*, denoted $R_G^D(F)$, is defined as the geodesic dilation of *F* with respect to *G*, iterated until stability is achieved; that is,

 $R_G^D(F) = D_G^{(k)}(F)$

with k such that $D_G^{(k)}(F) = D_G^{(k-1)}(F)$.

a b c d e f g h **FIGURE 9.28** Illustration of morphological reconstruction by dilation. F, G, B and $D_G^{(1)}(F)$ are from Fig. 9.26.

2/27/2014

Morphological Reconstruction by Erosion

Morphological reconstruction by erosion of a mask image *G* from a marker image *F*, denoted $R_G^E(F)$, is defined as the geodesic erosion of *F* with respect to *G*, iterated until stability is achieved; that is,

$$R_G^E(F) = E_G^{(k)}(F)$$

with k such that $E_G^{(k)}(F) = E_G^{(k-1)}(F)$.

Opening by Reconstruction

The opening by reconstruction of size n of an image F is defined as the reconstruction by dilation of F from the erosion of size n of F; that is

$$O_R^{(n)}(F) = R_F^D \left[\left(F \ominus nB \right) \right]$$

where $(F \ominus nB)$ indicates *n* erosions of *F* by *B*.

ponents or broken connection paths. There is no poir tion past the level of detail required to identify those Segmentation of nontrivial images is one of the mos processing. Segmentation accuracy determines the ev of computerized analysis procedures. For this reason, c be taken to improve the probability of rugged segment such as industrial inspection applications, at least some the environment is possible at times. The experienced i designer invariably pays considerable attention to suc

> p th Th р dt d tf th l fd t q t. p t th f th t t d t th t t р F th d р d t p th p b b l t t k b p d tl ppl d p t t h bl Th tt h p р bl bl tt d p d

a b c d

Filling Holes

Let I(x, y) denote a binary image and suppose that we form a marker image F that is 0 everywhere, except at the image border, where it is set to 1-I; that is

$$F(x, y) = \begin{cases} 1 - I(x, y) & \text{if } (x, y) \text{ is on the border of } I \\ 0 & \text{otherwise} \end{cases}$$

then

$$H = \left[R_{I^c}^D(F) \right]^c$$

SE: 3×3 1s.

a b c d e f g

FIGURE 9.30 Illustration of

hole filling on a simple image.

ponents or broken connection paths. There is no poir tion past the level of detail required to identify those Segmentation of nontrivial images is one of the mos processing. Segmentation accuracy determines the ev of computerized analysis procedures. For this reason, c be taken to improve the probability of rugged segment such as industrial inspection applications, at least some the environment is possible at times. The experienced i designer invariably pays considerable attention to suc

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Segmentation of nontrivial images is one of the mos processing. Segmentation accuracy determines the ev of computerized analysis procedures. For this reason, c be taken to improve the probability of rugged segment. such as industrial inspection applications, at least some the environment is possible at times. The experienced i designer invariably pays considerable attention to sucl a b c d

cu

FIGURE 9.31

(a) Text image of size 918×2018 pixels. (b) Complement of (a) for use as a mask image. (c) Marker image. (d) Result of hole-filling using Eq. (9.5-29).

ponents or broken connection paths. There is no point tion past the level of detail required to identify those a Segmentation of nontrivial images is one of the most processing. Segmentation accuracy determines the evol of computerized analysis procedures. For this reason, a be taken to improve the probability of rugged segments such as industrial inspection applications, at least some the environment is possible at times. The experienced i designer invariably pays considerable attention to such a such as a such as considerable attention to such a such as possible at times. The experienced is designer invariably pays considerable attention to such a such a such a such a such as a such as possible at times. The experience is possible attention to such a such as possible at times. The experience is possible at times. The experience is possible attention to such a such as possible at times. The experience is possible at times. The experience is possible at times. The experience is possible attention to such a such a

 $H = \left\lceil R^{D}_{I^{c}}(F) \right\rceil^{c}$

Border Clearing

It can be used to screen images so that only complete objects remain for further processing; it can be used as a singal that partial objects are present in the field of view.

The original image is used as the mask and the following marker image:

 $F(x, y) = \begin{cases} I(x, y) & \text{if } (x, y) \text{ is on the border of } I \\ 0 & \text{otherwise} \end{cases}$ $X = I - R_I^D(F)$

ponents or broken connection paths. There is no poir tion past the level of detail required to identify those

Segmentation of nontrivial images is one of the mos processing. Segmentation accuracy determines the ev of computerized analysis procedures. For this reason, c be taken to improve the probability of rugged segment such as industrial inspection applications, at least some the environment is possible at times. The experienced designer invariably pays considerable attention to suc

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a b

2/27/2014

FIGURE 9.32

Border clearing. (a) Marker image. (b) Image with no objects touching the border. The original image is Fig. 9.29(a).

Summary (1)

FIGURE 9.33 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the \times 's indicate "don't care" values.

Summary (2)

		Comments
Operation	Equation	(The Roman numerals refer to the structuring elements in Fig. 9.33.)
Translation	$(B)_z = \{w w = b + z, \\ \text{for } b \in B\}$	Translates the origin of B to point z .
Reflection	$\hat{B} = \{w w = -b, \text{ for } b \in B\}$	Reflects all elements of <i>B</i> about the origin of this set.
Complement	$A^c = \{w w \notin A\}$	Set of points not in A.
Difference	$egin{array}{lll} A &- B = \{w w \in A, w otin B^c \ &= A \cap B^c \end{array}$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \left\{ z (\hat{B}_z) \cap A \neq \emptyset \right\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \left\{ z (B)_z \subseteq A \right\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)

TABLE 9.1Summary ofmorphologicaloperations andtheir properties.

(*Continued*)

		Comments	
Operation	Equation	(The Roman numerals refer to the structuring elements in Fig. 9.33.)	
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)	
Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c	
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)	
Hole filling	$X_k = (X_{k-1} \oplus B) \cap A^c;$ $k = 1, 2, 3, \dots$	Fills holes in A ; $X_0 = array$ of 0s with a 1 in each hole. (II)	
Connected components	$X_k = (X_{k-1} \oplus B) \cap A;$ $k = 1, 2, 3, \dots$	Finds connected components in A ; X_0 = array of 0s with a 1 in each connected component. (I)	
Convex hull	$X_{k}^{i} = (X_{k-1}^{i} \circledast B^{i}) \cup A;$ i = 1, 2, 3, 4; k = 1, 2, 3,; $X_{0}^{i} = A;$ and $D^{i} = X_{conv}^{i}$	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)	
Thinning	$A \otimes B = A - (A \circledast B)$ = $A \cap (A \circledast B)^{c}$ $A \otimes \{B\} =$ $((\dots ((A \otimes B^{1}) \otimes B^{2}) \dots) \otimes B^{n})$ $\{B\} = \{B^{1}, B^{2}, B^{3}, \dots, B^{n}\}$	Thins set <i>A</i> . The first two equations give the basic defi- nition of thinning. The last equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)	
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\dots (A \odot B^{1}) \odot B^{2} \dots) \odot B^{n})$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0s and 1s reversed.	
Skeletons	$S(A) = \bigcup_{k=0}^{K} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{ (A \ominus kB) - [(A \ominus kB) \circ B] \}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosions of A by B. (I)	

TABLE 9.1(Continued)

2/27/2014

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.
Geodesic dilation of size 1	$D_G^{(1)}(F) = (F \oplus B) \cap G$	F and G are called the <i>marker</i> and <i>mask</i> images, respectively.
Geodesic dilation of size <i>n</i>	$D_G^{(n)}(F) = D_G^{(1)} [D_G^{(n-1)}(F)];$ $D_G^{(0)}(F) = F$	
Geodesic erosion of size 1	$E_G^{(1)}(F) = (F \ominus B) \cup G$	
Geodesic erosion of size <i>n</i>	$E_G^{(n)}(F) = E_G^{(1)} [E_G^{(n-1)}(F)];$ $E_G^{(0)}(F) = F$	
Morphological reconstruction by dilation	$R^D_G(F) = D^{(k)}_G(F)$	k is such that $D_G^{(k)}(F) = D_G^{(k+1)}(F)$
Morphological reconstruction by erosion	$R_G^E(F) = E_G^{(k)}(F)$	k is such that $E_G^{(k)}(F) = E_G^{(k+1)}(F)$
Opening by reconstruction Closing by	$O_R^{(n)}(F) = R_F^D[(F \ominus nB)]$	$(F \ominus nB)$ indicates n erosions of F by B .
reconstruction	$C_R^{(n)}(F) = R_F^E[(F \oplus nB)]$	$(F \oplus nB)$ indicates n dilations of F by B .
Hole filling	$H = \left[R_{I^c}^D(F) \right]^c$	H is equal to the input image I , but with all holes filled. See Eq. (9.5-28) for the definition of the marker image F .
Border clearing	$X = I - R_I^D(F)$	X is equal to the input image I, but with all objects that touch (are connected to) the boundary removed. See Eq. $(9.5-30)$ for the definition of the marker image F.

TABLE 9.1(Continued)

Gray-Scale Morphology

f(x, y): gray-scale image b(x, y): structuring element

a b c d

FIGURE 9.34 Nonflat and flat structuring elements, and corresponding horizontal intensity profiles through their center. All examples in this section are based on flat SEs.

Gray-Scale Morphology: Erosion and Dilation by Flat Structuring

$$[f \ominus b](x, y) = \min_{(s,t)\in b} \{f(x+s, y+t)\}$$

$$[f \oplus b](x, y) = \max_{(s,t) \in b} \{f(x-s, y-t)\}$$

a b c

FIGURE 9.35 (a) A gray-scale X-ray image of size 448×425 pixels. (b) Erosion using a flat disk SE with a radius of two pixels. (c) Dilation using the same SE. (Original image courtesy of Lixi, Inc.)

Gray-Scale Morphology: Erosion and Dilation by Nonflat Structuring

$$\left[f \ominus b_N\right](x, y) = \min_{(s,t) \in b} \left\{f(x+s, y+t) - b_N(s, t)\right\}$$

$$\left[f \oplus b_N\right](x, y) = \max_{(s,t) \in b} \left\{f(x-s, y-t) + b_N(s, t)\right\}$$

Duality: Erosion and Dilation

$$\left[f \ominus b\right]^{c} (x, y) = \left(f^{c} \oplus \hat{b}\right)(x, y)$$

where
$$f^{c} = -f(x, y)$$
 and $\hat{b} = b(-x, -y)$
 $[f \ominus b]^{c} = (f^{c} \oplus \hat{b})$
 $(f \oplus b)^{c} = (f^{c} \ominus \hat{b})$

Opening and Closing

$$f \circ b = (f \ominus b) \oplus b$$
$$f \bullet b = (f \oplus b) \ominus b$$

$$(f\Box b)^{c} = f^{c} \circ \hat{b} = -f \circ \hat{b}$$

$$(f \circ b)^c = f^c \square b = -f \square b$$

а b c d e

FIGURE 9.36

Opening and closing in one dimension. (a) Original 1-D signal. (b) Flat structuring element pushed up underneath the signal. (c) Opening. (d) Flat structuring element pushed

down along the top of the signal. (e) Closing.

Properties of Gray-scale Opening

(a)
$$f \circ b \dashv f$$

(b) $\text{if } f_1 \dashv f_2$, then $(f_1 \circ b) \dashv (f_2 \circ b)$
(c) $(f \circ b) \circ b = f \circ b$

where $e \downarrow r$ denotes *e* is a subset of *r* and also $e(x, y) \leq r(x, y)$.

Properties of Gray-scale Closing

(a) $f \dashv f \square b$ (b) $\text{if } f_1 \dashv f_2, \text{ then } (f_1 \square b) \dashv (f_2 \square b)$ (c) $(f \square b) \square b = f \square b$

a b c

Morphological Smoothing

- Opening suppresses bright details smaller than the specified SE, and closing suppresses dark details.
- Opening and closing are used often in combination as morphological filters for image smoothing and noise removal.

Morphological Smoothing

a b c d

FIGURE 9.38

(a) 566×566 image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)-(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii. 1, 3, and 5, respectively. (Original image courtesy of NASA.)

Morphological Gradient

Dilation and erosion can be used in combination with image subtraction to obtain the morphological gradient of an image, denoted by g,

$$g = (f \oplus b) - (f \ominus b)$$

The edges are enhanced and the contribution of the homogeneous areas are suppressed, thus producing a "derivative-like" (gradient) effect.

Morphological Gradient

a b c d

FIGURE 9.39 (a) 512×512 image of a head CT scan. (b) Dilation. (c) Erosion. (d) Morphological gradient, computed as the difference between (b) and (c). (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)

Top-hat and Bottom-hat Transformations

The top-hat transformation of a grayscale image f is defined as f minus its opening:

$$T_{hat}(f) = f - (f \circ b)$$

The bottom-hat transformation of a grayscale image f is defined as its closing minus f:

$$B_{hat}(f) = (f \bullet b) - f$$

Top-hat and Bottom-hat Transformations

One of the principal applications of these transformations is in removing objects from an image by using structuring element in the opening or closing operation

Example of Using Top-hat Transformation in Segmentation

a b c d e

FIGURE 9.40 Using the top-hat transformation for *shading correction*. (a) Original image of size $_{600 \times 600}$ pixels. (b) Thresholded image. (c) Image opened using a disk SE of radius 40. (d) Top-hat transformation (the image minus its opening). (e) Thresholded top-hat image.

Granulometry

- Granulometry deals with determining the size of distribution of particles in an image
- Opening operations of a particular size should have the most effect on regions of the input image that contain particles of similar size
- For each opening, the sum (surface area) of the pixel values in the opening is computed

Example

FIGURE 9.41 (a) 531 × 675 image of wood dowels. (b) Smoothed image. (c)–(f) Openings of (b) with disks of radii equal to 10, 20, 25, and 30 pixels, respectively. (Original image courtesy of Dr. Steve Eddins, The MathWorks, Inc.)

FIGURE 9.42 Differences in surface area as a function of SE disk radius, *r*. The two peaks are indicative of two dominant particle sizes in the image.

Textual Segmentation

Segmentation: the process of subdividing an image into regions.

Textual Segmentation

a b c d

FIGURE 9.43 Textural segmentation. (a) A 600 \times 600 image consisting of two types of blobs. (b) Image with small blobs removed by closing (a). (c) Image with light patches between large blobs removed by opening (b). (d) Original image with boundary between the two regions in (c) superimposed. The boundary was obtained using a morphological gradient

operation.

Gray-Scale Morphological Reconstruction (1)

Let f and g denote the marker and mask image with the same size, respectively and f ≤ g. The geodesic dilation of size 1 of f with respect to g is defined as

 $D_g^{(1)}(f) = (f \oplus g) \wedge g$

where \land denotes the point-wise minimum operator.

The geodesic dilation of size n of f with respect to g is defined as

$$D_g^{(n)}(f) = D_g^{(1)} \left[D_g^{(n-1)}(f) \right]$$
 with $D_g^{(0)}(f) = f$

Gray-Scale Morphological Reconstruction (2)

The geodesic erosion of size 1 of f with respect to g is defined as

 $E_g^{(1)}(f) = (f \ominus g) \lor g$

where \vee denotes the point-wise maximum operator.

The geodesic erosion of size n of f with respect to g is defined as

$$E_g^{(n)}(f) = E_g^{(1)} \left[E_g^{(n-1)}(f) \right]$$
 with $E_g^{(0)}(f) = f$

Gray-Scale Morphological Reconstruction (3)

The morphological reconstruction by dilation of a grayscale mask image g by a gray-scale marker image f, is defined as the geodesic dilation of f with respect to g, iterated until stability is reached, that is,

$$R_g^D(f) = D_g^{(k)}(f)$$

with k such that $D_g^{(k)}(f) = D_g^{(k+1)}(f)$

The morphological reconstruction by erosion of g by f is defined as F(x) = F(x) = F(x)

$$R_g^E(f) = E_g^{(k)}(f)$$

with k such that $E_g^{(k)}(f) = E_g^{(k+1)}(f)$

2/27/2014

Gray-Scale Morphological Reconstruction (4)

The opening by reconstruction of size n of an image f is defined as the reconstruction by dilation of f from the erosion of size n of f; that is,

$$O_R^{(n)}(f) = R_f^D[f \ominus nb]$$

The closing by reconstruction of size n of an image f is defined as the reconstruction by erosion of f from the dilation of size n of f; that is,

$$C_R^{(n)}(f) = R_f^E[f \oplus nb]$$

FIGURE 9.44 (a) Original image of size 1134 × 1360 pixels. (b) Opening by reconstruction of (a) using a horizontal line 71 pixels long in the erosion. (c) Opening of (a) using the same line. (d) Top-hat by reconstruction. (e) Top-hat. (f) Opening by reconstruction of (d) using a horizontal line 11 pixels long. (g) Dilation of (f) using a horizontal line 21 pixels long. (h) Minimum of (d) and (g). (i) Final reconstruction result. (Images courtesy of Dr. Steve Eddins, The MathWorks, Inc.)

Steps in the Example

1. Opening by reconstruction of the original image using a horizontal line of size 1x71 pixels in the erosion operation $a(r) \in \mathbb{R} \setminus \mathbb{R}$

$$O_R^{(n)}(f) = R_f^D \left[f \ominus nb \right]$$

2. Subtract the opening by reconstruction from original image

$$f' = f - O_R^{(n)}(f)$$

 Opening by reconstruction of the f' using a vertical line of size 11x1 pixels

$$f1 = O_R^{(n)}(f') = R_f^D \left[f \ominus nb' \right]$$

4. Dilate f1 with a line SE of size 1x21, get f2.

Steps in the Example

- 5. Calculate the minimum between the dilated image f2 and and f', get f3.
- 6. By using f3 as a marker and the dilated image f2 as the mask,

 $R_{f2}^{D}(f3) = D_{f2}^{(k)}(f3)$ with k such that $D_{f2}^{(k)}(f3) = D_{f2}^{(k+1)}(f3)$