EE247
Lecture 3

- Summary- last week
- Integrator based filters
  - Signal flowgraph concept
  - First order integrator based filter
  - Second order integrator based filter & biquads
- High order & high Q filters
  - Cascaded biquads
    - Cascaded biquad sensitivity
  - Ladder type filters

Summary
Last Week

- Major success in CMOS technology scaling:
  → Inexpensive DSPs technology
  → Resulted in the need for high performance Analog/Digital interface circuitry
- Analog/Digital interface building blocks includes
  - Analog filters
  - D/A converters
  - A/D converters

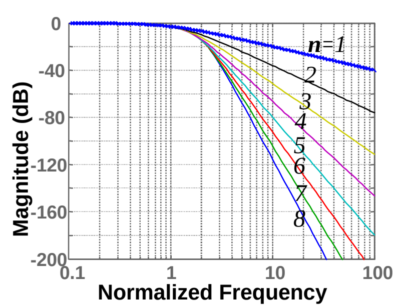
Summary
Last Week

Analog filters:
- Filter specifications
  - Quality factor
  - Frequency characteristics
  - Group delay- no phase distortion→ constant group delay
- Filter types
  - Butterworth→ Maximally flat passband amplitude
  - Chebyshev I → Ripple in the passband
  - Chebyshev II → Ripple in the stopband
  - Elliptic → Ripple in both passband & stopband – highest rolloff/pole
  - Bessel→ Maximally flat group delay
- Group delay comparison example → phase distortion effects on BER

Summary
LPF Out-of-Band Signal Attenuation Versus Filter Order

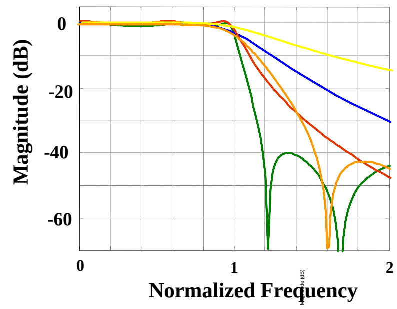
- Example: Butterworth low-pass filter
  - Magnitude roll-off versus filter order (n):

Out of band magnitude rolloff→ nx20dB/Decade



Summary

Comparison of Various LPF Magnitude Response



All 5th order filters with same corner freq.

Bessel
Butterworth
Chebyshev I
Chebyshev II
Elliptic

Summary Last Week
Monolithic LC Filters

- Monolithic inductor in CMOS tech.
  - Integrated L<10nH with Q<10 combined with max. cap. 10pF

  → *LC filters in the monolithic form feasible: freq>500MHz*

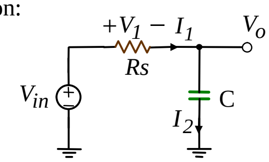
- Analog/Digital interface circuitry require fully integrated filters with critical frequencies << 500MHz
- Good alternative:

⇨Integrator based filters

Integrator Based Filters
First Order LPF

- Start from RC prototype

•Use KCL & KVL to derive state space description:



$$\frac{V_o}{V_{in}} = \frac{1}{1 + sRC}$$

- Use state space description to draw signal flowgraph (SFG)

→ What is Signal Flowgraph (SFG)?

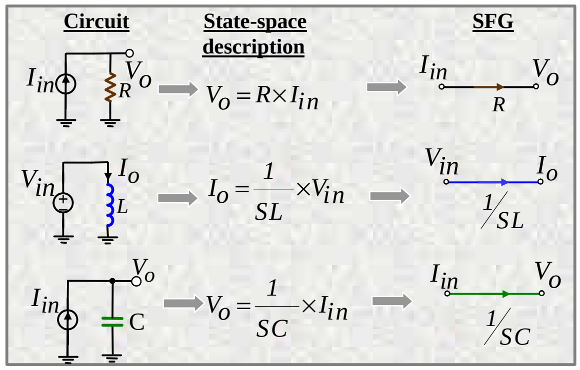
What is a Signal Flowgraph (SFG)?

- SFG → Topological network representation consisting of nodes & branches- used to convert one form of network to a more suitable form (e.g. passive RLC filters to integrator based filters)
- Any network described by a set of linear diff. equations can be expressed in SFG form.
- For a given network, many different SFGs exists.
- Choice of a particular SFG is based on practical considerations such as type of available components.

*Ref: W.Heinlein & W. Holmes, “Active Filters for Integrated Circuits”, Prentice Hall, Chap. 8, 1974.

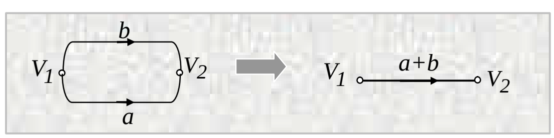
What is a Signal Flowgraph (SFG)?

- SFG nodes represent variables (*V & I* in our case), branches represent transfer functions (we will call these transfer functions branch multiplication factor *BMF*)
- To convert a network to its SFG form, *KCL* & *KVL* is used to derive state space description:
- Example:

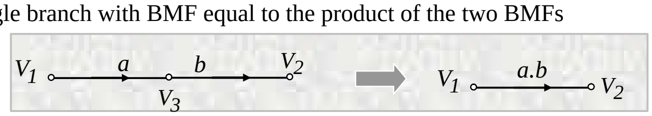


Signal Flowgraph (SFG) Rules

- Two parallel branches can be replaced by a single branch with BMF equal to sum of two BMFs



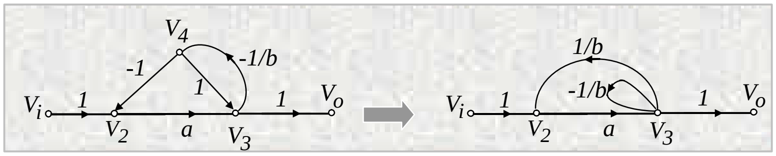
•A node with only one incoming branch & one outgoing branch can be replaced by a single branch with BMF equal to the product of the two BMFs



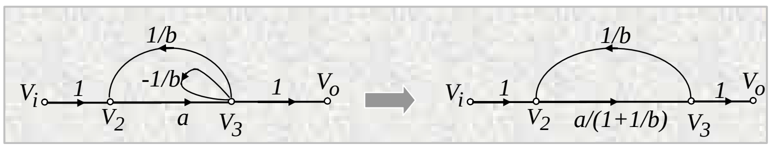
•An intermediate node can be multiplied by a factor (x). BMFs for incoming branches have to be multiplied by x and outgoing branches divided by x

Signal Flowgraph (SFG) Rules

- Simplifications can often be achieved by shifting or eliminating nodes



- A self-loop branch with BMF y can be eliminated by multiplying the BMF of incoming branches by 1/(1-y)



Integrator Based Filters

First Order LPF

- Start from RC prototype
- Use KCL & KVL to derive state space description:

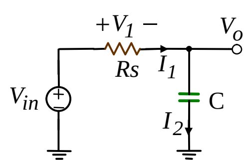
$$V_1 = V_{in} - V_o$$

$$V_o = \frac{I_2}{sC}$$

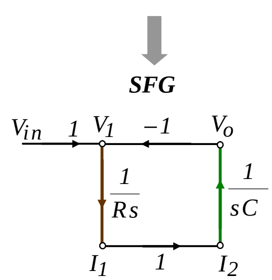
$$I_1 = \frac{V_1}{Rs}$$

$$I_2 = I_1$$

- Use state space description to draw signal flowgraph (SFG)



SFG



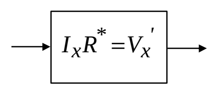
Normalize

- Since integrators - the main building blocks- require in & out signals in the voltage form (not current)

  → Convert all currents to voltages by multiplying current nodes by a scaling resistance R^*

  → Branches should then be scaled accordingly

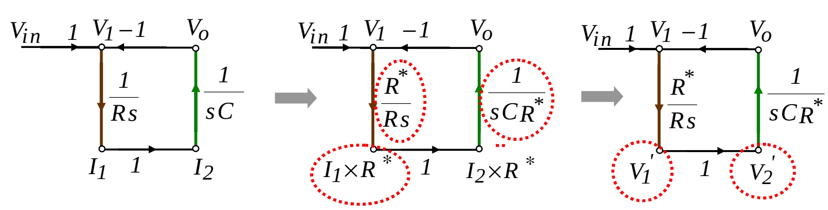
$$V_1 = V_{in} - V_o \qquad I_1 = \frac{V_1}{Rs} \qquad V_o = \frac{I_2}{sC} \qquad I_2 = I_1$$



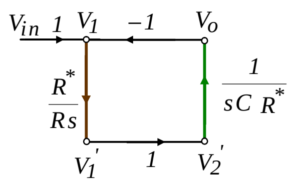
$$V_1 = V_{in} - V_o \qquad I_1 R^* = \frac{R^*}{Rs} V_1 \qquad V_o = \frac{I_2 R^*}{sC\ R^*} \qquad I_2 R^* = I_1 R^*$$

$$V_1 = V_{in} - V_o \qquad V_1' = \frac{R^*}{Rs} V_1 \qquad V_o = \frac{V_2'}{sC\ R^*} \qquad V_1' = V_2'$$

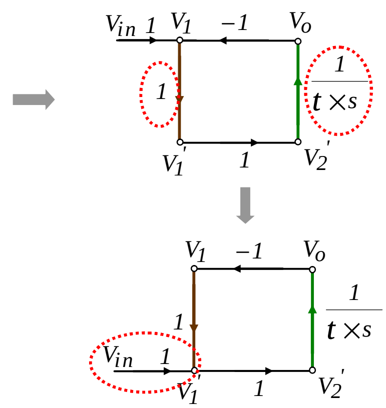
Normalize



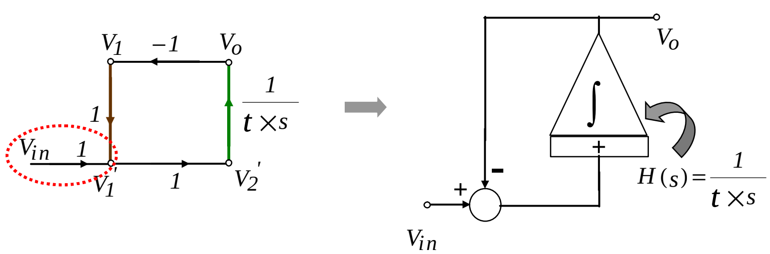
Synthesis



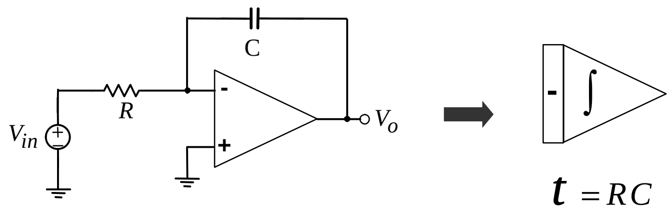
$$R^* = Rs \quad , \; \tau = R^* \times C$$



First Order Integrator Based Filter



Opamp-RC Single-Ended Integrator

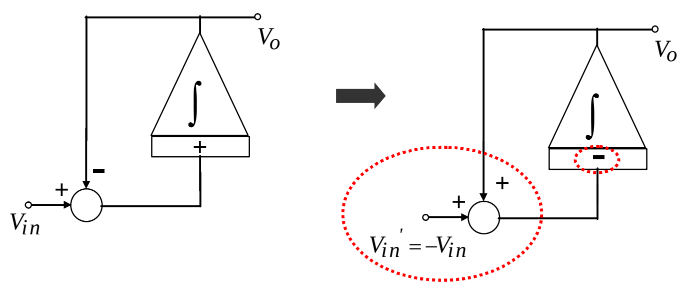


$$V_o = -\frac{1}{RC}\int V_{in}\,dt \quad , \quad \frac{V_o}{V_{in}} = -\frac{1}{sRC}$$

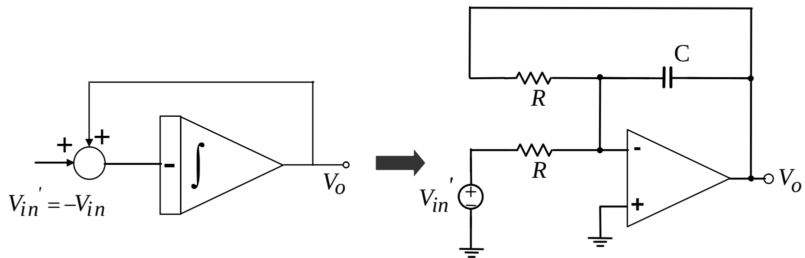
Opamp-RC Integrator

- Single-ended Opamp-RC integrator has a sign inversion from input to output

→ Convert SFG accordingly by modifying BMF



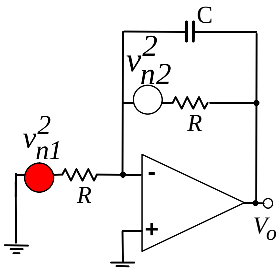
Opamp-RC 1st Order Filter



$$\frac{V_o}{V_{in}'} = -\frac{1}{1+sRC}$$

Opamp-RC First Order Filter Noise

Identify noise sources (here it is resistors & opamp)
Find transfer function from each noise source
to the output (opamp noise next page)



$$\overline{v_o^2} = \sum_{m=1}^{n} \int_0^{\infty} |H_m(f)|^2 S_i(f)\, df$$

$S_i(f) \rightarrow$ *Input referred noise spectral density*

$$|H_1(f)|^2 = |H_2(f)|^2 = \frac{1}{1+(2p\, fRC)^2}$$

$$v_{n1}^2 = v_{n2}^2 = 4KTR\Delta f$$

$$\sqrt{\overline{v_o^2}} = \sqrt{2\frac{kT}{C}}$$

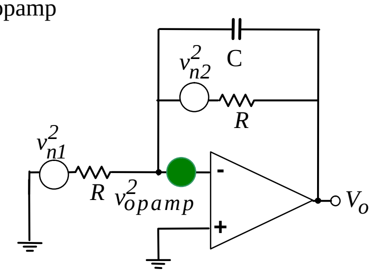
$a = 2$

α

Typically, α increases as filter order increases

Opamp-RC Filter Noise
Opamp Contribution

So far only the fundamental noise sources are considered.
In reality, noise associated with the opamp increases the overall noise.
The bandwidth of the opamp affects the opamp noise contribution to the total noise



Integrator Based Filters
Second Order RLC Filter

•State space description:

$$V_R = V_L = V_o$$

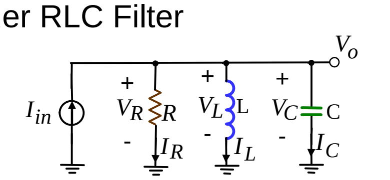
$$V_C = \frac{I_C}{sC}$$

$$I_R = \frac{V_R}{R}$$

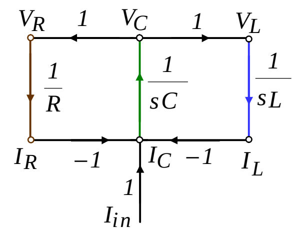
Integrator form

$$I_L = \frac{V_L}{sL}$$

$$I_C = I_{in} - I_R - I_L$$



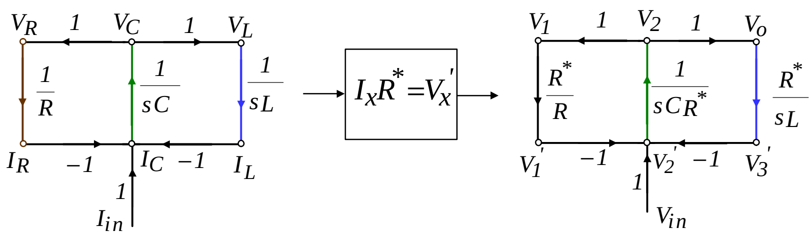
SFG



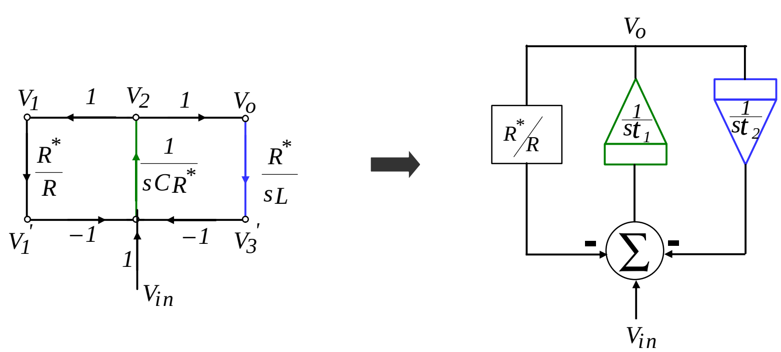
• Draw signal flowgraph (SFG)

Normalize

- Convert currents to voltages by multiplying all current nodes by the scaling resistance R^*

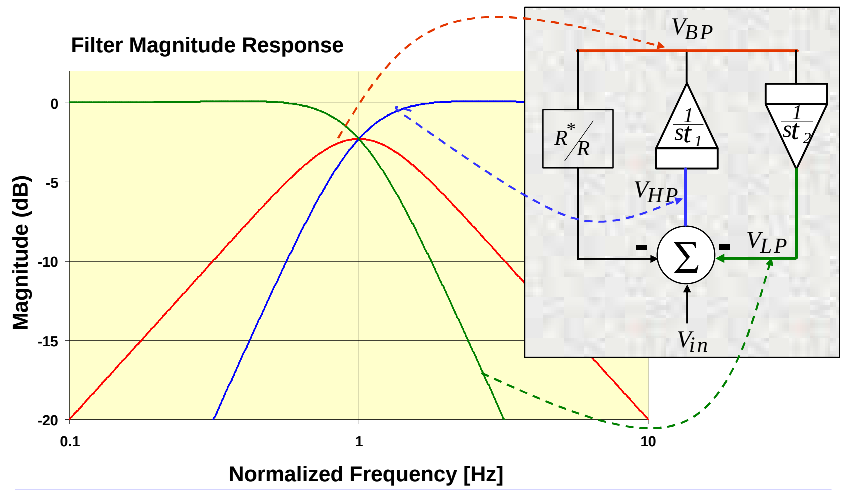


Synthesis



$$t_1 = R^* \times C \qquad t_2 = L \big/ R^*$$

Second Order Integrator Based Filter



Second Order Integrator Based Filter

$$\frac{V_{BP}}{V_{in}} = \frac{t_2 s}{t_1 t_2 s^2 + b\, t_2 s + 1}$$

$$\frac{V_{LP}}{V_{in}} = \frac{1}{t_1 t_2 s^2 + b\, t_2 s + 1}$$

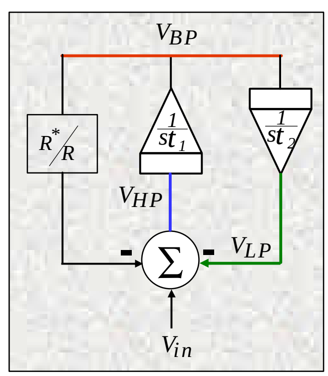
$$\frac{V_{HP}}{V_{in}} = \frac{t_1 t_2 s^2}{t_1 t_2 s^2 + b\, t_2 s + 1}$$

$$t_1 = R^* \times C \qquad t_2 = L \big/ R^*$$

$$b = R^* \big/ R$$

$$w_0 = 1 \big/ \sqrt{t_1 t_2} = 1 \big/ \sqrt{L\,C}$$

$$Q = 1/b \times \sqrt{t_1 \big/ t_2}$$



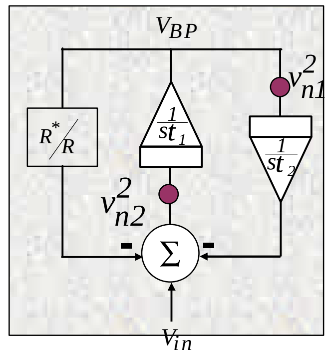
From matching point of view desirable:

$$t_1 = t_2 \rightarrow Q = R \big/ R^*$$

Second Order Bandpass Filter Noise

$$\overline{v_o^2} = \sum_{m=1}^{n} \int_0^\infty |H_m(f)|^2 S_i(f)$$

- Find transfer function of each noise source to the output
- Integrate contribution of all noise sources
- Here it is assumed that opamps are noise free (not usually the case!)



$$v_{n1}^2 = v_{n2}^2 = 4KTR$$

$$\sqrt{\overline{v_o^2}} = \sqrt{2Q\frac{kT}{C}}$$

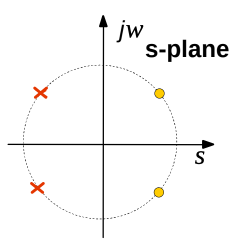
α

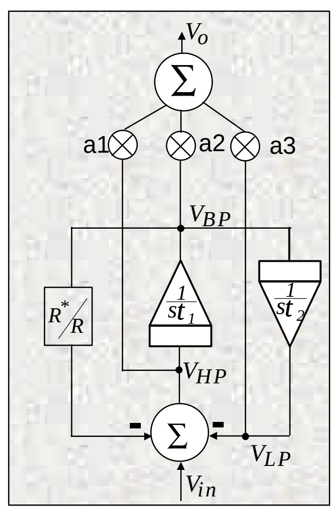
Typically, α increases as filter order increases
Note the noise power is directly proportion to Q

Second Order Integrator Based Filter Biquad

- By combining outputs can generate biquad function:

$$\frac{V_0}{V_{in}} = \frac{a_1 t_1 t_2 s^2 + a_2 t_2 s + a_3}{t_1 t_2 s^2 + b t_2 s + 1}$$





Summary
Integrator Based Monolithic Filters

- Signal flowgraph techniques utilized to convert RLC networks to all integrator active filters
- Each reactive element (L& C) replaced by an integrator
- Fundamental noise limitation determined by integrating capacitor:

  – For lowpass filter: $\sqrt{\overline{v_o^2}} = \sqrt{a \frac{k\,T}{C}}$

  – Bandpass filter: $\sqrt{\overline{v_o^2}} = \sqrt{a\ Q \frac{k\,T}{C}}$

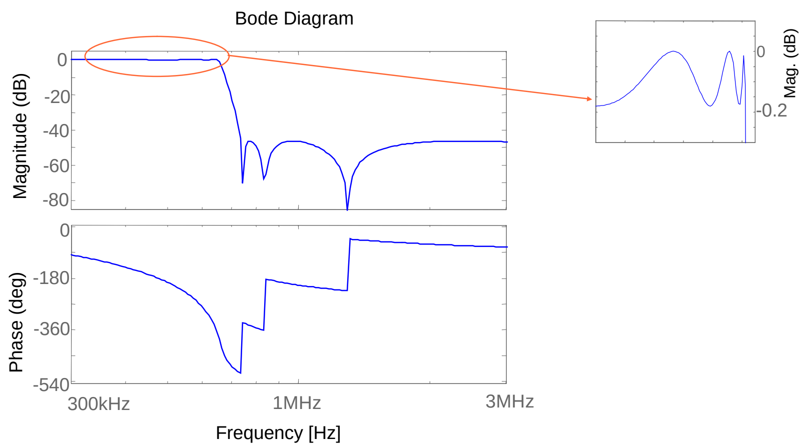
  where a is a function of filter order and topology

Higher Order Filters
Cascade of Biquads

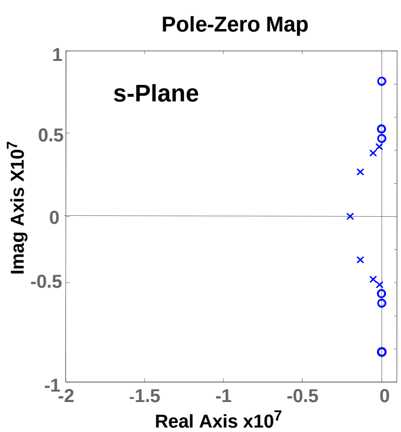
Example: LPF filter for CDMA RX baseband

- LPF with
  – fpass = 650 kHz Rpass = 0.2 dB
  – fstop = 750 kHz Rstop = 45 dB
- 7th order Elliptic Filter
- Implementation with Biquads
  Goal: Maximize dynamic range
  – Pair poles and zeros
    highest Q poles with closest zeros is a good starting point, but not necessarily optimum
  – Ordering:
    Lowest Q poles first is a good start

Filter Frequency Response



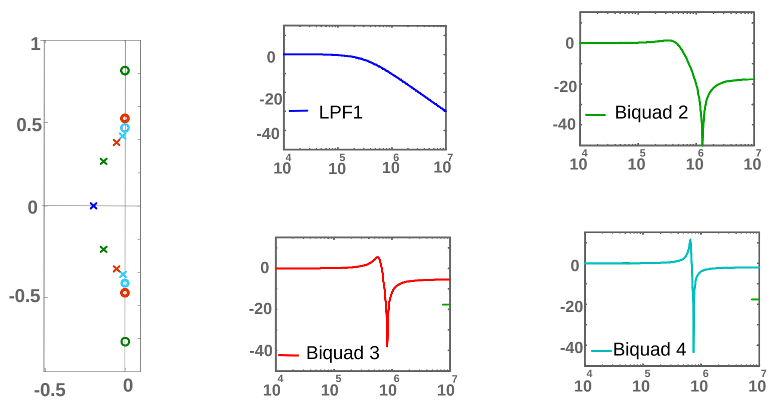
Pole-Zero Map



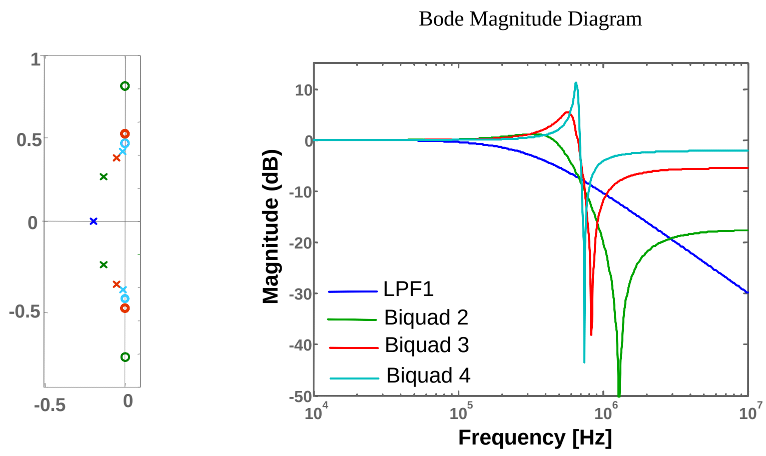
Q_{pole}	f_{pole} [kHz]
16.7902	659.496
3.6590	611.744
1.1026	473.643
	319.568

f_{zero} [kHz]
1297.5
836.6
744.0

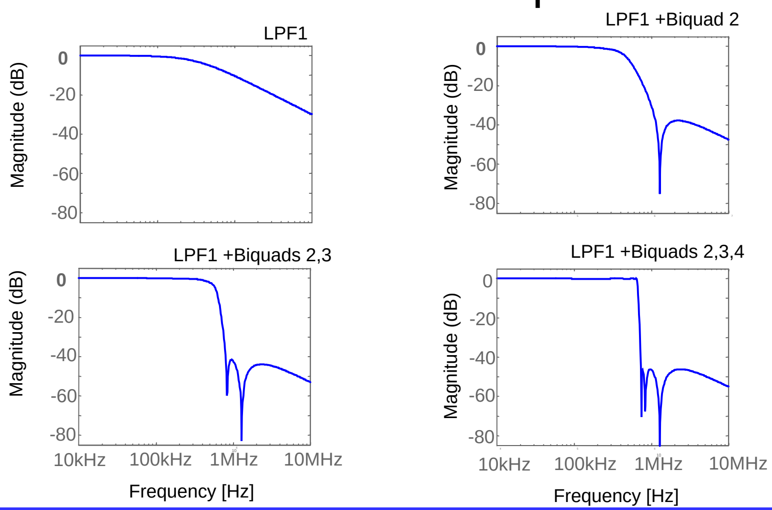
Biquad Response



Biquad Response



Intermediate Outputs

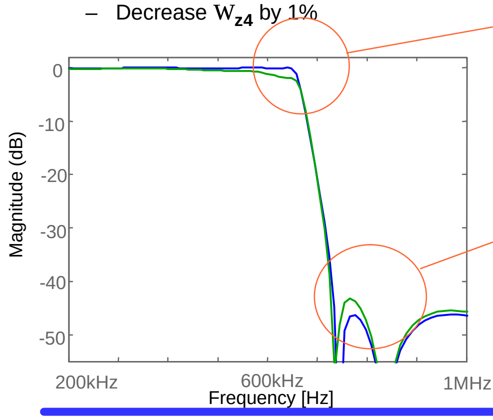


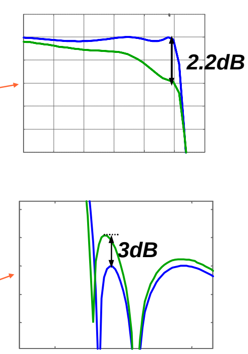
Sensitivity

Component variation in Biquad 4 (highest Q pole):

– Increase W_{p4} by 1%

– Decrease W_{z4} by 1%



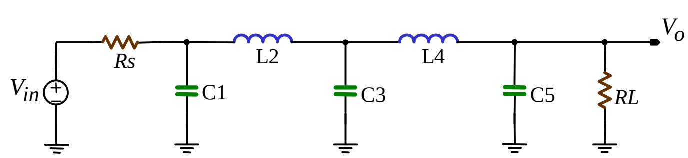


High Q poles → High sensitivity in Biquad realizations

High Q & High Order Filters

- Cascade of biquads
  - Highly sensitive to component variations → not suitable for implementation of high Q & high order filters
  - Cascade of biquads only used in cases where required Q for all biquads <4 (e.g. filters for disk drives)
- LC ladder filters more appropriate for high Q & high order filters (next topic)
  - Less sensitive to component variations

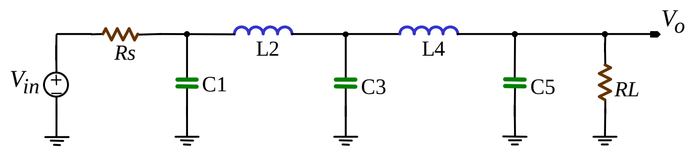
LC Ladder Filters



- Made of resistors, inductors, and capacitors
- Doubly terminated or singly terminated

Doubly terminated LC ladder filters ⇨ Lowest sensitivity to component variations

LC Ladder Filters



- Design:
  - CAD tools
    - Matlab
    - Spice
  - Filter tables
    - A. Zverev, *Handbook of filter synthesis*, Wiley, 1967.
    - A. B. Williams and F. J. Taylor, *Electronic filter design*, 3rd edition, McGraw-Hill, 1995.

LC Ladder Filter Design Example

Design a LPF with maximally flat passband:

$f_{-3dB} = 10MHz$, $f_{stop} = 20MHz$

$R_s > 27dB$

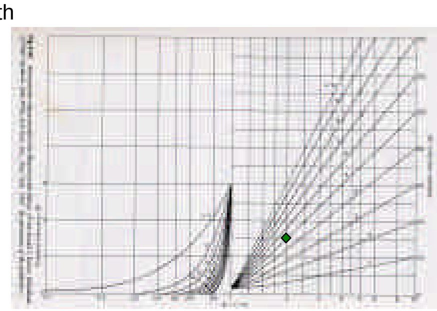
Maximally flat passband ⇨ Butterworth

$f_{stop} / f_{-3dB} = 2$

$R_s > 27dB$

Determine minimum filter order :
- -Use of Matlab
- -or Tables

⇨5th order Butterworth



Stopband Attenuation dB

Normalized w

From: Williams and Taylor, p. 2-37

LC Ladder Filter Design Example

Find values for L & C from Table:

Note L &C values normalized to

$\omega_{-3dB} = 1$

Denormalization:

Multiply all L_{Norm}, C_{Norm} by:

$L_r = R/\omega_{-3dB}$

$C_r = 1/(R \times \omega_{-3dB})$

R is the value of the source and termination resistor (choose both 1Ω for now)

Then: $L = L_r \times L_{Norm}$

$C = C_r \times C_{Norm}$

NORMALIZED FILTER DESIGN TABLES 11.3

TABLE 11-2 Butterworth *LC* Element Values (*Continued*)

n	R_s	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	1.0000	0.6180	1.6180	2.0000	1.6180	0.6180		
	0.9000	0.4416	1.0265	1.9095	1.7562	1.3887		
	0.8000	0.4698	0.8660	2.0605	1.5443	1.7380		
	0.7000	0.5173	0.7313	2.2849	1.3326	2.1083		
	0.6000	0.5860	0.6094	2.5998	1.1255	2.5524		
	0.5000	0.6857	0.4955	3.0510	0.9237	3.1331		
	0.4000	0.8378	0.3877	3.7357	0.7274	3.9648		
	0.3000	1.0937	0.2848	4.8835	0.5367	5.3073		
	0.2000	1.6077	0.1861	7.1849	0.3518	7.9345		
	0.1000	3.1522	0.0912	14.0945	0.1727	15.7103		
	Inf.	1.5451	1.6944	1.3820	0.8944	0.3090		
6	1.0000	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176	
	1.1111	0.2890	1.0403	1.3217	2.0539	1.7443	1.3347	
	1.2500	0.2445	1.1163	1.1257	2.2389	1.5498	1.6881	
	1.4286	0.2072	1.2363	0.9567	2.4991	1.3464	2.0618	
	1.6667	0.1732	1.4071	0.8011	2.8580	1.1431	2.5092	
	2.0000	0.1412	1.6531	0.6542	3.3687	0.9423	3.0938	
	2.5000	0.1108	2.0275	0.5139	4.1408	0.7450	3.9305	
	3.3333	0.0816	2.6559	0.3788	5.4325	0.5517	5.2804	
	5.0000	0.0535	3.9170	0.2484	8.0201	0.3628	7.9216	
	10.0000	0.0263	7.7053	0.1222	15.7855	0.1788	15.7375	
	Inf.	1.5529	1.7593	1.5529	1.2016	0.7579	0.2588	
7	1.0000	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450
	0.9000	0.2985	0.7111	1.4043	1.4891	2.1249	1.7268	1.2961
	0.8000	0.3215	0.6057	1.5174	1.2777	2.3338	1.5461	1.6520
	0.7000	0.3571	0.5154	1.6883	1.0910	2.6177	1.3498	2.0277
	0.6000	0.4075	0.4322	1.9284	0.9170	3.0050	1.1503	2.4771
	0.5000	0.4799	0.3536	2.2726	0.7512	3.5532	0.9513	3.0640
	0.4000	0.5899	0.2782	2.7950	0.5917	4.3799	0.7542	3.9037
	0.3000	0.7745	0.2055	3.6706	0.4373	5.7612	0.5600	5.2583
	0.2000	1.1448	0.1350	5.4267	0.2874	8.5263	0.3692	7.9079
	0.1000	2.2571	0.0665	10.7004	0.1417	16.8222	0.1823	15.7480
	Inf.	1.5576	1.7988	1.6588	1.3972	1.0550	0.6560	0.2225
n	$1/R_s$	L_1	C_2	L_3	C_4	L_5	C_6	L_7

From: Williams and Taylor, p. 11.3

LC Ladder Filter Design Example

Find values for L & C from Table:

Normalized values:

$C1_{Norm} = C5_{Norm} = 0.618$

$C3_{Norm} = 2.0$

$L2_{Norm} = L4_{Norm} = 1.618$

Denormalization:

Since $\omega_{-3dB} = 2\pi \times 10MHz$

$L_r = R/\omega_{-3dB} = 14.1\ nH$

$C_r = 1/(R \times \omega_{-3dB}) = 14.1\ nF$

R =1

⇨C1=C5=9.836nF, C3=31.83nF

⇨L2=L4=25.75nH

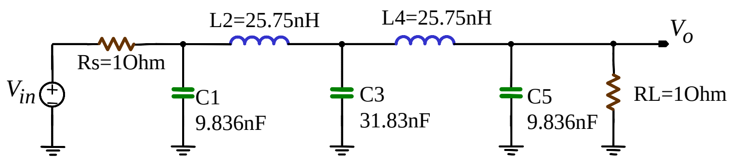
NORMALIZED FILTER DESIGN TABLES 11.3

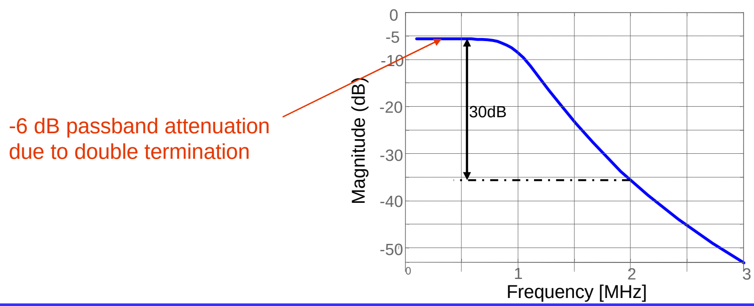
TABLE 11-2 Butterworth *LC* Element Values (*Continued*)

n	R_s	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	1.0000	0.6180	1.6180	2.0000	1.6180	0.6180		
	0.9000	0.4416	1.0265	1.9095	1.7562	1.3887		
	0.8000	0.4698	0.8660	2.0605	1.5443	1.7380		
	0.7000	0.5173	0.7313	2.2849	1.3326	2.1083		
	0.6000	0.5860	0.6094	2.5998	1.1255	2.5524		
	0.5000	0.6857	0.4955	3.0510	0.9237	3.1331		
	0.4000	0.8378	0.3877	3.7357	0.7274	3.9648		
	0.3000	1.0937	0.2848	4.8835	0.5367	5.3073		
	0.2000	1.6077	0.1861	7.1849	0.3518	7.9345		
	0.1000	3.1522	0.0912	14.0945	0.1727	15.7103		
	Inf.	1.5451	1.6944	1.3820	0.8944	0.3090		
6	1.0000	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176	
	1.1111	0.2890	1.0403	1.3217	2.0539	1.7443	1.3347	
	1.2500	0.2445	1.1163	1.1257	2.2389	1.5498	1.6881	
	1.4286	0.2072	1.2363	0.9567	2.4991	1.3464	2.0618	
	1.6667	0.1732	1.4071	0.8011	2.8580	1.1431	2.5092	
	2.0000	0.1412	1.6531	0.6542	3.3687	0.9423	3.0938	
	2.5000	0.1108	2.0275	0.5139	4.1408	0.7450	3.9305	
	3.3333	0.0816	2.6559	0.3788	5.4325	0.5517	5.2804	
	5.0000	0.0535	3.9170	0.2484	8.0201	0.3628	7.9216	
	10.0000	0.0263	7.7053	0.1222	15.7855	0.1788	15.7375	
	Inf.	1.5529	1.7593	1.5529	1.2016	0.7579	0.2588	
7	1.0000	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450
	0.9000	0.2985	0.7111	1.4043	1.4891	2.1249	1.7268	1.2961
	0.8000	0.3215	0.6057	1.5174	1.2777	2.3338	1.5461	1.6520
	0.7000	0.3571	0.5154	1.6883	1.0910	2.6177	1.3498	2.0277
	0.6000	0.4075	0.4322	1.9284	0.9170	3.0050	1.1503	2.4771
	0.5000	0.4799	0.3536	2.2726	0.7512	3.5532	0.9513	3.0640
	0.4000	0.5899	0.2782	2.7950	0.5917	4.3799	0.7542	3.9037
	0.3000	0.7745	0.2055	3.6706	0.4373	5.7612	0.5600	5.2583
	0.2000	1.1448	0.1350	5.4267	0.2874	8.5263	0.3692	7.9079
	0.1000	2.2571	0.0665	10.7004	0.1417	16.8222	0.1823	15.7480
	Inf.	1.5576	1.7988	1.6588	1.3972	1.0550	0.6560	0.2225
n	$1/R_s$	L_1	C_2	L_3	C_4	L_5	C_6	L_7

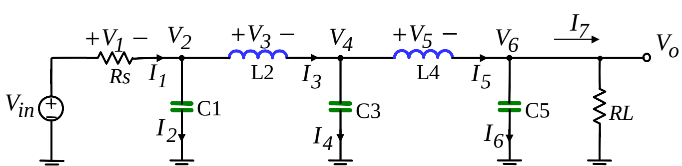
From: Williams and Taylor, p. 11.3

Magnitude Response Simulation





LC Ladder Filter
Conversion to Integrator Based Active Filter



- Use KCL & KVL to derive equations:

$V_1 = V_{in} - V_2$, $V_2 = \frac{I_2}{sC_1}$, $V_3 = V_2 - V_4$

$V_4 = \frac{I_4}{sC_3}$, $V_5 = V_4 - V_6$, $V_6 = \frac{I_6}{sC_5}$ $V_o = V_6$

$I_1 = \frac{V_1}{Rs}$, $I_2 = I_1 - I_3$, $I_3 = \frac{V_3}{sL_2}$

$I_4 = I_3 - I_5$, $I_5 = \frac{V_5}{sL_4}$, $I_6 = I_5 - I_7$, $I_7 = \frac{V_6}{RL}$

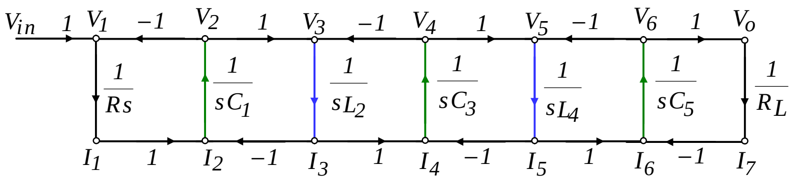
LC Ladder Filter
Signal Flowgraph

$$V_1 = V_{in} - V_2 \quad , \quad V_2 = \frac{I_2}{sC_1} \quad , \quad V_3 = V_2 - V_4$$

$$V_4 = \frac{I_4}{sC_3} \quad , \quad V_5 = V_4 - V_6 \quad , \quad V_6 = \frac{I_6}{sC_5} \quad V_o = V_6$$

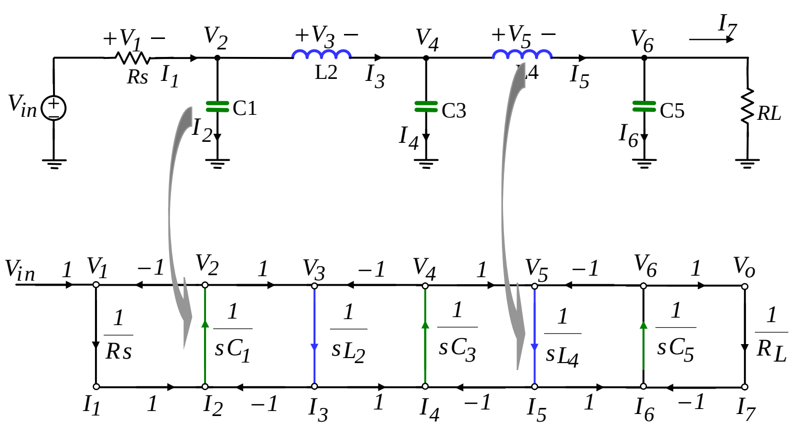
$$I_1 = \frac{V_1}{Rs} \quad , \quad I_2 = I_1 - I_3 \quad , \quad I_3 = \frac{V_3}{sL_2}$$

$$I_4 = I_3 - I_5 \quad , \quad I_5 = \frac{V_5}{sL_4} \quad , \quad I_6 = I_5 - I_7 \quad , \quad I_7 = \frac{V_6}{RL}$$



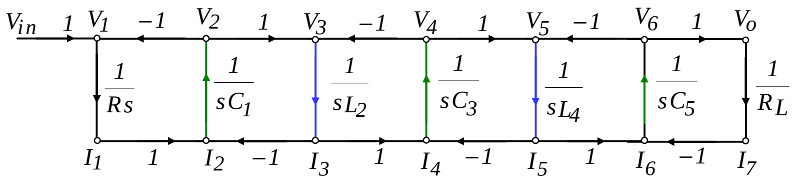
SFG

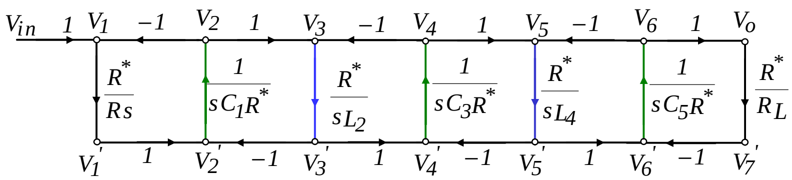
LC Ladder Filter
Signal Flowgraph



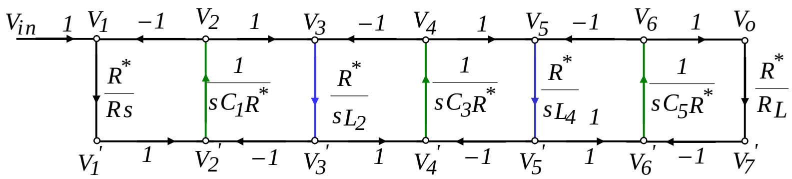
SFG

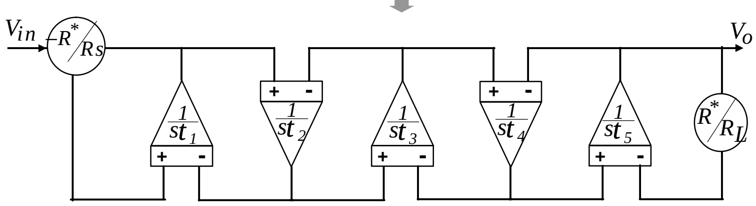
LC Ladder Filter
Normalize



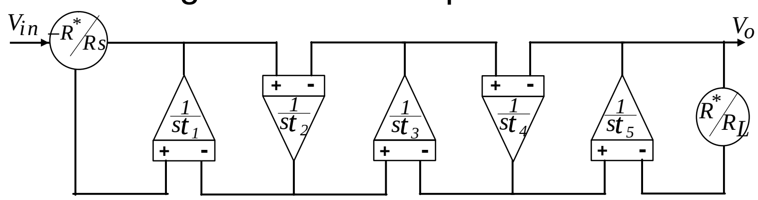


LC Ladder Filter
Synthesize



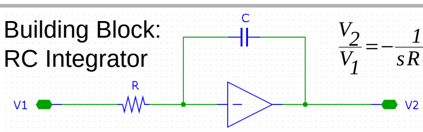


LC Ladder Filter
Integrator Based Implementation



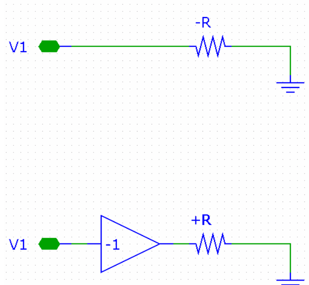
$$t_1 = C_1.R^* \quad , \quad t_2 = \frac{L_2}{R^*} = C_2.R^* \quad , \quad t_3 = C_3.R^* \quad , \quad t_4 = \frac{L4}{R^*} = C_4.R^* \quad , \quad t_5 = C_5.R^*$$

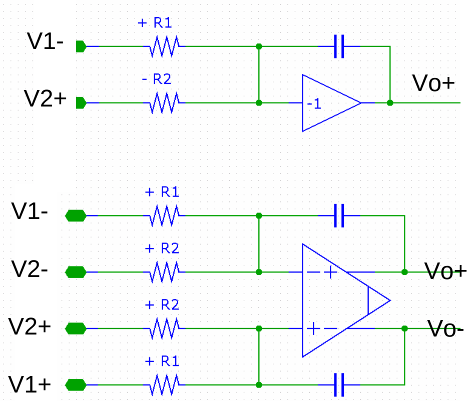
Building Block:
RC Integrator



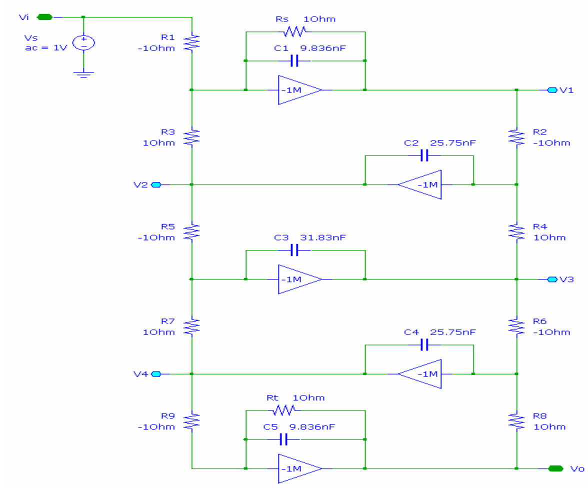
$$\frac{V_2}{V_1} = -\frac{1}{sRC}$$

Negative Resistors

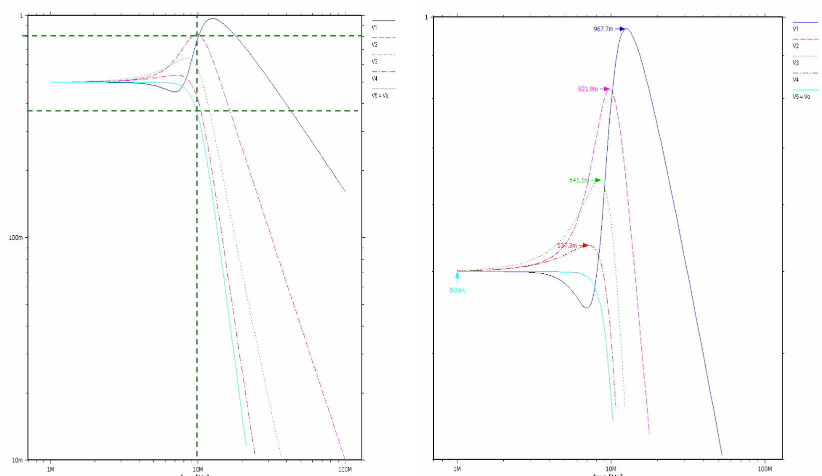




Synthesize

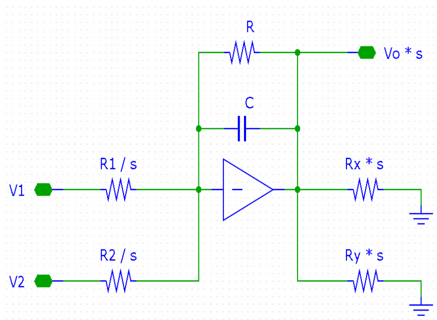


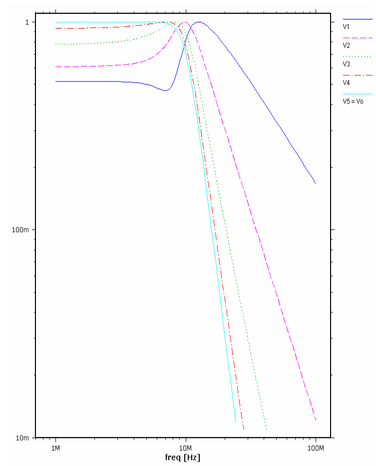
Frequency Response



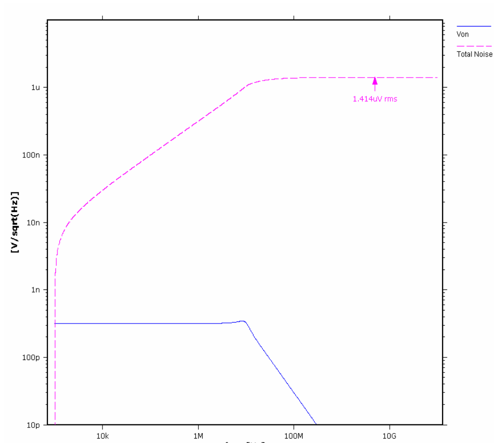
Scale Node Voltages

Scale V_o by factor “s”





Noise



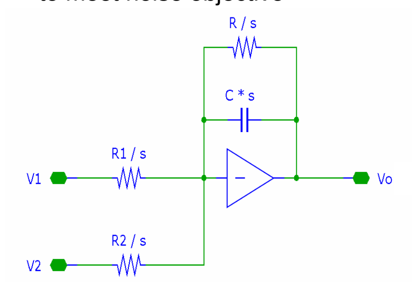
Total noise: 1.4 μV rms (noiseless opamps)

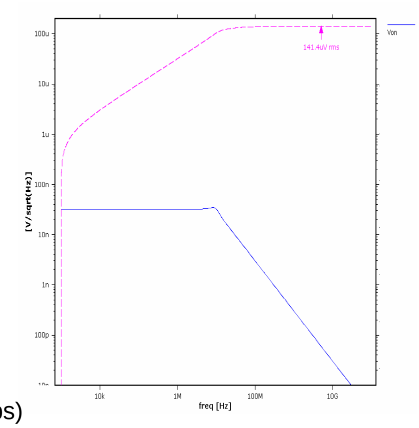
That’s excellent, but the capacitors are very large (and the resistors small). Not possible to integrate.

Suppose our application calls for 140 μV rms …

Scale to Meet Noise Target

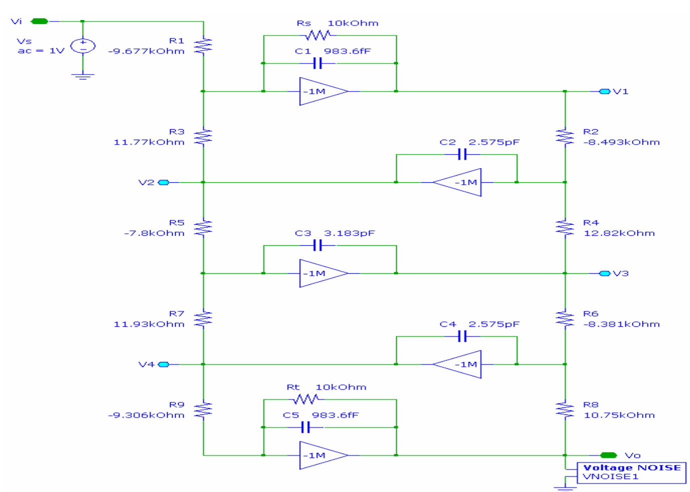
Scale capacitors and resistors
to meet noise objective



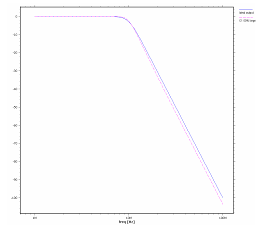


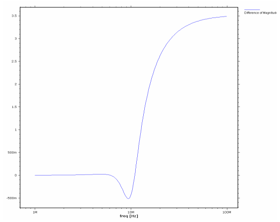
$s = 10^{-4}$
Noise: 141 μV rms (noiseless opamps)

Completed Design



Sensitivity





- C_1 made (arbitrarily) 50% (!) larger than its nominal value
- 0.5 dB error at band edge
- 3.5 dB error in stopband
- Looks like <u>very low sensitivity</u>

