Lecture 3

- Last time:
  - Imaginary exponentials: simplify the math
  - Phasor: complex "prefactor" for $e^{j\omega t}$
- Today:
  - Complex number review
  - Circuit analysis with phasors

Impedance of a Capacitor

Definition: the impedance $Z$ of a two-terminal circuit element is the ratio of the phasor voltage to the phasor current (positive reference convention)

\[
Z = \frac{V}{I}
\]

Admittance:

\[
Y = \frac{1}{Z} = \frac{1}{jX} = \frac{1}{j\omega C}
\]
Using Phasors: Inductor Voltage

\[ v_L(t) = \text{Re}[L \dot{i}_L e^{j\omega t}] \]

Result:

\[ V_L = (j\omega L) i_L \]

\[ Z_L = j\omega L \]

Inductor Impedance

Admittance:

\[ Y_L = \frac{1}{Z_L} = \frac{1}{j\omega L} \]
Kirchhoff's Current Law Example

At node a:

\[ I_s = I_i \Rightarrow I_a = \frac{V_a - V_b}{Z_a} \]

\[ V_a = V_i \left( 1 - e^{-t/\tau} \right) \]

\[ V_b = \frac{V_i e^{-t/\tau}}{Z_a} \]

\[ Z_a = \frac{1}{R} \]

Resistor path
Circuit Analysis with Phasors

Assumption: sources are sinusoidal, steady-state.

\[ v_s(t) = V_s \cos(\omega t) \]
\[ v_c(t) = V_c \cos(\omega t + \phi) \]
Redrawing the Circuit with Impedances

Note: this is not a "real" circuit that could be built and tested!

Transfer Function

\[ H(j\omega) = \frac{1}{1+j\omega C} \]

Ratio of output to input phasor is called the transfer function of the circuit.
Bode Plots for Low-Pass Filter

1. Plot magnitude $|H|$ in dB vs. $\omega$ (log scale)
2. Plot phase $\angle H$ in degrees vs. $\omega$ (log scale)

$$\angle H = \angle \frac{1}{1 + j\omega C} = \angle \frac{1}{\sqrt{1 + (j\omega R)^2}}$$

Why?
Sketching the Magnitude Plot

\[ |H|_{dB} = 20 \log \left( \frac{1}{1 + j \omega \tau} \right) \]

Low-frequency (\( \omega \tau \ll 1 \)) asymptote

High-frequency (\( \omega \tau \gg 1 \)) asymptote

\[ |H|_{dB} = -20 \log \sqrt{\frac{1}{1 + (\omega \tau)^2}} \]

\[ |H|_{dB} = -20 \log \sqrt{1 + (\omega \tau)^2} \]

The Break Frequency \( \omega_{0.3dB} = \frac{1}{\tau} \)