Logistical Changes and Notes

• Friday Lunch is now Monday lunch (starting next Monday)
  – Email me by Saturday evening if you’d like to come: JHUG aat eecs.berkeley.edu

• My office hours will be Wednesday and Friday, 11:00-12:00, room TBA

• Google calendar with important dates now online

• Did anybody not get my email sent out Monday (that said no discussion yesterday)?

• Will curate the reading a little more carefully next time
Lab/HW Deadlines and Dates

• Discussions start Friday
• Labs start next Tuesday
• HW0 Due Today
• Homework 1 will be posted by 3PM, due Friday at 5 PM
• Tuesday homeworks now due at 2PM, not 5PM in Cory 240 HW box
Summary From Last Time

- **Current** = rate of charge flow
- **Voltage** = energy per unit charge created by charge separation
- **Power** = energy per unit time

**Ideal Basic Circuit Element**
- 2-terminal component that cannot be sub-divided
- Described mathematically in terms of its terminal voltage and current

**Circuit Schematics**
- Networks of ideal basic circuit elements
- Equivalent to a set of algebraic equations
- Solution provides voltage and current through all elements of the circuit
Heating Elements

• Last time we posed a question:
  – Given a fixed voltage, should we pick a thick or thin wire to maximize heat output
  – Note that resistance decreases with wire radius

• Most of you said that we’d want a thin wire to maximize heat output, why is that?
  – Believed that low resistance wire would give the most heat?
  – Didn’t believe me that thick wire has low resistance?
  – General intuition?
Intuitive Answer

• I blasted through some equations and said “thicker is better, Q.E.D.”, but I’m not sure you guys were convinced, so here’s another view

• You can think of a big thick *wire* as a bunch of small *wires* connected to a *source*
  – The thicker the wire, the more little *wires*
  – Since they are all connected directly to the *source*, they all have same voltage and current and hence power
  – Adding more *wires* gives us more total current flow (same voltage), and hence more power
Then Why Don’t Toasters and Ovens Have Thicker Elements?

- Thicker elements mean hotter elements
  - Will ultimately reach higher max temperature
  - Will get to maximum faster [see message board after 6 or 7 PM tonight for why]
- Last time, you guys asked “Well if thickness gives you more heat, why aren’t toaster elements thicker?”
- The answer is most likely:
Toaster Element Design Goals

• Make heating element that can:
  – Can reach a high temperature, but not too high
  – Can reach that temperature quickly
  – Isn’t quickly oxidized into oblivion by high temperature
  – Doesn’t cost very much money
  – Will not melt at desired temperature

• Nichrome is a typical metal alloy in elements:
  – Low oxidation
  – High resistance (so normal gauge wire will not draw too much power and get too hot)

• Size was tweaked to attain desired temperature
Continue the Discussion on BSpace

• Let’s get working on some more complicated circuits than this:
Topic 2

Setting Up and Solving Resistive Circuit Models
Circuit Schematics

• Many circuit elements can be approximated as simple ideal two terminal devices or **ideal basic circuit elements**

• These elements can be combined into **circuit schematics**

• Circuit schematics can be converted into algebraic equations

• These algebraic equations can be solved, giving voltage and current through any element of the circuit
Today

• We’ll enumerate the types of ideal basic circuit elements
• We’ll more carefully define a circuit schematic
• We’ll discuss some basic techniques for analyzing circuit schematics
  – Kirchoff’s voltage and current laws
  – Current and voltage divider
  – Node voltage method
Circuit Elements

• There are 5 ideal basic circuit elements (in our course):
  – voltage source
  – current source
  – resistor
  – inductor
  – capacitor

• Many practical systems can be modeled with just sources and resistors

• The basic analytical techniques for solving circuits with inductors and capacitors are the same as those for resistive circuits
Electrical Sources

• An *electrical source* is a device that is capable of converting non-electric energy to electric energy and *vice versa*.

  Examples:
  – battery: chemical $\leftrightarrow$ electric
  – dynamo (generator/motor): mechanical $\leftrightarrow$ electric

$\rightarrow$ Electrical sources can either deliver or absorb power.
The Big Three

Constant current, unknown voltage
Circuit Schematics

- A circuit schematic is a diagram showing a set of interconnected circuit elements, e.g.
  - Voltage sources
  - Current sources
  - Resistors
- Each element in the circuit being modeled is represented by a symbol
- Lines connect the symbols, which you can think of as representing zero resistance wires
**Terminology: Nodes and Branches**

**Node:** A point where two or more circuit elements are connected – *entire wire*

Can also think of as the “vertices” of our schematic.
Terminology: Nodes and Branches

**Branch**: A path that connects exactly two nodes

![Diagram of an electrical circuit with labeled branches and nodes]

- **Branch**: Not a branch
Terminology: Loops

• A **loop** is formed by tracing a closed path in a circuit through selected basic circuit elements without passing through any intermediate node more than once.

• Example: (# nodes, # branches, # loops)

![Diagram of a circuit with nodes, branches, and loop annotations](image-url)

6 nodes
7 branches
3 loops
Kirchhoff’s Laws

- **Kirchhoff’s Current Law (KCL):**
  - The algebraic sum of all the currents at any node in a circuit equals zero.
  - “What goes in, must come out”
  - Basically, law of charge conservation
Using Kirchhoff’s Current Law (KCL)

Often we’re considering unknown currents and only have reference directions:

\[ i_1 + i_2 = i_3 + i_4 \]

or

\[ i_1 + i_2 - i_3 - i_4 = 0 \]

or

\[ -i_1 - i_2 + i_3 + i_4 = 0 \]

• Use reference directions to determine whether reference currents are said to be “entering” or “leaving” the node – with no concern about actual current directions
KCL Example

\[5 + (-10) = 15 + i\]

\[i = -20 \text{mA}\]
A Major Implication of KCL

• KCL tells us that all of the elements along a single uninterrupted* path carry the same current.

• We say these elements are connected in series.

\[
i_1 = i_2
\]

*: To be precise, by uninterrupted path I mean all branches along the path connected EXACTLY two nodes.
Generalization of KCL

- The sum of currents entering/leaving a **closed surface** is zero. Circuit branches can be inside this surface, *i.e.* the surface can enclose more than one node!

This could be a big chunk of a circuit, *e.g.* a “black box”
Generalized KCL Examples

\[ i = 50 \text{ mA} \]

\[ 7 \mu A \]

\[ 2 \mu A \]

\[ 5 \mu A \]
Kirchhoff’s Laws

- **Kirchhoff’s Voltage Law (KVL):**
  - The algebraic sum of all the voltages around any loop in a circuit equals zero.
  - “What goes up, must come down”

\[ 80 = 20 + 50 + 10 \]
A Major Implication of KVL

- KVL tells us that any set of elements which are connected at both ends carry the same voltage.
- We say these elements are connected in parallel.

Applying KVL, we have that:

\[ v_b - v_a = 0 \implies v_b = v_a \]
KVL Example

Three closed paths:

Path 1: \( V_a = V_2 + V_b \)

If you want a mechanical rule:

Path 2: \( V_b + V_3 = V_c \)

If you hit a – first, LHS
If you hit a + first, RHS

Path 3: \( V_a + V_3 = V_2 + V_c \)

LHS is left hand side
An Underlying Assumption of KVL

- No time-varying magnetic flux through the loop
  Otherwise, there would be an induced voltage (Faraday’s Law)
  Voltage around a loop would sum to a nonzero value

- **Note**: Antennas are designed to “pick up” electromagnetic waves; “regular circuits” often do so undesirably.

How do we deal with antennas (EECS 117A)?
Include a voltage source as the circuit representation of the induced voltage or “noise”.
(Use a lumped model rather than a distributed (wave) model.)
Mini-Summary

• KCL tells us that all elements on an uninterrupted path have the same current.
  – We say they are “in series”

• KVL tells us that a set of elements whose terminals are connected at the same two nodes have the same voltage.
  – We say they are “in parallel”
Nonsense Schematics

• Just like equations, it is possible to write nonsense schematics:
  – $1 = 7$

• A schematic is nonsense if it violates KVL or KCL
Verifying KCL and KVL

Is this schematic valid? Yes

How much power is consumed/provided by each source?

Voltage source: \( P_v = 5V \times 20A = 100W \) (consumed)

Current source: \( P_i = -20A \times 5V = 100W \) (provided)
Verifying KCL and KVL

Is this valid? Yes

KCL:
- Top left node: $I_{100} = 10A$
- Top right node: $10A = 5A + 5A$
- Bottom node: $5A + 5A = I_{100}$

No contradiction

KVL:
- Left loop: $100V = V_{10} + V_5$
- Right loop: $V_5 = V_5$
- Big loop: $100V = V_{10} + V_5$

No contradiction
Verifying KCL and KVL

Is this valid?  Yes

KCL:
Top left node: \( I_{100} = 10A \)

KVL:
Left loop: \( 100V = V_{10} + V_5 \)

So what are \( V_{10} \) and \( V_5 \)?
Whatever we want that sums to 100V
Multiple circuit solutions
iClicker #1

- Are these interconnections permissible?

A. Both are bad
B. Left is ok, right is bad
C. Left is bad, right is ok
D. Both are ok
On to Solving Circuits

• Next we’ll talk about a general method for solving circuits
  – The book calls this the “basic method”
  – It’s a naïve way of solving circuits, and is way more work than you need
    • Basic idea is to write every equation you can think of to write, then solve
  – However, it will build up our intuition for solving circuits, so let’s start here
Solving Circuits (naïve way)

- Label every branch with a reference voltage and current
  - If two branches are in parallel, share voltage label
  - If in series, share same current label
- For each branch:
  - Write Ohm’s law if resistor
  - Get branch voltage “for free” if known voltage source
  - Get branch current “for free” if known current source
Solving Circuits (naïve way)

- Label every branch with a reference voltage and current
  - If two branches are in parallel, share voltage label
  - If in series, share same current label
- For each branch:
  - Write Ohm’s law if resistor
  - Get branch voltage “for free” if known voltage source
  - Get branch current “for free” if known current source
- For each node touching at least 2 reference currents:
  - Write KCL – gives reference current relationships
  - Can omit nodes which contain no new currents
- For each loop:
  - Write KVL – gives reference voltage relationships
  - Can omit loops which contain no new voltages

Could also call this the “kitchen sink” approach
Example: KCL and KVL applied to circuits

- Find the current through the resistor
- Use KVL, we see we can write:

\[
\begin{align*}
V_1 &= V_R + V_2 \\
V_1 &= 5V \\
V_2 &= 3V \\
I_R &= \frac{V_R}{20\Omega} \\
\text{4 equations} \\
\text{4 “unknowns”}
\end{align*}
\]

Now solving, we have:

\[
\begin{align*}
5V &= V_R + 3V \\
2V &= V_R \\
I_R &= \frac{2V}{20\Omega} = 0.1 \text{ Amps}
\end{align*}
\]

Note: We had no node touching 2 ref currents, so no reference current relationships
Bigger example

Branches:
\[ V_1 = i_a \times 80 \Omega \]
\[ V_{30} = 1.6A \times 30 \Omega \]
\[ V_g = 1.6A \times 90 \Omega \]

Two nodes which touch two different reference currents:
\[ i_g = i_a + 1.6 \]
\[ i_a + 1.6 = i_g \quad \text{[no new currents]} \]

Three loops, but only one needed to touch all voltages:
\[ V_1 = V_{30} + V_g \]
\[ V_{30} = 48V \]
\[ V_g = 144V \]
\[ V_1 = 192V \]
\[ i_a = 2.4A \]
\[ i_g = 4A \]

5 equations
5 unknowns
How many KCL and KVL equations will we need to cover every branch voltage and branch current?

2 KVL, 1 KCL

Top node: \( I_1 = I_2 + I_3 \)

Bottom node: \( I_3 + I_2 = I_1 \)
There are better ways to solve circuits

• The kitchen sink method works, but we can do better
  – Current divider
  – Voltage divider
  – Lumping series and parallel elements together (circuit simplification)
  – Node voltage
Voltage Divider

• Voltage divider
  – Special way to handle N resistors in series
  – Tells you how much voltage each resistor consumes
  – Given a set of N resistors $R_1, \ldots, R_k, \ldots, R_N$ in series with total voltage drop $V$, the voltage through $R_k$ is given by

\[
V_k = V \frac{R_k}{R_1 + R_2 + \cdots + R_k + \cdots + R_n}
\]

Or more compactly:

\[
V_k = \frac{VR_k}{\sum_{i=1}^{N} R_i}
\]

Can prove with kitchen sink method (see page 78)
Voltage Divider Example

\[ V_k = V \frac{R_k}{R_1 + R_2 + \ldots + R_k + \ldots R_n} \]

\[ V_{85} = 100V \frac{85\Omega}{5\Omega + 85\Omega + 10\Omega} \]

\[ V_{85} = 100V \frac{85\Omega}{100\Omega} \]

\[ V_{85} = 85V \]

And likewise for other resistors
Current Divider

• Current divider
  – Special way to handle N resistors in parallel
  – Tells you how much current each resistor consumes
  – Given a set of N resistors \( R_1, \ldots, R_k, \ldots, R_N \) in parallel with total current \( I \) the current through \( R_k \) is given by

\[
I_k = I \frac{G_k}{G_1 + G_2 + \cdots + G_k + \cdots G_n}
\]

Where:

\[
G_p = \frac{1}{R_p}
\]

We call \( G_p \) the conductance of a resistor, in units of Mhos (℧)

-Sadly, not units of Shidnevacs (づ ͡° ͜ʖ ͡°)

Can prove with kitchen sink method (see http://www.elsevierdirect.com/companions/9781558607354/casestudies/02~Chapter_2/Example_2_20.pdf)
Current Divider Example

5Ω

10Ω

5Ω

2Ω

20A

Conductances are:

\[
\frac{1}{5Ω}=0.2 \, Ω
\]

\[
\frac{1}{10Ω}=0.1 \, Ω
\]

\[
\frac{1}{5Ω}=0.2 \, Ω
\]

\[
\frac{1}{2Ω}=0.5 \, Ω
\]

Sum of conductances is 1Ω (convenient!)

Current through 5Ω resistor is:

\[
I_2 = 20A \cdot \frac{0.2}{1} = 4A
\]
Circuit Simplification

• Next we’ll talk about some tricks for combining multiple circuit elements into a single element
  • Many elements in series → One single element
  • Many elements in parallel → One single element
Circuit Simplification Example Combining Voltage Sources

• KVL trivially shows voltage across resistor is 15 V
• Can form equivalent circuit as long as we don’t care about individual source behavior
  – For example, if we want power provided by each source, we have to look at the original circuit
Example – Combining Resistances

• Can use kitchen sink method or voltage divider method to show that current provided by the source is equivalent in the two circuits below.
Source Combinations

• Voltage sources in series combine additively
• Voltage sources in parallel
  – This is like crossing the streams – “Don’t cross the streams”
  – Mathematically nonsensical if the voltage sources are not exactly equal
• Current sources in parallel combine additively
• Current sources in series is bad if not the same current
Resistor Combinations

- Resistors in series combine additively
  \[ R_{eq} = R_1 + R_2 + \cdots + R_N \]

- Resistors in parallel combine weirdly
  \[ \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n} \]
  
  - More natural with conductance:
    \[ G_{eq} = G_1 + G_2 + \cdots + G_n \]

- N resistors in parallel with the same resistance R have equivalent resistance
  \[ R_{eq} = \frac{R}{N} \]
Algorithm For Solving By Combining Circuit Elements

• Check circuit diagram
  – If two or more elements of same type in series
    • Combine using series rules
  – If two or more elements of same type in parallel
    • Combine using parallel rules
• If we combined anything, go back to
• If not, then solve using appropriate method (kitchen sink if complicated, divider rule if possible)
Using Equivalent Resistances

Example: Find $I$

Are there any circuit elements in parallel?
No!

Are there any circuit elements in series?
Yes!

\[ \begin{align*}
30 \text{ V} & \\
15 & \\
15 & \\
10 & \\
40 & \\
50 & \\
\end{align*} \]

\[ I \rightarrow \]
Using Equivalent Resistances

Example: Find $I$

Are there any circuit elements in parallel?

Yes!

Are there any circuit elements in series?

Yes!
Example: Find $I$

Are there any circuit elements in parallel?

Yes!

Are there any circuit elements in series?

No!
Using Equivalent Resistances

Example: Find $I$

Are there any circuit elements in parallel?
Yes!

Are there any circuit elements in series?
No!
Using Equivalent Resistances

Example: Find $I$

Are there any circuit elements in parallel?
No!

Are there any circuit elements in series?
Yes!
Example: Find $I$

$\begin{array}{c}
30 \text{ V} \\
\hline
30 \Omega
\end{array}$

$\frac{30 \text{ V}}{30 \Omega} = 1 \text{ A}$

Are there any circuit elements in parallel?

No!

Are there any circuit elements in series?

No!
Working Backwards

• Assume we’ve combined several elements to understand large scale behavior

• Now suppose we want to know something about one of those circuit elements that we’ve combined
  – For example, current through a resistor that has been combined into equivalent resistance

• We undo our combinations step by step
  – At each step, use voltage and current divider tricks
  – Only undo enough so that we get the data we want
Suppose we want to know the voltage across the 40Ω Resistor.
Using Equivalent Resistances

I=1 Amp

Starting from here…

\[ V = 30 \text{ V} \]
Using Equivalent Resistances

I = 1 Amp

\[ V_{25} = I \times 25 \Omega = 25V \]

We back up one step...

Then another...
Using Equivalent Resistances

I=1 Amp

We back up one step…
\[ V_{25} = I \times 25 \Omega = 25\text{V} \]

Then another…

Then one more…

\[ V_{25} \]
Using Equivalent Resistances

I = 1 Amp

$V_{25} = I \times 25 \Omega = 25V$

Then another...

We back up one step...

Then one more...

Now we can use the voltage divider rule, and get

$$V_{40} = \frac{40\Omega}{10\Omega + 40\Omega} \times 25V = 20V$$
Using Equivalent Resistances

\[ I = 1 \text{ Amp} \]

\[ V_{25} = I \times 25 \Omega = 25 \text{V} \]

We back up one step…

Then another…

Then one more…

Now we can use the voltage divider rule, and get

\[ V_{40} = \frac{40 \Omega}{10 \Omega + 40 \Omega} \times 25 \text{V} = 20 \text{V} \]
Equivalent Resistance Between Two Terminals

• We often want to find the equivalent resistance of a network of resistors with no source attached

\[ R_{eq} = 10 \Omega \]

• Tells us the resistance that a hypothetical source would “see” if it were connected

  • e.g. In this example, the resistance that provides the correct source current
Equivalent Resistance Between Two Terminals

- Pretend there is a source of some kind between the circuits
- Perform the parallel/series combination algorithm as before

\[
\begin{align*}
10\Omega & \quad 10\Omega \\
10\Omega & \quad 10\Omega \\
10\Omega & \quad 10\Omega
\end{align*}
\]

\[
\begin{align*}
10\Omega & \quad 5\Omega \\
10\Omega & \quad 10\Omega
\end{align*}
\]

\[
25\Omega
\]
Can Pick Other Pairs of Terminals

These resistors do nothing (except maybe confuse us)

Combine these parallel resistors

5Ω
There are better ways to solve circuits

• The kitchen sink method works, but we can do better
  – Current divider
  – Voltage divider
  – Lumping series and parallel elements together (circuit simplification)
  – Node voltage
The Node Voltage Technique

• We’ll next talk about a general technique that will let you convert a circuit schematic with N nodes into a set of N-1 equations

• These equations will allow you to solve for every single voltage and current

• Works on any circuit, linear or nonlinear!

• Much more efficient than the kitchen sink
Definition: Node Voltage and Ground Node

- Remember that voltages are always defined in terms of TWO points in a circuit.
- It is convenient to label one node in our circuit the “Ground Node”
  - Any node can be “ground”, it doesn’t matter which one you pick.
- Once we have chosen a ground node, we say that each node has a “node voltage”, which is the voltage between that node and the arbitrary ground node.
- Gives each node a universal single valued voltage level.
Node Voltage Example

- Pick a ground, say the bottom left node.
- Label nodes a, b, c, d. Node voltages are:
  - $V_d$ = voltage between node d and d=0V
  - $V_c$ = voltage between node c and d=$V_{10}$=20V
  - $V_b$ = voltage between node b and d=$V_{85}+V_{10}$=190V
  - $V_a$ = voltage between node a and d=200V

\[\begin{align*}
V_5 &= 10V \\
V_{85} &= 170V \\
V_{10} &= 20V
\end{align*}\]
What is $V_a$?

A. 200V  
B. 20V  
C. 160V  
D. 180V

$V_a = V_5 + V_{85} = 180V$
Relationship: Node and Branch Voltages

- Node voltages are useful because:
  - The branch voltage across a circuit element is simply the difference between the node voltages at its terminals
  - It is easier to find node voltages than branch voltages

Example:

\[ V_{85} = V_b - V_c = 190V - 20V = 170V \]
Why are Node Voltages Easier to Find?

- KCL is easy to write in terms of node voltages
- For example, at node a:
  - $4A = \frac{V_a}{80\Omega} + \frac{(V_a - V_b)}{30\Omega}$
- And at node b:
  - $\frac{(V_b - V_a)}{30\Omega} = \frac{V_b}{90\Omega}$
- Well look, two equations, two unknowns. We’re done.
- Better than 5 equations, 5 unknowns with kitchen sink method
(Almost) The Node Voltage Method

- Assign a ground node
- For every node except the ground node, write the equation given by KCL in terms of the node voltages
  - Be very careful about reference directions
- This gives you a set of N-1 linearly independent algebraic equations in N-1 unknowns
  - Solvable using whatever technique you choose
What about Voltage Sources?

• Suppose we have the circuit below

• When we try to write KCL at node a, what happens?

• How do we get around this?
  – Write fixed node voltage relationship:
    \[ V_a = V_d + 200 \]
Full Node Voltage Method

• Assign a ground node
• For every node (except the ground node):
  – If there is no voltage source connected to that node, then write the equation given by KCL in terms of the node voltages
  – If there is a voltage source connecting two nodes, write down the simple equation giving the difference between the node voltages
  – Be very careful about reference directions (comes with practice)
• This gives you a set of N-1 linearly independent algebraic equations in N-1 unknowns
• Solvable using whatever technique you choose

More Examples Next Time!
Next Class

• Node voltage practice and examples
• Why we are bothering to understand so deeply the intricacies of purely resistive networks
  – Things we can build other than the most complicated possible toaster
• How we actually go about measuring voltages and currents
• More circuit tricks
  – Superposition
  – Source transformations
Quick iClicker Question

• How was my pacing today?
  A. Way too slow
  B. A little too slow
  C. Pretty good
  D. Too fast
  E. Way too fast
Summary (part one)

- There are five basic circuit elements
  - Voltage Sources
  - Current Sources
  - Resistors
  - Capacitors
  - Inductors
- Circuit schematics are a set of interconnect ideal basic circuit elements
- A connection point between elements is a node, and a path that connects two nodes is a branch
- A loop is a path around a circuit which starts and ends at the same node without going through any circuit element twice
Summary (part two)

- Kirchoff’s current law states that the sum of the currents entering a node is zero.
- Kirchoff’s voltage law states that the sum of the voltages around a loop is zero.
- From these laws, we can derive rules for combining multiple sources or resistors into a single equivalent source or resistor.
- The current and voltage divider rules are simple tricks to solve simple circuits.
- The node voltage technique provides a general framework for solving any circuit using the elements we’ve used so far.
Short Circuit and Open Circuit

**Wire** (“short circuit”):

- \( R = 0 \) \( \rightarrow \) no voltage difference exists
  (all points on the wire are at the same potential)
- Current can flow, as determined by the circuit

**Air** (“open circuit”):

- \( R = \infty \) \( \rightarrow \) no current flows
- Voltage difference can exist, as determined by the circuit

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*Figure 2.10* Circuit symbols. (a) Short circuit. (b) Open circuit. (c) Switch.
Ideal Voltage Source

- Circuit element that maintains a prescribed voltage across its terminals, regardless of the current flowing in those terminals.
  - Voltage is known, but current is determined by the circuit to which the source is connected.
- The voltage can be either independent or dependent on a voltage or current elsewhere in the circuit, and can be constant or time-varying.

Circuit symbols:

\[ v_s = \mu \ v_x \]

independent

\[ v_s = \rho \ i_x \]

current-controlled

\[ v_s = \mu \ v_x \]

voltage-controlled
Ideal Current Source

• Circuit element that maintains a prescribed current through its terminals, regardless of the voltage across those terminals.
  – Current is known, but voltage is determined by the circuit to which the source is connected.

• The current can be either independent or dependent on a voltage or current elsewhere in the circuit, and can be constant or time-varying.

  **Circuit symbols:**

\[
\begin{align*}
  i_s & : \text{independent} \\
  i_s &= \alpha v_x & : \text{voltage-controlled} \\
  i_s &= \beta i_x & : \text{current-controlled}
\end{align*}
\]