CS10
The Beauty and Joy of Computing

Lecture #11 : Recursion II

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VERY SNEAKEY

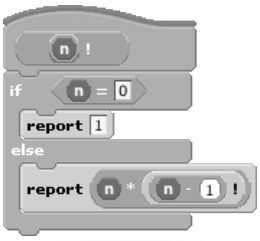
Back in 2008, researchers at UCSD demonstrated that they could make a working copy of a key based on a photo taken from 195 feet away. The shape of a key is actually a secret that can be stolen and needs to be protected!

`http://vision.ucsd.edu/~blaxton/sneakey.html`

How the Computer Works … n!

- **Factorial(n) = n!**
- **Informal Definition**
  n! = [1 * 2 * 3 * … * n]
- **Inductive Definition**

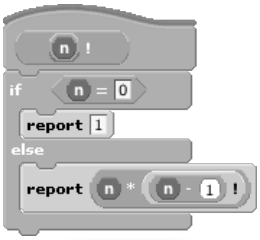
$$n! = \begin{cases} 1 & \text{, if } n = 0 \\ n * (n-1)! & \text{, if } n > 0 \end{cases}$$



How the Computer Works … n!

- **Let's act it out…**
  - subcontractor model
- **5!**

n	n!
0	1
1	1
2	2
3	6
4	24
5	120



Order of growth of # of calls of n!

a) Constant
b) Logarithmic
c) Linear
d) Quadratic
e) Exponential

(source: FallingFifth.com)

Fibonacci

en.wikipedia.org/wiki/Fibonacci_number
www.ics.uci.edu/~eppstein/161/960109.html

How the Computer Works … fib(n)

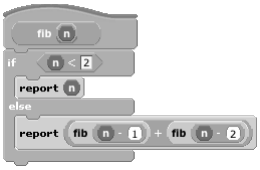
en.wikipedia.org/wiki/Fibonacci_number
www.ics.uci.edu/~eppstein/161/960109.html

- **Inductive definition**

$$fib(n) = \begin{cases} n & \text{, } n < 2 \\ fib(n-1)+fib(n-2) & \text{, } n > 1 \end{cases}$$

- **Let's act it out…**
  - subcontractor model
  - fib(5)

n	fib(n)
0	0
1	1
2	1
3	2
4	3
5	5



Let's now: trace… (gif from Ybungalobill@wikimedia)

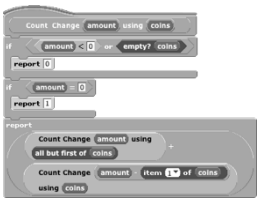
Order of growth of # of calls of fib(n)

a) Constant
b) Logarithmic
c) Linear
d) Quadratic
e) Exponential

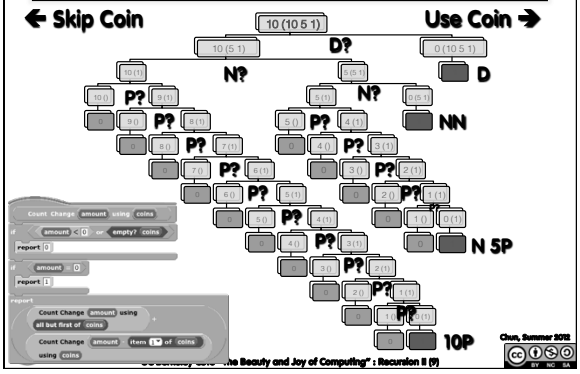
Chimney of Turku Energia, Turku, Finland featuring Fibonacci sequence in 2m high neon lights. By Italian artist Mario Merz for an environmental art project. (Wikipedia)

Counting Change (thanks to BH)

- Given coins {50, 25, 10, 5, 1} how many ways are there of making change?
  - 5: 2 (N,5 P)
  - 10
    - 4 (D, 2N, N 5P, 10P)
  - 15
    - 6 (DN,D5P,3N,2N5P,1N10P, 15P)
  - 100?



Call Tree for "Count Change 10 (10 5 1)"



Summary

- It's important to understand the machine model
- It's often the cleanest, simplest way to solve many problems
  - Esp those recursive in nature!
- Recursion is a very powerful idea, and one way to separate good from great

Menger Cube