EE247
Lecture 7

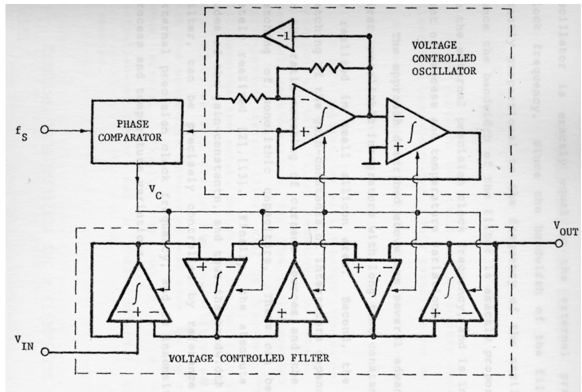
- Summary last lecture
- Continuous-time filters
  - Bandpass filters
    - Example Gm-C BP filter using simple diff. pair
      - Linearity
      - Noise
  - Various Gm-C Filter implementations
  - Comparison of continuous-time filter topologies

Summary last lecture

- Automatic on-chip filter tuning
  - Continuous tuning
    - Master-slave tuning
  - Periodic off-line tuning
    - Systems where filter is followed by ADC & DSP, existing hardware can be used to periodically update filter freq. response

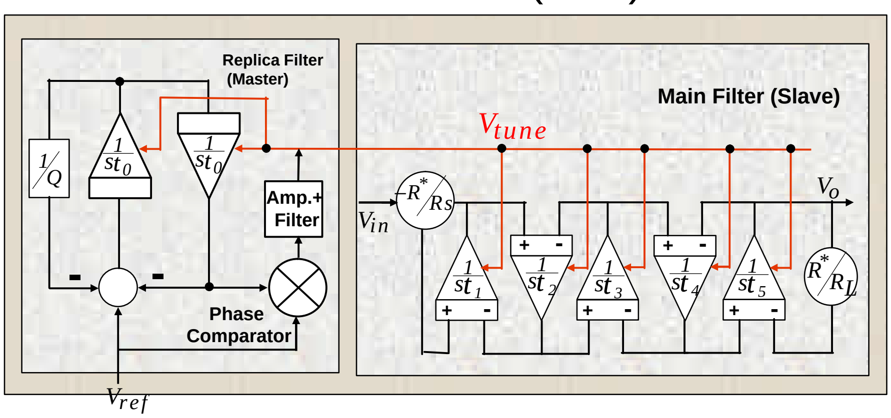
Master-Slave Frequency Tuning
Reference Voltage-Controlled-Oscillator (VCO)

- Instead of VCF a voltage-controlled-oscillator (VCO) is used
- VCO made or replica integrators
- Tuning circuit operates exactly as a conventional phase-locked loop (PLL)
- Tuning signal used to tune main filter



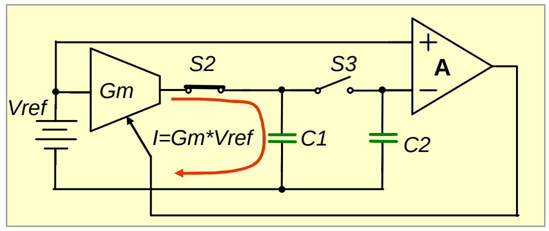
Ref: K.S. Tan and P.R. Gray, "Fully integrated analog filters using bipolar FET technology," IEEE, J. Solid-State Circuits, vol. SC-13, no.6, pp. 814-821, December 1978..

Master-Slave Frequency Tuning
Reference Filter (VCF)

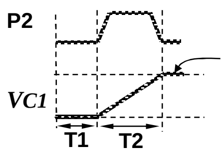


Ref: H. Khorramabadi and P.R. Gray, "High Frequency CMOS continuous-time filters," IEEE Journal of Solid-State Circuits, Vol.-SC-19, No. 6, pp.939-948, Dec. 1984.

Reference C/Gm Locked to Ref. Frequency
P2 high → S2 closed

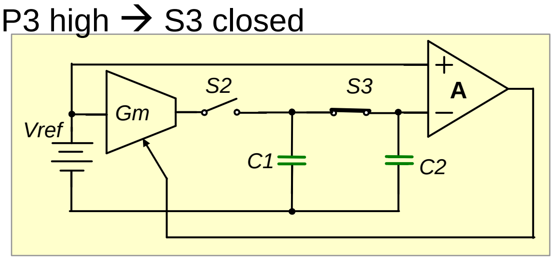


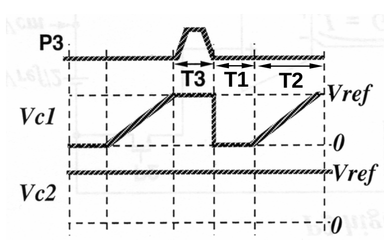
Charge *C1 with I=Gm*Vref*



$$V_{C1} \approx Gm \times V_{ref} \times T2 / C1$$

Reference C/Gm Locked to Ref. Frequency
P3 high → S3 closed





Charge on *C1 shared with C2*
Feedback forces Gm to assume a value:

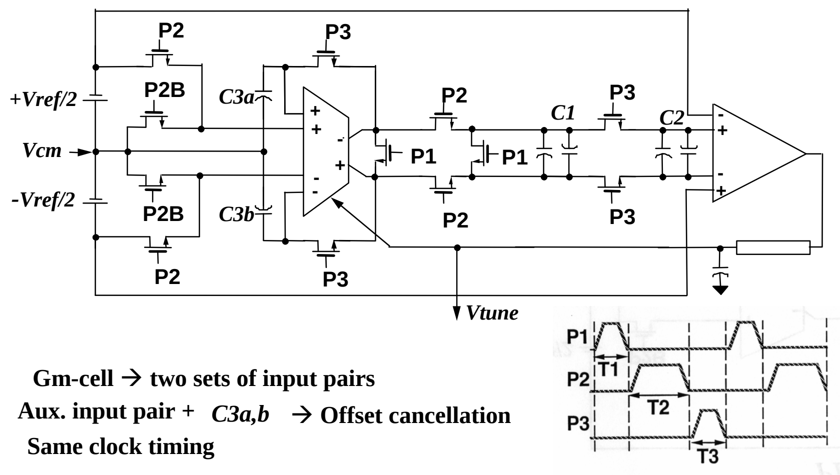
$$V_{C1} = V_{C2} = Vref$$

$$\text{since: } V_{C1} = Gm \times V_{ref} \times T2 / C1$$

$$\text{then: } V_{ref} = Gm \times V_{ref} \times T2 / C1$$

$$\text{or: } \boxed{C1/Gm = T2 = N/fclk}$$

Reference C/Gm Locked to Ref. Frequency Incorporating Offset Cancellation



Gm-cell → two sets of input pairs

Aux. input pair + ***C3a,b*** **→ Offset cancellation**

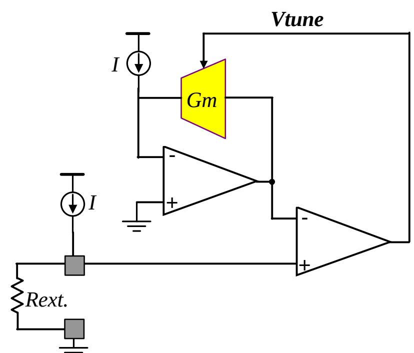
Same clock timing

DC Tuning of Resistive Timing Element

Rext used to lock Gm or on-chip R

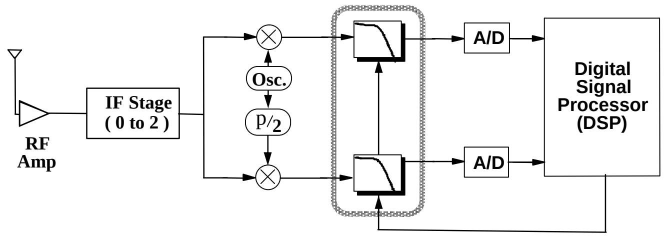
Feedback forces *Gm=1/Rext*

Account for Cap. variations in the gm-C implementation by trimming



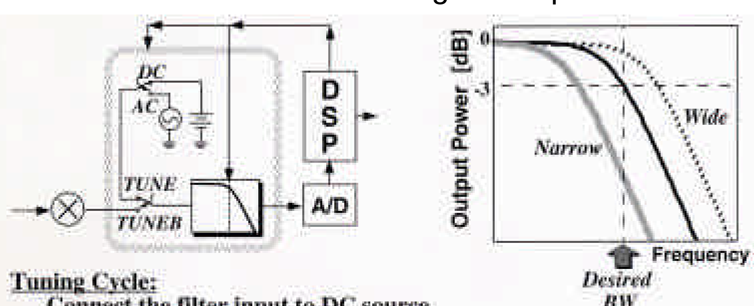
Ref: C. Laber and Gray, "A 20MHz 6th Order BiCOM Parasitic Insensitive Continuous-time Filter and Second Order Equalizer Optimized for Disk Drive Read Channels," *IEEE Journal of Solid State Circuits*, Vol. 28, pp. 462-470, April 1993

Off-line Frequency Tuning
Example:Wireless Receiver Baseband Filters



- Systems where filter is followed by ADC & DSP
  - Take advantage of existing digital signal processor to periodically update the filter critical frequency
  - Filter tuned only at the outset of each data transmission session (off-line tuning)

Offline Filter Tuning Concept



Tuning Cycle:

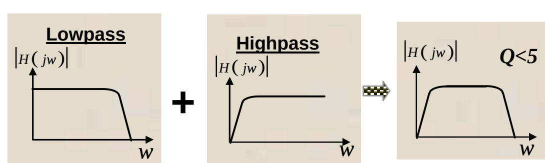
Connect the filter input to DC source
DSP measures the DC power level
Connect the filter input to AC souce (freq. -> desired -3dB freq.)
DSP measures the AC signal power level
If DC = 4*AC
    Then filter is tuned
    Else If DC > 4*AC
        Then widen the filter bandwidth & repeat
        Else narrow the filter bandwidth & repeat

Summary
Filter Frequency Tuning

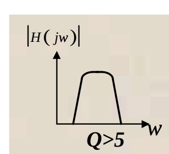
- Trimming
    - Expensive
    - Does not account for variations associated with temperature and supply etc…
- Automatic frequency tuning
    - Continuous tuning
        - Master VCF used in tuning loop
            - Tuning quite accurate
            - Issue → reference signal feedthrough to the filter output
        - Master VCO used in tuning loop
            - Design of reliable & stable VCO challenging
            - Issue → reference signal feedthrough
        - Single integrator in negative feedback loop forces time-constant to be a function of accurate clock frequency
            - More flexibility in choice of reference frequency → less feedthrough issues
        - Locking a replica of the Gm-cell to an external resistor
            - DC offset issues
            - Does not account for integrating capacitor variations
- Periodic tuning
    - Requires digital capability + minimal additional hardware
    - Advantage of no reference signal feedthrough since tuning performed off-line

Bandpass Filters

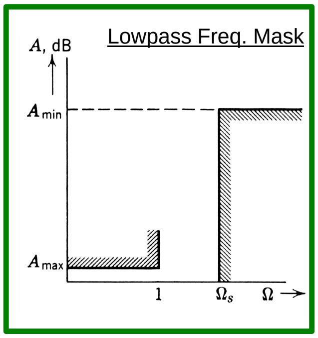
- Bandpass Filters:
    - $Q < 5$ → Combination of lowpass & highpass

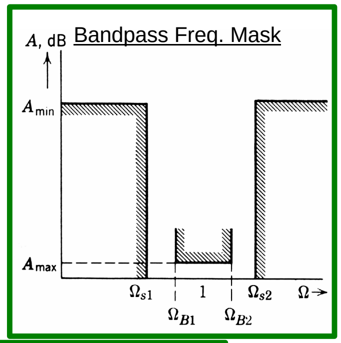


- $Q > 5$ → Direct implementation



Direct Implementation
Narrow Band Bandpass Filters

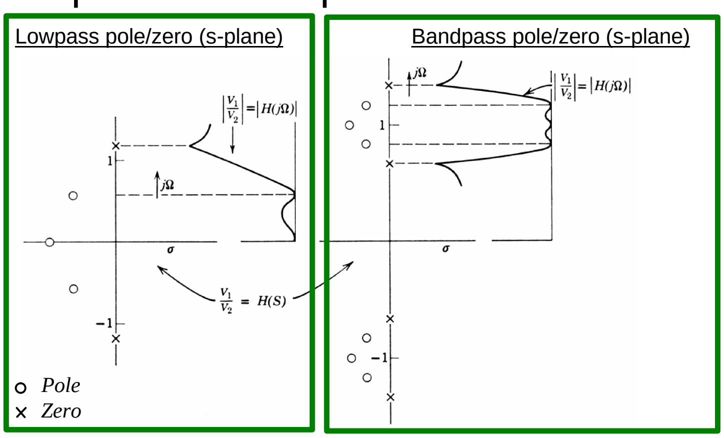




$$s \Longrightarrow = Q \times \left[\frac{s}{w_C} + \frac{w_C}{s} \right] \qquad \frac{\Omega_S}{\Omega_C} = \frac{\Omega_{S2} - \Omega_{S1}}{\Omega_{B2} - \Omega_{B1}}$$

- Design based on lowpass prototype for narrow band filters
- Same lowpass tables used

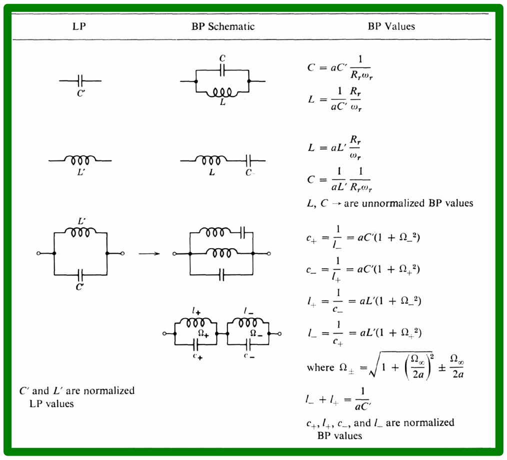
Lowpass to Bandpass Transformation



From: Zverev, *Handbook of filter synthesis*, Wiley, 1967- p.156.

Lowpass to Bandpass Transformation Table

$$a = (\Omega_{B2} - \Omega_{B1})^{-1}$$
$$a = Q_{filter}$$

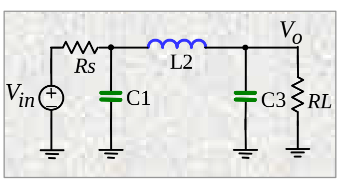


LP	BP Schematic	BP Values
C'	C, L	$C = aC' \frac{1}{R_r \omega_r}$ $L = \frac{1}{aC'} \frac{R_r}{\omega_r}$
L'	L, C	$L = aL' \frac{R_r}{\omega_r}$ $C = \frac{1}{aL'} \frac{1}{R_r \omega_r}$ $L, C \rightarrow$ are unnormalized BP values
L', C'	l_+, Ω_+, c_+, l_-, Ω_-, c_-	$c_+ = \frac{1}{l_-} = aC'(1 + \Omega_-^2)$ $c_- = \frac{1}{l_+} = aC'(1 + \Omega_+^2)$ $l_+ = \frac{1}{c_-} = aL'(1 + \Omega_-^2)$ $l_- = \frac{1}{c_+} = aL'(1 + \Omega_+^2)$ where $\Omega_\pm = \sqrt{1 + \left(\frac{\Omega_\infty}{2a}\right)^2} \pm \frac{\Omega_\infty}{2a}$ $l_- + l_+ = \frac{1}{aC'}$ c_+, l_+, c_-, and l_- are normalized BP values
C' and L' are normalized LP values		

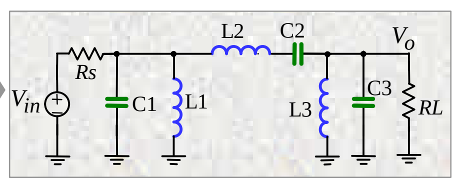
From:
Zverev,
Handbook of filter synthesis,
Wiley, 1967- p.157.

Lowpass to Bandpass Transformation

<u>Lowpass</u>



<u>Bandpass</u>



- Each capacitor replaced by parallel L& C
- Each inductor replaced by series L&C

Lowpass to Bandpass Transformation

$$C_1 = QC_1' \times \frac{1}{Rw_0}$$

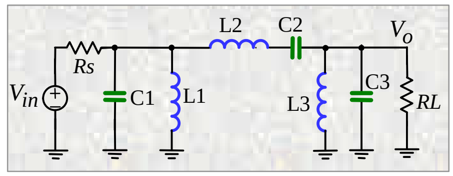
$$L_1 = \frac{1}{QC_1'} \times \frac{R}{w_0}$$

$$C_2 = \frac{1}{QL_2'} \times \frac{1}{Rw_0}$$

$$L_2 = QL_2' \times \frac{R}{w_0}$$

$$C_3 = QC_3' \times \frac{1}{Rw_0}$$

$$L_3 = \frac{1}{QC_3'} \times \frac{R}{w_0}$$



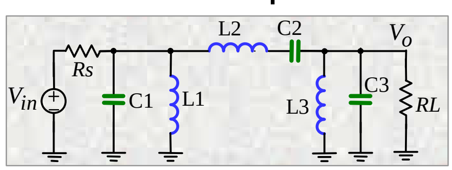
Where:

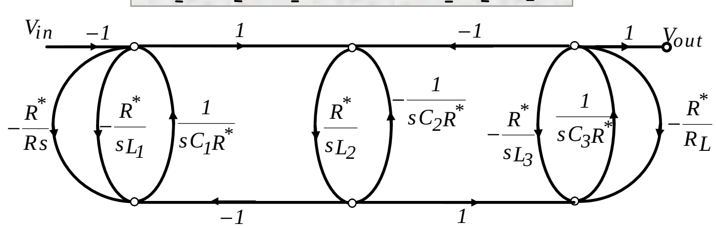
C_1' , L_2' , C_3' , → normalized lowpass values

Q → bandpass filter quality factor &

w_0 → filter center frequency

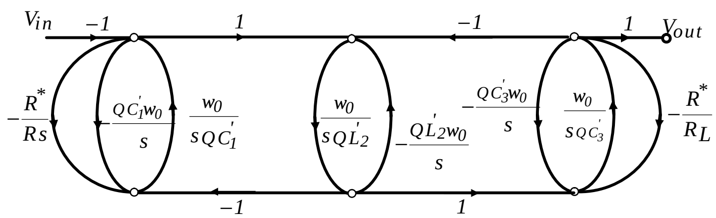
Signal Flowgraph
6th Order Bandpass Filter





Note each C & L in the original lowpass prototype → replaced by a resonator
Substituting the bandpass L1, C1,….. by their normalized lowpass equivalent previous page
The resulting SFG is:

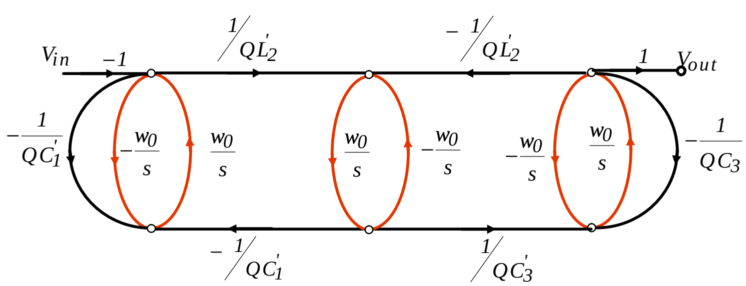
Signal Flowgraph
6^{th} Order Bandpass Filter



•Note the integrators have different time constants
- • Ratio of time constants for each resonator $\sim 1/Q^2$
  - → typically, requires high component ratios
  - → poor matching

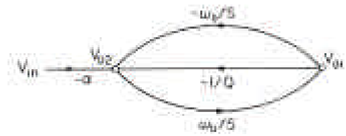
•Desirable to convert SFG so that all integrators have equal time constants for optimum matching.
- •Scale nodes to obtain equal integrator time constant

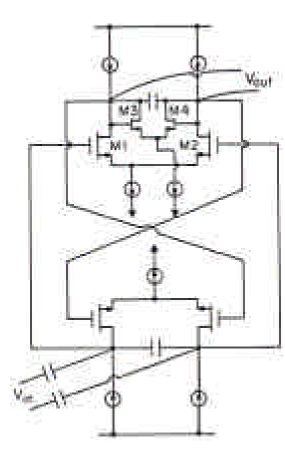
Signal Flowgraph
6^{th} Order Bandpass Filter



•Note: Three resonators
•All integrator time-constants are equal
•Let us try to build this bandpass filter using the simple Gm-C structure

Second Order Gm-C Filter
Using Simple Source-Couple Pair Gm-Cell





- Center frequency:

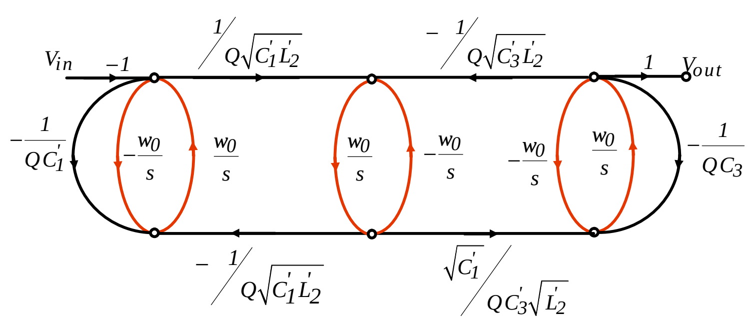
$$w_o = \frac{g_m^{M1,2}}{2 \times C_{intg}}$$

- Q function of:

$$Q = \frac{g_m^{M1,2}}{g_m^{M3,4}}$$

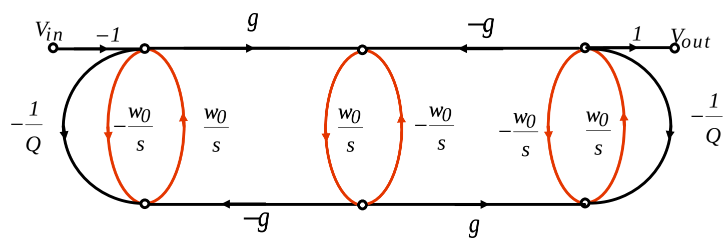
To use this structure it is easier to couple resonators through capacitive coupling

Signal Flowgraph
6th Order Bandpass Filter



Modified signal flowgraph to have equal coupling between resonators
•In most filter cases $C_1' = C_3'$
•Example: For a butterworth lowpass filter $C_1' = C_3' = 1$ & $L_2' = 2$
•Assume overall bandpass filter $Q=10$

Sixth Order Bandpass Filter Signal Flowgraph

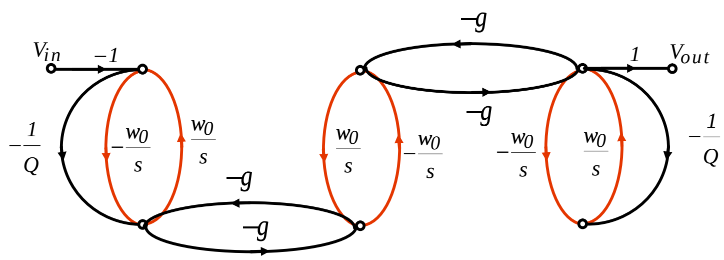


•Where for a Butterworth shape $g = \frac{1}{Q\sqrt{2}}$

•Since $Q=10$ then: $g \approx \frac{1}{14}$

Sixth Order Bandpass Filter Signal Flowgraph

For narrow band filters (high Q) where frequencies within the passband are close to ω_0 narrow-band approximation can be used:
The resulting SFG:



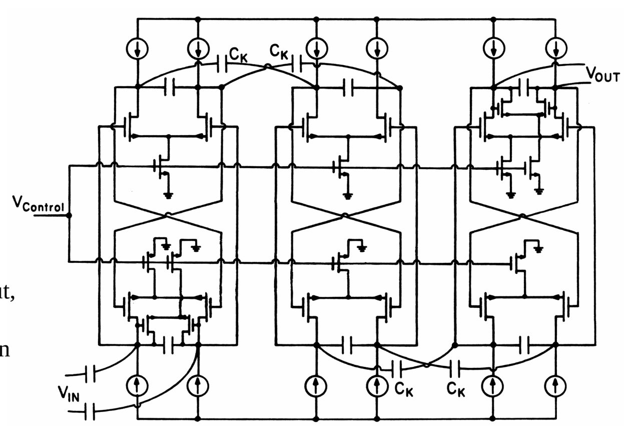
Sixth Order Gm-C Bandpass Filter
Utilizing Simple Source-Coupled Pair Gm-Cell

$$g = \frac{C_k}{2 \times C_{intg}}$$

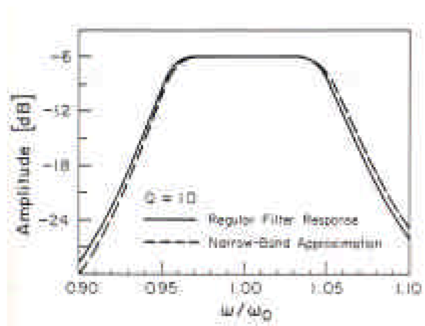
$$g = 1/14$$

$$\rightarrow C_k = \frac{1}{7 \times C_{intg}}$$

Parasitic C at integrator output, if unaccounted for, will result in inaccuracy in g



Sixth Order Gm-C Bandpass Filter
Frequency Response Simulation

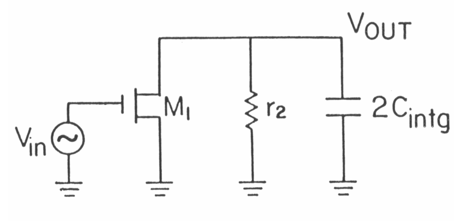


Simplest Form of CMOS Gm-Cell Nonidealities

- DC gain (integrator Q)

$$a = \frac{g_m^{M1,2}}{g_0^{M1,2} + g_{load}}$$

$$a = \frac{2L}{q\left(V_{gs} - V_{th}\right)_{M1,2}}$$



Small Signal Differential Mode Half-Circuit

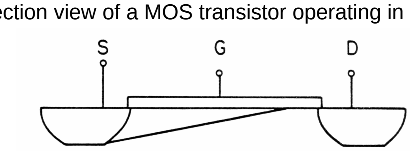
- Where θ is related to channel length modulation by:

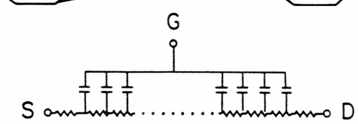
$$l = \frac{q}{L}$$

- Seems no extra poles!

CMOS Gm-Cell High-Frequency Poles

Cross section view of a MOS transistor operating in saturation

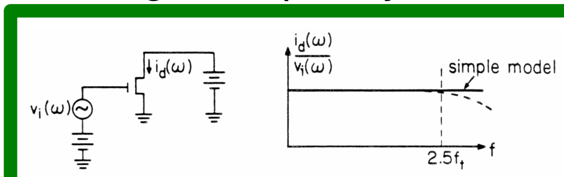




Distributed channel resistance & gate capacitance

- Distributed nature of gate capacitance & channel resistance results in infinite no. of high-frequency poles

CMOS Gm-Cell High-Frequency Poles



High frequency behavior of an MOS transistor

$$P_2^{effective} = \frac{1}{\sum_{i=2}^{\infty}\frac{1}{P_i}}$$

$$\boxed{P_2^{effective} \approx 2.5 w_t^{M1,2}}$$

$$w_t^{M1,2} = \frac{g_m^{M1,2}}{C_{ox}WL} = \frac{3}{2}\frac{m\left(V_{gs}-V_{th}\right)_{M1,2}}{L^2}$$

- Distributed nature of gate capacitance & channel resistance results in an effective pole at 2.5 times input device cut-off frequency

CMOS Gm-Cell Quality Factor

$$a = \frac{2L}{q\left(V_{gs}-V_{th}\right)_{M1,2}}$$

$$P_2^{effective} = \frac{15}{4}\frac{m\left(V_{gs}-V_{th}\right)_{M1,2}}{L^2}$$

$$Q_{real}^{intg.} \approx \frac{1}{\frac{1}{a} - w_o\sum_{i=2}^{\infty}\frac{1}{p_i}}$$

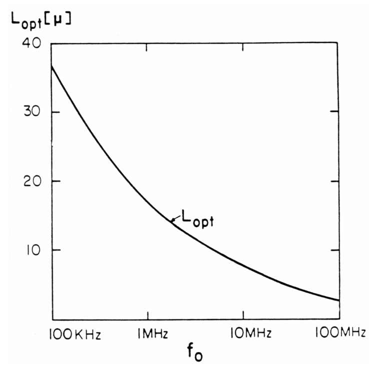
$$\boxed{\frac{1}{Q_{real}^{intg.}} \approx \frac{q\left(V_{gs}-V_{th}\right)_{M1,2}}{2L} - \frac{4}{15}\frac{w_o L^2}{m\left(V_{gs}-V_{th}\right)_{M1,2}}}$$

- Note that the phase lead associated with DC gain is inversely prop. to L
- The phase lag due to high-freq. poles directly prop. to L

  → For a given ω_o there exists an optimum *L* which cancel the lead/lag phase error resulting in high integrator *Q*

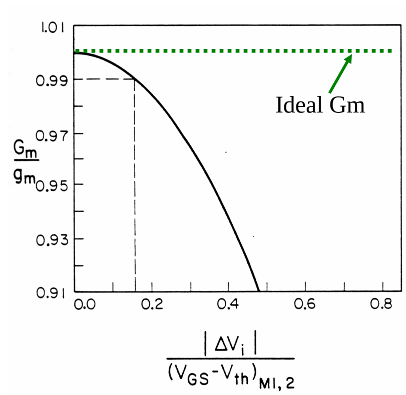
CMOS Gm Cell Channel Length for Optimum Quality Factor

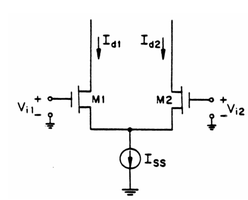
$$L_{opt.} \approx \left[\frac{15}{4} \frac{q m \left(V_{gs} - V_{th}\right)^2_{M1,2}}{w_o} \right]^{1/3}$$



- Optimum channel length computed based on process parameters

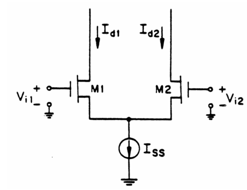
Linearity of the Source-Coupled Pair CMOS Gm-Cell





- Large signal Gm drops as input voltage increases
  → Gives rise to filter nonlinearity

Source-Coupled Pair CMOS Gm-Cell Linearity



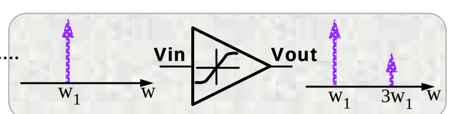
$$\Delta I_d = I_{ss}\left[\frac{\Delta v_i}{\left(V_{gs}-V_{th}\right)_{M1,2}}\right]\left\{1-\frac{1}{4}\left[\frac{\Delta v_i}{\left(V_{gs}-V_{th}\right)_{M1,2}}\right]^2\right\}^{1/2} \qquad (1)$$

$$\Delta I_d = a_1\times\Delta v_i + a_2\times\Delta v_i^2 + a_3\times\Delta v_i^3 + \ldots\ldots\ldots\ldots$$

Series expansion used in (1)

Measure of Linearity

$$Vout = a_1 Vin + a_2 Vin^2 + a_3 Vin^3 + \ldots\ldots\ldots\ldots$$

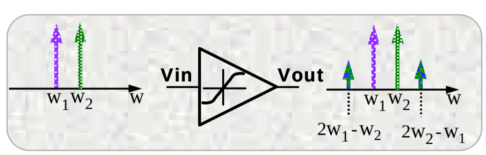


$$HD3 = \frac{amplitude\ 3rd\ harmonic\ dist.\ comp.}{amplitude\ fundamental}$$

$$= \frac{1}{4}\frac{a_3}{a_1}Vin^2 + \ldots\ldots$$

$$IM_3 = \frac{amplitude\ 3rd\ order\ IM\ comp.}{amplitude\ fundamental}$$

$$= \frac{3}{4}\frac{a_3}{a_1}Vin^2 + \frac{25}{8}\frac{a_5}{a_1}Vin^4 + \ldots\ldots$$

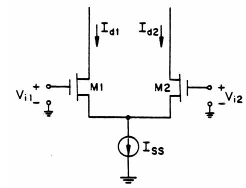


Linearity of the Source-Coupled Pair CMOS Gm-Cell

$$a_1 = \frac{I_{ss}}{(V_{GS} - V_{th})} \qquad a_2 = 0$$

$$a_3 = -\frac{I_{ss}}{8\,(V_{GS} - V_{th})^3} \qquad a_4 = 0$$

$$a_5 = -\frac{I_{ss}}{128\,(V_{GS} - V_{th})^5} \qquad a_6 = 0$$



$$IM3 \approx \frac{3}{4}\frac{a_3}{a_1}\hat{v}_i^{\,2} + \frac{25}{8}\frac{a_5}{a_1}\hat{v}_i^{\,4} + \ldots\ldots$$

$$\boxed{\hat{v}_{i\,max} \approx 4\,(V_{GS} - V_{th})\left[\frac{2}{3}\,IM3\right]^{1/2}}$$

$$IM_3 = 1\% \;\; \& \;\; (V_{GS} - V_{th}) = 1V \;\; \Rightarrow \hat{V}_{in}^{rms} \approx 230mV$$

- Key point: Max. signal handling capability function of gate-overdrive

Simplest Form of CMOS Gm Cell Disadvantages

•Max. signal handling capability function of gate-overdrive

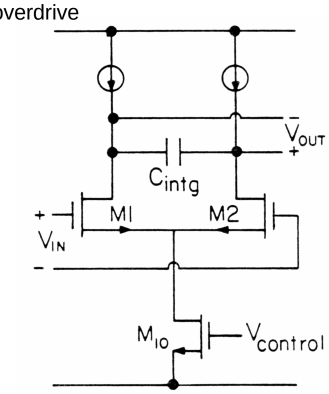
$$IM_3 \propto (V_{GS} - V_{th})^{-2}$$

•Critical freq. function of gate-overdrive too

$$w_o = \frac{g_m^{M1,2}}{2 \times C_{int\,g}}$$

$$\text{since} \quad g_m = mC_{ox}\frac{W}{L}(V_{gs} - V_{th})$$

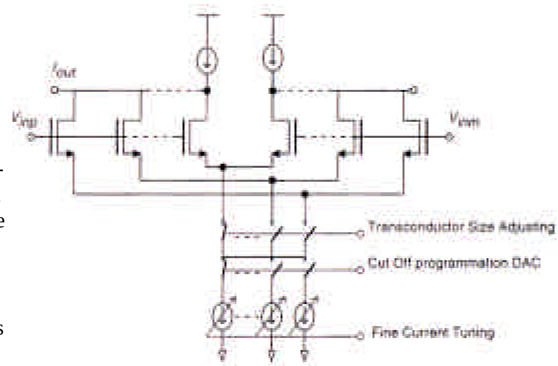
$$\text{then} \quad w_o \propto (V_{gs} - V_{th})$$



→ Filter tuning affects max. signal handling capability!

Simplest Form of CMOS Gm Cell
Removing Dependence of Maximum Signal Handling Capability on Tuning

- Can overcome problem of max. signal handling capability a function of tuning by providing tuning through :
  - Coarse tuning via switching in/out binary-weighted cross-coupled pairs→ Try to keep gate overdrive voltage constant
  - Fine tuning through varying current sources

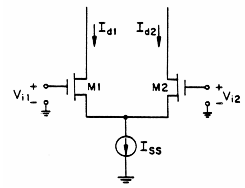


→ Dynamic range dependence on tuning removed (to 1st order)

Ref: R.Castello ,I.Bietti, F. Svelto , "High-Frequency Analog Filters in Deep Submicron Technology , "*International Solid State Circuits Conference*, pp 74-75, 1999.

Dynamic Range for Source-Coupled Pair Based Filter

$$IM_3 = 1\% \;\; \& \;\; (V_{GS} - V_{th}) = 1V \;\; \Rightarrow V_{in}^{rms} \approx 230mV$$



- It can be shown for the 6^{th} order Butterworth bandpass filter noise is given by:

$$\sqrt{v_o^2} \approx \sqrt{3Q\frac{k\,T}{C_{intg}}}$$

$$Assuming \quad Q = 10 \quad C_{intg} = 5pF$$

$$v_{noise}^{rms} \approx 160mV$$
$$since \;\; v_{max}^{rms} = 230mV$$

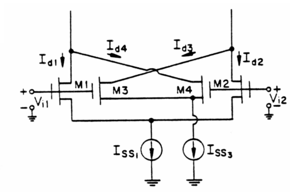
$$Dynamic\;Range \approx 63dB$$

Improving the Max. Signal Handling Capability of the Source-Coupled Pair Gm-Cell

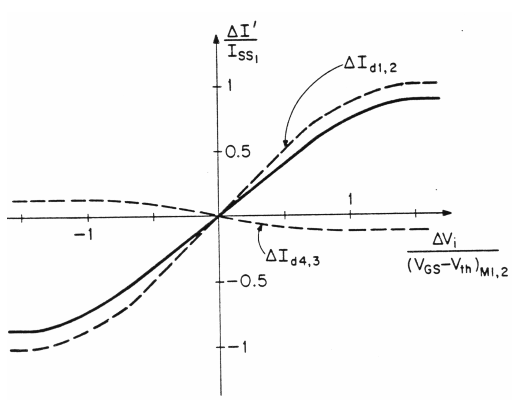
- 2nd source-coupled pair added to subtract current from the main SCP

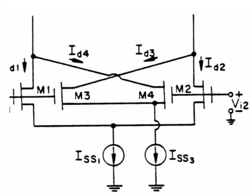
$$\frac{I_{ss\,1}}{I_{ss\,3}} = b \text{ and } \frac{(V_{GS}-V_{th})_{M1,2}}{(V_{GS}-V_{th})_{M3,4}} = a \text{ and thus}$$

$$\frac{(\frac{W}{L})_{M1,2}}{(\frac{W}{L})_{M3,4}} = \frac{b}{a^2}$$



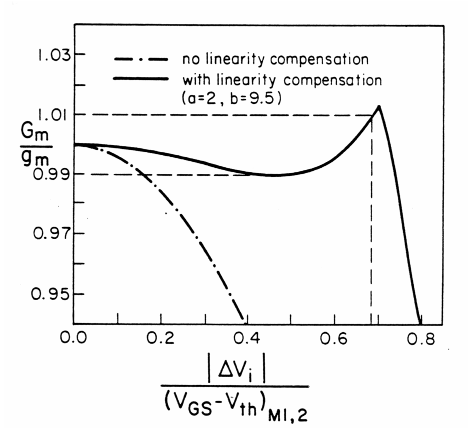
Improving the Max. Signal Handling Capability of the Source-Coupled Pair Gm





Ref: H. Khorramabadi, "High-Frequency CMOS Continuous-Time Filters," U. C. Berkeley, Department of Electrical Engineering, Ph.D. Thesis, February 1985 (ERL Memorandum No. UCB/ERL M85/19).

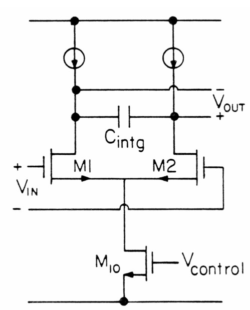
Improving the Max. Signal Handling Capability of the Source-Coupled Pair Gm



- Improves maximum signal handling capability by about 12dB
  -> Dynamic range theoretically improved to 63+12=75dB

Simplest Form of CMOS Gm-Cell

- Pros
  - Capable of very high frequency performance (highest?)
  - Simple design
- Cons
  - Tuning affects power dissipation
  - Tuning affects max. signal handling capability (can overcome)
  - Limited linearity (possible to improve)



Ref: H. Khorramabadi and P.R. Gray, "High Frequency CMOS continuous-time filters," *IEEE Journal of Solid-State Circuits,* Vol.-SC-19, No. 6, pp.939-948, Dec. 1984.

Gm-Cell
Source-Coupled Pair with Degeneration

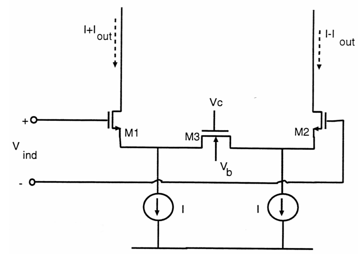
$$I_d = \frac{mC_{ox}}{2}\frac{W}{L}\left[2\left(V_{gs}-V_{th}\right)V_{ds}-V_{ds}^2\right]$$

$$g_{ds} = \frac{\partial I_d}{\partial V_{ds}} \approx mC_{ox}\frac{W}{L}\left(V_{gs}-V_{th}\right)\Big|_{V_{ds}\ small}$$

$$g_{eff} = \frac{1}{\frac{1}{g_{ds}^{M3}}+\frac{2}{g_m^{M1,2}}}$$

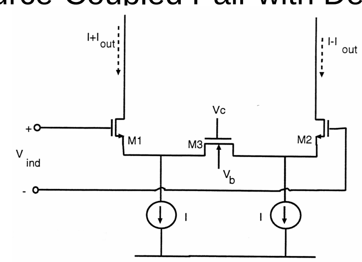
$$for\ g_m^{M1,2} \gg g_{ds}^{M3}$$

$$g_{eff} \approx g_{ds}^{M3}$$



M3 operating in triode mode, determines the gm

Gm-Cell
Source-Coupled Pair with Degeneration



- Pros
  - Moderate linearity
  - Continuous tuning provided by Vc
  - Tuning does not affect power dissipation
- Cons
  - Extra poles associated with the source of M1,2 → Low frequency applications only

Ref: Y. Tsividis, Z. Czarnul and S.C. Fang, "MOS transconductors and integrators with high linearity," *Electronics Letters*, vol. 22, pp. 245-246, Feb. 27, 1986

BiCMOS Gm-Cell

- MOSFET in triode mode:

$$I_d = \frac{mC_{ox}}{2}\frac{W}{L}\left[2\left(V_{gs} - V_{th}\right)V_{ds} - V_{ds}^2\right]$$

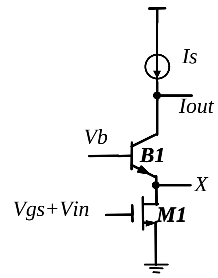
- Note that if Vds is kept constant:

$$g_m = \frac{\partial I_d}{\partial V_{gs}} = mC_{ox}\frac{W}{L}V_{ds}$$

- Linearity performance function of how constant *Vds* can be held
  - Gain @ Node X must be minimized

$$A_x = g_m^{M1} \big/ g_m^{B1}$$

- Since for a given current, *gm* of BJT is larger compared to MOS preferable to have BJT
- Extra pole at node X

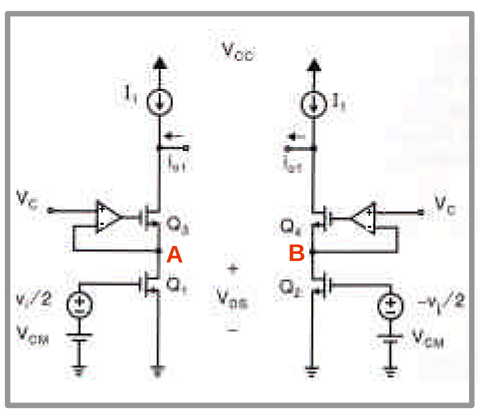


gm can be varied by changing *Vb* and thus *Vds*

CMOS Alternative Gm-Cell

- BJT replaced by a MOS transistor with boosted gm

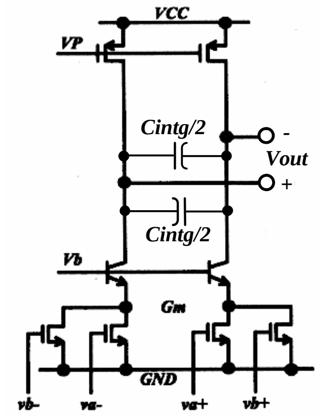
- Lower frequency of operation compared to the BiCMOS version due to more parasitic capacitance at node A & B



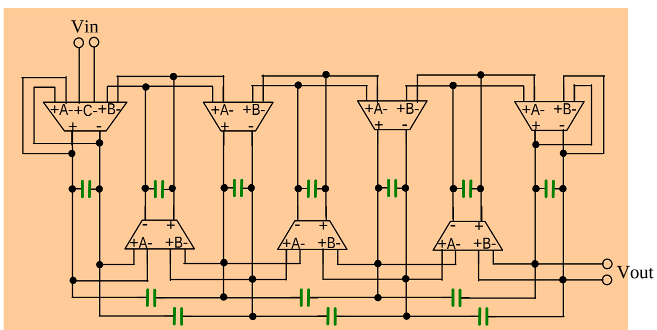
Ref: D.A. Johns, K. Martin "Analog Integrated Circuits" *Wiley*, *1997*, p.606

BiCMOS Gm-C Integrator

- Differential- needs common-mode feedback ckt
- Freq. tuned by varying Vb
- Design tradeoffs:
  - Extra poles at the input device drain junctions
  - Input devices have to be small to minimize parasitic poles
  - Results in high input-referred offset voltage → could drive ckt into non-linear region
  - Small devices → high 1/f noise



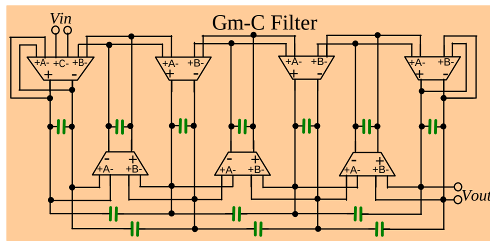
7th Order Elliptic Gm-C LPF For CDMA RX Baseband Application

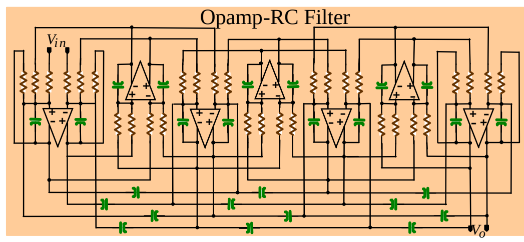


- Gm-Cell in previous page used to build a 7th order elliptic filter for CDMA baseband applications (650kHz corner frequency)
- In-band dynamic range of <50dB achieved

Comparison of 7^{th} Order Gm-C versus Opamp-RC LPF

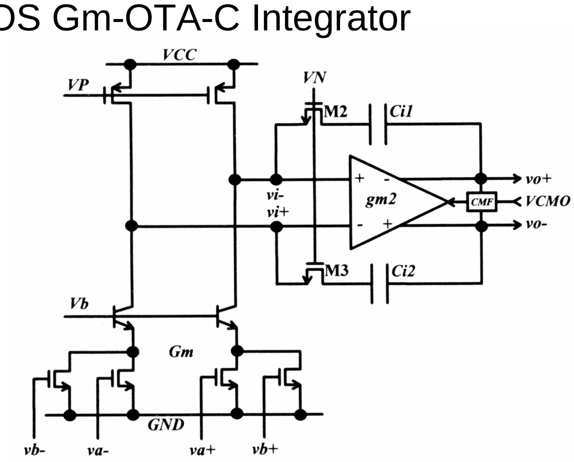
- Gm-C filter requires 4 times less intg. cap. area compared to Opamp-RC
  - → For low-noise applications where filter area is dominated by cap. area could make a significant difference in the total area
- Opamp-RC linearity superior compared to Gm-C
- Power dissipation tends to be lower for Gm-C since output is high impedance and thus no need for buffering





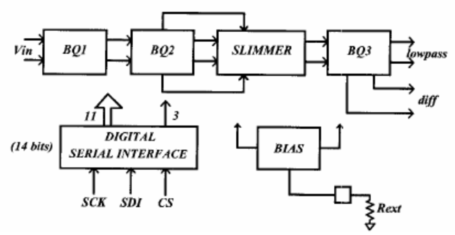
BiCMOS Gm-OTA-C Integrator

- Used to build filter for disk-drive applications
- Since high frequency of operation, time-constant sensitivity to parasitic caps significant.
  - → Opamp used
- M2 & M3 added to compensate for phase lag (provides phase lead)



Ref: C. Laber and P.Gray, "A 20MHz 6th Order BiCMOS Parasitic Insensitive Continuous-time Filter & Second Order Equalizer Optimized for Disk Drive Read Channels," *IEEE Journal of Solid State Circuits*, Vol. 28, pp. 462-470, April 1993.

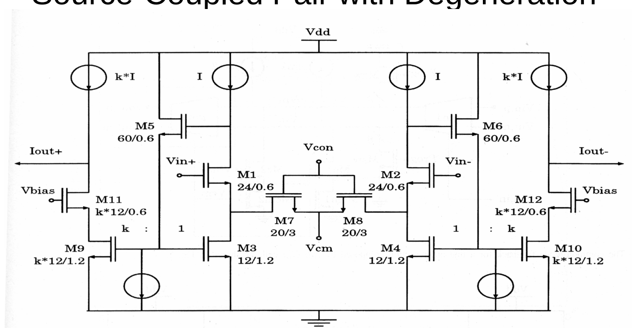
6th Order BiCMOS Continuous-time Filter & Second Order Equalizer for Disk Drive Read Channels



- Gm-C-opamp of the previous page used to build a 6th order filter for Disk Drive
- Filter consists of 3 Biquad with max. Q of 2 each
- Performance in the order of 40dB SNDR achieved for up to 20MHz corner frequency

Ref: C. Laber and P.Gray, "A 20MHz 6th Order BiCMOS Parasitic Insensitive Continuous-time Filter & Second Order Equalizer Optimized for Disk Drive Read Channels," *IEEE Journal of Solid State Circuits*, Vol. 28, pp. 462-470, April 1993.

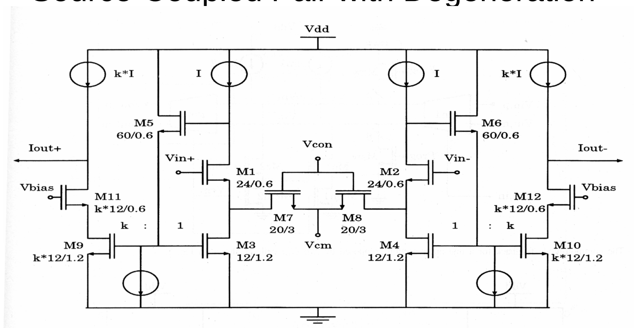
Gm-Cell Source-Coupled Pair with Degeneration



– Gm-cell intended for low Q disk drive filter

Ref: I.Mehr and D.R.Welland, "A CMOS Continuous-Time Gm-C Filter for PRML Read Channel Applications at 150 Mb/s and Beyond", IEEE Journal of Solid-State Circuits, April 1997, Vol.32, No.4, pp. 499-513.

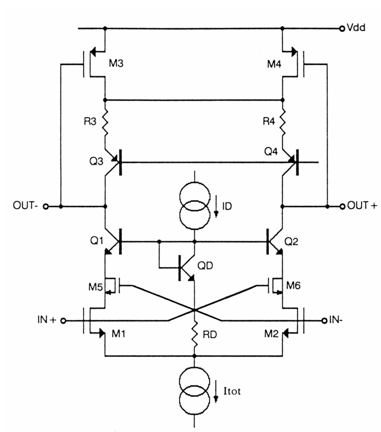
Gm-Cell
Source-Coupled Pair with Degeneration



- M7,8 operating in triode mode determine the gm of the cell
- Feedback provided by M5,6 maintains the gate-source voltage of M1,2 constant by forcing their current to be constant→ helps linearize *rds* of M7,8
- Current mirrored to the output via M9,10 with a factor of k
- Performance level of about 50dB SNDR at fcorner of 25MHz achieved

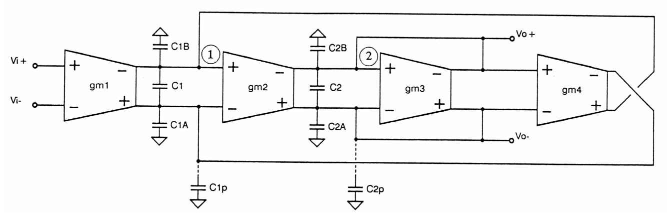
BiCMOS Gm-C Integrator

- Needs higher supply voltage compared to the previous design
- M5 & M6 configured as capacitors added to compensate for RHP zero due to Cgd of M1 & M2 (moves it to LHP) size of M5-6 is 1/3 of M1-2
- Current ID used to tune filter critical frequency
- M3, M4 operate in triode mode and added to provide CMFB



Ref: R. Alini, A. Baschirotto, and R. Castello, "Tunable BiCMOS Continuous-Time Filter for High-Frequency Applications," *IEEE Journal of Solid State Circuits,* Vol. 27, No. 12, pp. 1905-1915, Dec. 1992.

BiCMOS Gm-C Filter For Disk-Drive Application



- Using the integrators shown in the previous page
- Biquad filter for disk drives
- *gm1=gm2=gm4=2gm3*
- *Q=2*
- Tunable from 8MHz to 32MHz

Ref: R. Alini, A. Baschirotto, and R. Castello, "Tunable BiCMOS Continuous-Time Filter for High-Frequency Applications," *IEEE Journal of Solid State Circuits,* Vol. 27, No. 12, pp. 1905-1915, Dec. 1992.

Summary Continuous-Time Filters

- Opamp RC filters
  - Good linearity → High dynamic range (60-90dB)
  - Only discrete tuning possible
  - Medium usable signal bandwidth (<10MHz)
- Opamp MOSFET-C
  - Linearity compromised (typical dynamic range 40-60dB)
  - Continuous tuning possible
  - Low usable signal bandwidth (<5MHz)
- Opamp MOSFET-RC
  - Improved linearity compared to Opamp MOSFET-C (D.R. 50-90dB)
  - Continuous tuning possible
  - Low usable signal bandwidth (<5MHz)
- Gm-C
  - Highest frequency performance (at least an order of magnitude higher compared to the rest <100MHz)
  - Dynamic range not as high as Opamp RC but better than Opamp MOSFET-C (40-70dB)