#### <u>EECS150 – Digital Design</u> <u>Lecture 2 – Combinational Logic</u> <u>Review and FPGAs</u>

January 19, 2012

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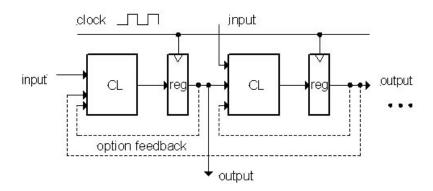
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#### **General Model for Synchronous Systems**



- All synchronous digital systems fit this model:
  - Collections of combinational logic blocks and state elements connected by signal wires. These form a directed graph with only two types of nodes (although the graph need not be bi-partite.)
  - Instead of simple registers, sometimes the state elements are large memory blocks.

# <u>Outline</u>

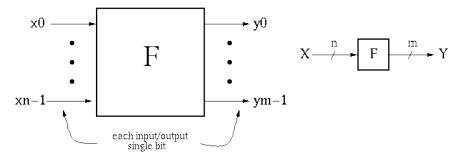
- Review of three representations for combinational logic: truth tables, gate diagrams, algebraic equations
  - Laws of Boolean Algebra
  - Canonical Forms
  - Boolean Simplification
  - multi-level logic
  - NAND/NOR Networks
- Field Programmable Gate Arrays (FPGAs) Introduction

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# **Combinational Logic (CL) Defined**



- y<sub>i</sub> = f<sub>i</sub>(x0,..., xn-1), where x, y are {0,1}.
  Y is a function of only X.
- If we change X, Y will change immediately (well almost!).
- There is an implementation dependent delay from X to Y.

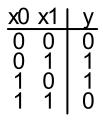
# CL Block Example #1

output 1 if either 0x $x_{0=1} \text{ or } x_{1=1}$ ⊷ У but not both x1

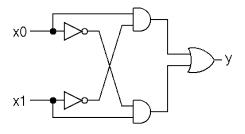
**Boolean Equation:** 

 $y_0 = (x_0 AND not(x_1))$ OR (not( $x_0$ ) AND  $x_1$ )  $y_{n} = x_{n}x_{1}' + x_{n}'x_{1}$ 

**Truth Table Description:** 



Gate Representation:



How would we prove that all three representations are equivalent?

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# **Boolean Algebra/Logic Circuits**

- Why are they called "logic circuits"? ٠
- Logic: The study of the principles of reasoning. •
- The 19th Century Mathematician, George Boole, developed a math. system (algebra) involving logic, Boolean Algebra.
- His variables took on TRUE, FALSE
- Later Claude Shannon (father of information theory) showed (in his Master's thesis!) how to map Boolean Algebra to digital circuits:
- Primitive functions of Boolean Algebra: ٠ a b AND









a b OR

0

00

01 1

10 1

11 1

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00

01

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0

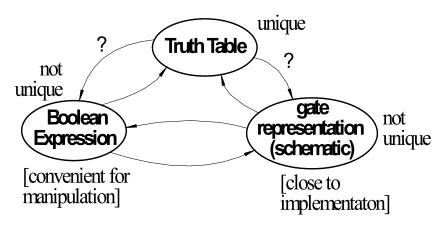
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0 11| 1

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# **Relationship Among Representations**

\* Theorem: Any Boolean function that can be expressed as a truth table can be written as an expression in Boolean Algebra using AND, OR, NOT.



How do we convert from one to the other?

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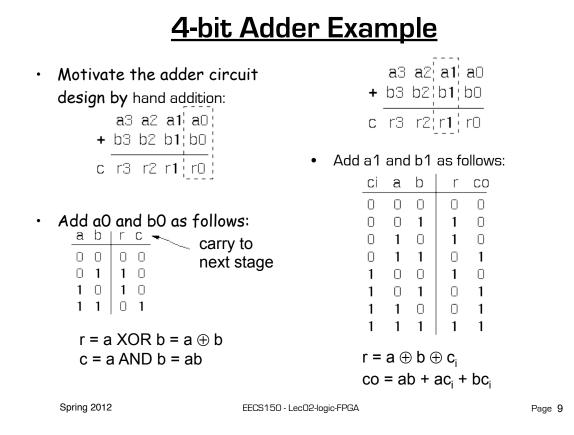
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# CL Block Example #2

• 4-bit adder:	Truth Table Representation:								
	a3 a2 a1 a0 b3 b2 b1 b0 r3 r2 r1 r0 c								
$A \xrightarrow{4} R$ add $B \xrightarrow{4} c$	0       1       0       0       1       0       0       0       1       0								
R = A + B, c is carry out									
In general: 2ª rows for n in	• • • • • • • • • • • • • • • • • • • •								

In general: 2<sup>n</sup> rows for n inputs. 256 rows! Is there a more efficient (compact) way to specify this function?

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<u>4-bit Adder Example</u>

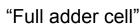
• In general:

$$\mathbf{r}_{i} = \mathbf{a}_{i} \oplus \mathbf{b}_{i} \oplus \mathbf{c}_{in}$$

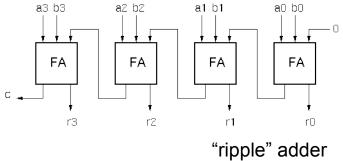
 $c_{out} = a_i c_{in} + a_i b_i + b_i c_{in} = c_{in}(a_i + b_i) + a_i b_i$ 

FA сO

a b ci



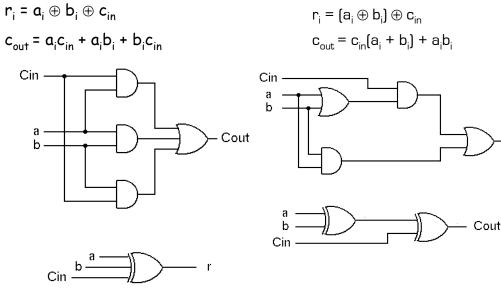
Now, the 4-bit adder: **a**3 b3 a2 b2 a1 b1



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# 4-bit Adder Example

- Graphical Representation of FAcell
- Alternative Implementation (with 2-input gates):



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Cout

# **Boolean Algebra**

Defined as:

Set of elements *B*, binary operators  $\{+, \bullet\}$ , unary operation  $\{'\}$ , such that the following axioms hold :

- 1. *B* contains at least two elements a, b such that  $a \neq b$ .
- 2. Closure : a, b in B,
  - a + b in B,  $a \bullet b$  in B, a' in B.

#### 3. Communitive laws :

a + b = b + a,  $a \bullet b = b \bullet a$ .

4. Identities : 0, 1 in B

 $a + 0 = a, \quad a \bullet 1 = a.$ 

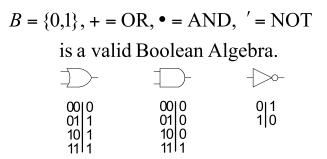
5. Distributive laws:

 $a + (b \bullet c) = (a + b) \bullet (a + c), \ a \bullet (b + c) = a \bullet b + a \bullet c.$ 

6. Complement :

$$a + a' = 1, a \bullet a' = 0.$$

#### Logic Functions



#### Do the axioms hold?

- Ex: communitive law: a+b = b+a?

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# Some Laws of Boolean Algebra

Duality: A dual of a Boolean expression is derived by interchanging OR and AND operations, and Os and 1s (literals are left unchanged).

 $\{F(x_1, x_2, ..., x_n, 0, 1, +, \bullet)\}^D = \{F(x_1, x_2, ..., x_n, 1, 0, \bullet, +)\}$ 

Any law that is true for an expression is also true for its dual.

```
Operations with 0 and 1:

1. x + 0 = x x * 1 = x

2. x + 1 = 1 x * 0 = 0

Idempotent Law:

3. x + x = x x = x

Involution Law:

4. [x']' = x

Laws of Complementarity:

5. x + x' = 1 x = x' = 0

Commutative Law:

6. x + y = y + x x = y = y
```

# Laws of Boolean Algebra (cont.)

Associative Laws: (x + y) + z = x + (y + z)	x y z = x (y z)
Distributive Laws: x (y + z) = (x y) + (x z)	x +(y z) = (x + y)(x + z)
"Simplification" Theorems: x y + x y' = x x + x y = x	(x + y) (x + y') = x x (x + y) = x
DeMorgan's Law: (x + y + z +)' = x'y'z'	(x y z)' = x' + y' +z'
Theorem for Multiplying and Fac (x + y) (x' + z) = x z + x' y Consensus Theorem: x y + y z + x' z = (x + y) (y + z)	

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# <u>Proving Theorems via axioms of</u> <u>Boolean Algebra</u>

Ex: prove the theorem: x y + x y' = x

x y + x y' = x (y + y') distributive law

x (y + y') = x (1) complementary law

x(1) = x identity

x y + x' z = (x + y) (x' + z)

Ex: prove the theorem: X + X Y = X

x + xy = x1 + xy identity x1 + xy = x(1 + y) distributive law x(1 + y) = x(1) identity x(1) = x identity

#### **DeMorgan's Law** x y x' y' | (x + y)' x'y' (x + y)' = x' y'Exhaustive 0011 1 1 Proof 0110 0 0 Õ 1001 0 1100 Ó 0 (x y)' x' + y' хуху (x y)' = x' + y'Exhaustive 0011 1 1 Proof 0110 1 1 1001 1 1 1100 0 0

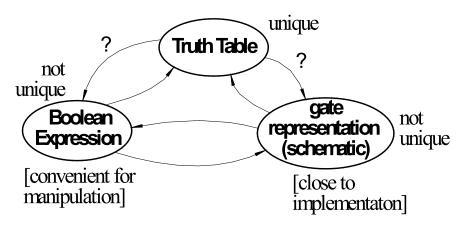
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# **Relationship Among Representations**

\* Theorem: Any Boolean function that can be expressed as a truth table can be written as an expression in Boolean Algebra using AND, OR, NOT.



How do we convert from one to the other?

# **Canonical Forms**

- Standard form for a Boolean expression unique algebraic expression directly from a true table (TT) description.
- Two Types:
  - \* Sum of Products (SOP)
  - \* Product of Sums (POS)
- <u>Sum of Products</u> (disjunctive normal form, <u>minterm</u> expansion). Example:

minterms a'b'c' a'b'c a'bc' a'bc ab'c'	a b c     f f'       0 0 0     0 1       0 0 1     0 1       0 1 0     0 1       0 1 0     0 1       0 1 1     1 0       1 0 0     1 0	One product ( <b>and</b> ) term for each 1 in f: f = a'bc + ab'c' + ab'c +abc' +abc
ab'c' ab'c	10010 10110	f = a'bc + ab'c' + ab'c +abc' +abc f' = a'b'c' + a'b'c + a'bc'
abc' abc	1 1 0   1 0 1 1 1   1 0	

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# **Canonical Forms**

• **<u>Product of Sums</u>** (conjunctive normal form, <u>maxterm</u> expansion). Example:

abc  f f'	
00001	
00101	
01001	
01110	
10010	
10110	One sum ( <b>or</b> ) term for each 0 in f:
11010	
11110	f = (a+b+c)(a+b+c')(a+b'+c)
·	f' = (a+b'+c')(a'+b+c)(a'+b+c')
	(a'+b'+c)(a+b+c')
	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$

Mapping from SOP to POS (or POS to SOP): Derive truth table then proceed.

#### **Algebraic Simplification Example**

Ex: full adder (FA) carry out function (in canonical form):

Cout = a'bc + ab'c + abc' + abc

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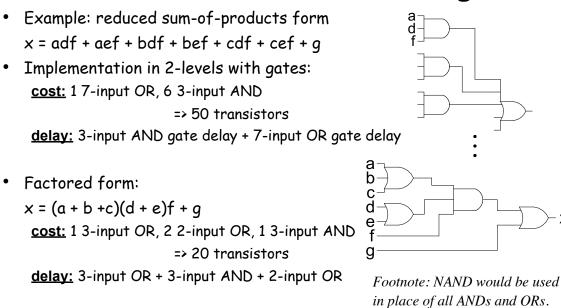
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# **Algebraic Simplification**

Cout = a'bc + ab'c + abc' + abc = a'bc + ab'c + abc' + abc + abc = a'bc + abc + ab'c + abc' + abc = (a' + a)bc + ab'c + abc' + abc= (1)bc + ab'c + abc' + abc= bc + ab'c + abc' + abc + abc= bc + ab'c + abc + abc' + abc= bc + a(b' + b)c + abc' + abc= bc + a(1)c + abc' + abc= bc + ac + ab(c' + c)= bc + ac + ab(1)= bc + ac + ab

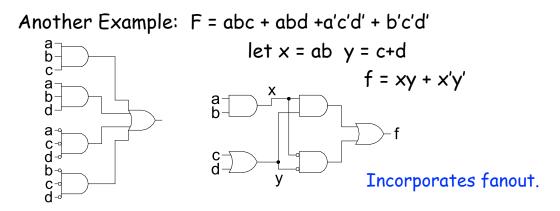
# **Multi-level Combinational Logic**



#### Which is faster?

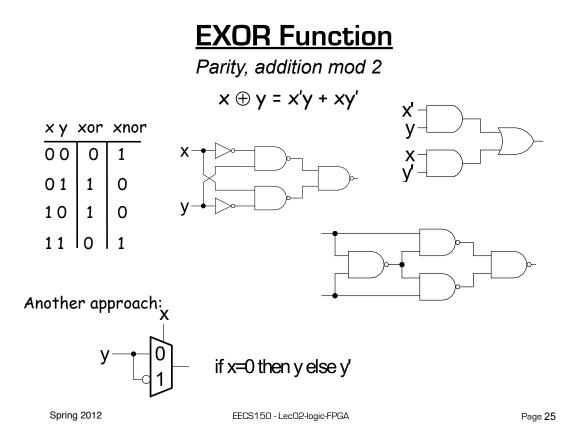
In general: Using multiple levels (more than 2) will reduce the cost. Sometimes also delay. Spring 2012 EECS150 - Lec02-Jogic-FPGA Page 23

# **Multi-level Combinational Logic**



No convenient hand methods exist for multi-level logic simplification:

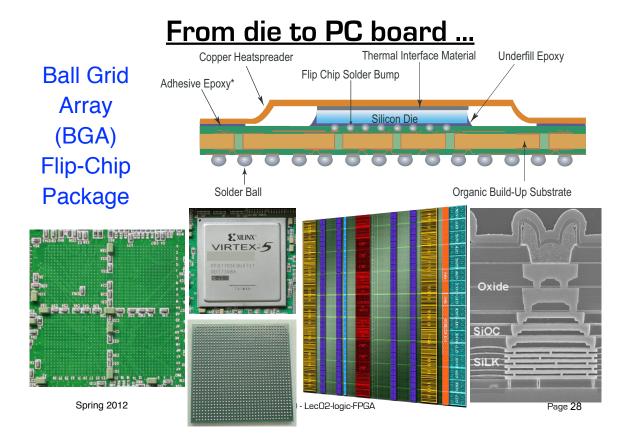
- a) CAD Tools use sophisticated algorithms and heuristics
- b) Humans and tools often exploit some special structure (example adder)



# Project platform: Xilinx ML505-110

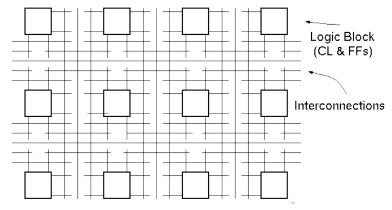






## **FPGA** Overview

- Basic idea: two-dimensional array of logic blocks and flip-flops with a means for the user to configure (program):
  - 1. the interconnection between the logic blocks,
  - 2. the function of each block.



Simplified version of FPGA internal architecture:

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#### Why are FPGAs Interesting?

- Technical viewpoint:
  - For hardware/system-designers, like ASICs only better! "Tape-out" new design every few minutes/hours.
  - Does the "reconfigurability" or "reprogrammability" offer other advantages over fixed logic?
  - Dynamic reconfiguration? In-field reprogramming? Self-modifying hardware, evolvable hardware?

#### Why are FPGAs Interesting?

• Staggering logic capacity growth (10000x):

Year Introduced	Device	Logic Cells	"logic gate equivalents"
1985	XC2064	128	1024
2011	XC7V2000T	1,954,560	15,636,480

 FPGAs have tracked Moore's Law better than any other programmable device.

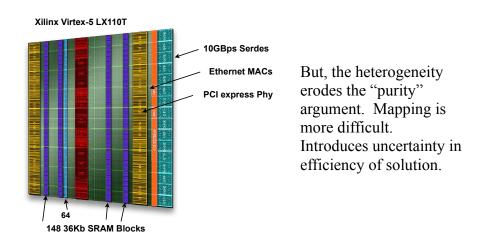
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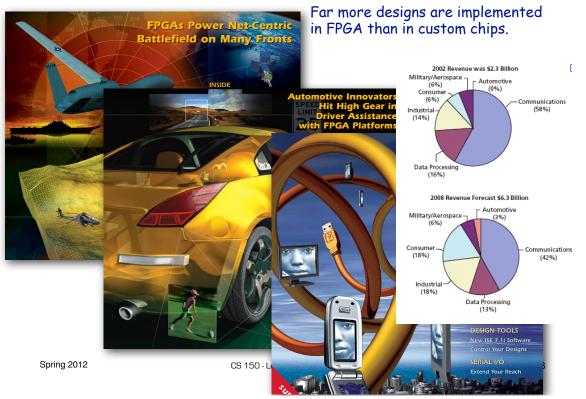
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#### Why are FPGAs Interesting?

- Logic capacity now only part of the story: on-chip RAM, high-speed I/Os, "hard" function blocks, ...
- Modern FPGAs are "reconfigurable systems"

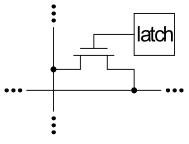


# FPGAs are in widespread use



# **User Programmability**

Latch-based (Xilinx, Altera, ...)

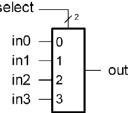


- + reconfigurable
- volatile
- relatively large.

- Latches are used to:
  - 1. control a switch to make or break cross-point connections in the interconnect
  - 2. define the function of the logic blocks
  - 3. set user options:
    - within the logic blocks
    - in the input/output blocks
    - global reset/clock
- "Configuration bit stream" is loaded under user control

# **Background (review) for upcoming**

 A <u>MUX</u> or multiplexor is a combinational logic circuit that chooses between 2<sup>N</sup> inputs under the control of N control signals.

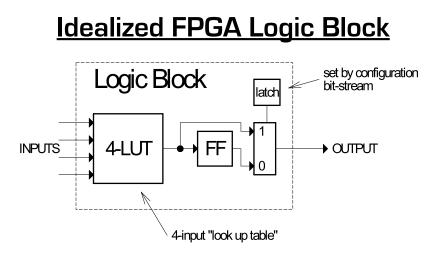


• A <u>latch</u> is a 1-bit memory (similar to a flip-flop).

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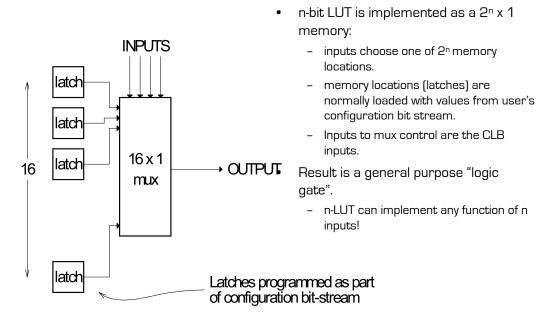
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- 4-input look up table (LUT)

   implements combinational logic functions
- Register
  - optionally stores output of LUT

## **4-LUT Implementation**



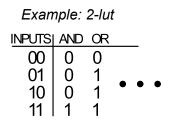
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#### LUT as general logic gate

- An n-lut as a direct implementation of a function **truth-table**.
- Each latch location holds the value of the function corresponding to one input combination.



Implements any function of 2 inputs.

How many of these are there? How many functions of n inputs? Example: 4-lut

INPUTS	
0000	F(0,0,0,0) < store in 1st latch
0001	$F(0,0,0,1) \leq $ store in 2nd latch
0010	F(0,0,1,0) <
0011	F(0,0,1,1) <
0011	
0100	•
0101	•
0110	•
0111	
1000	
1001	
1010	
1011	
1100	
1101	
1110	
1111	

#### FPGA Generic Design Flow

**Design Entry** 

Design

Implementation

- Design Entry:
  - Create your design files using:
    - · schematic editor or
    - HDL (hardware description languages: Verilog, VHDL)
- Design Implementation:
  - Logic synthesis (in case of using HDL entry) followed by,
  - Partition, place, and route to create configuration bit-stream file
- Design verification:
  - Optionally use simulator to check function,
  - Load design onto FPGA device (cable connects PC to development board), optional "logic scope" on FPGA
    - · check operation at full speed in real environment.

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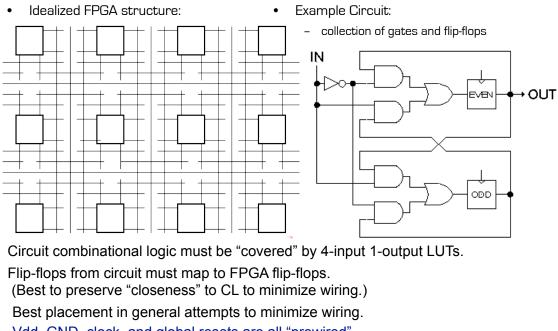
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Desian

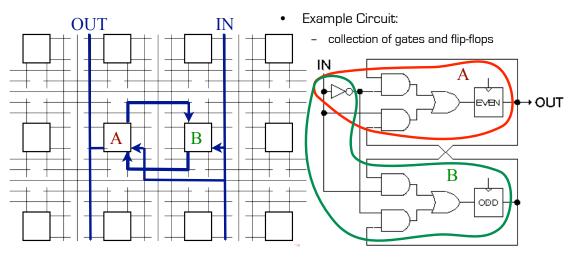
Verification

#### **Example Partition, Placement, and Route**



Vdd, GND, clock, and global resets are all "prewired".

#### **Example Partition, Placement, and Route**



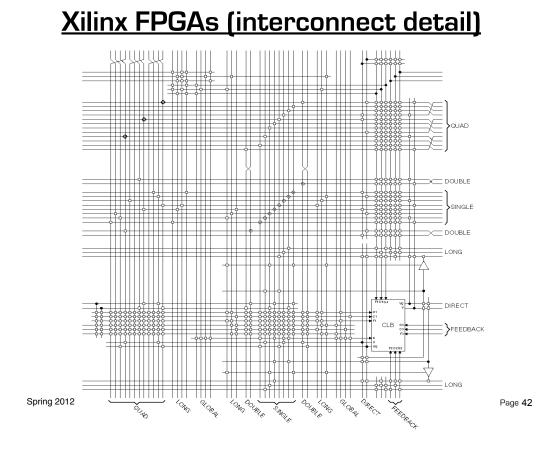
Two partitions. Each has single output, no more than 4 inputs, and no more than 1 flip-flop. In this case, inverter goes in both partitions.

Note: the partition can be arbitrarily large as long as it has not more than 4 inputs and 1 output, and no more than 1 flip-flop.

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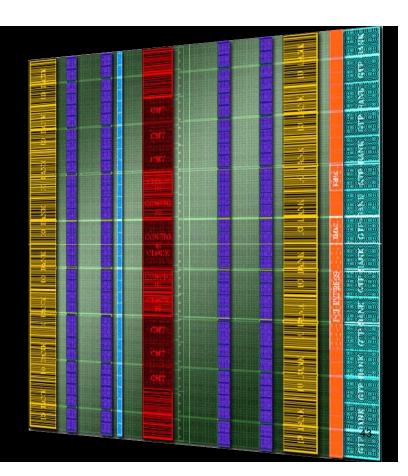
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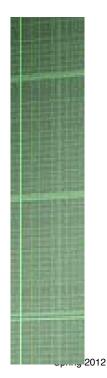
Colors represent different types of resources:

Logic Block RAM DSP (ALUs) Clocking I/O Serial I/O + PCI

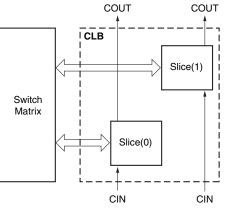
A routing fabric runs throughout the chip to wire everything together.



# **Configurable Logic Blocks (CLBs)**

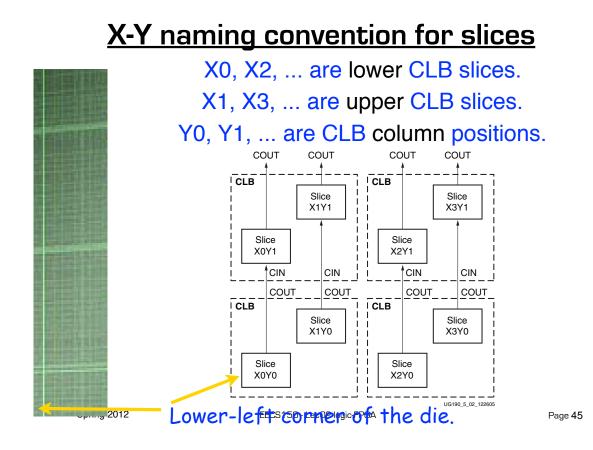


Slices define regular connections to the switching fabric, and to slices in CLBs above and below it on the die.

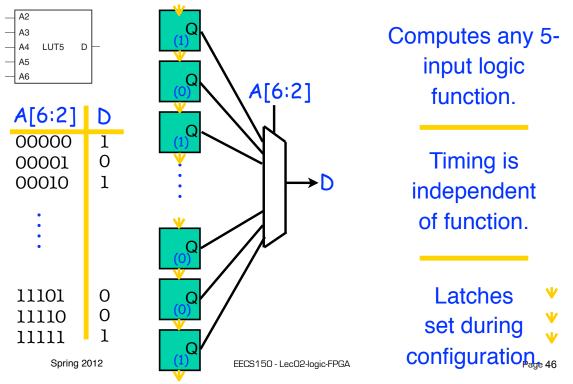




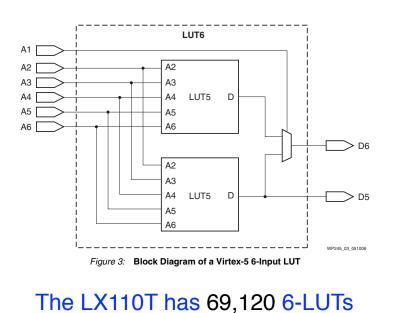
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# Atoms: 5-input Look Up Tables (LUTs)



## Virtex-5 6-LUTs: Composition of 5-LUTs



6-LUT delay is 0.9 ns

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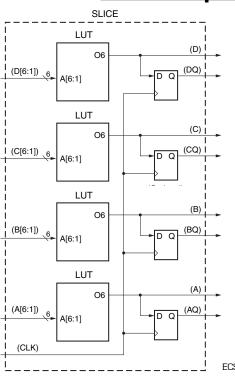
May be used as one 6-input LUT (D6 out) ...

... or as two 5-input LUTS (D6 and D5)

Combinational logic (post configuration) Page 47

# The simplest view of a slice

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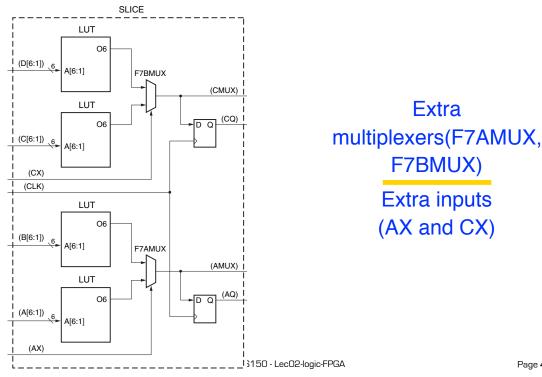
Four 6-LUTs

Four Flip-Flops

Switching fabric may see combinational and registered outputs.

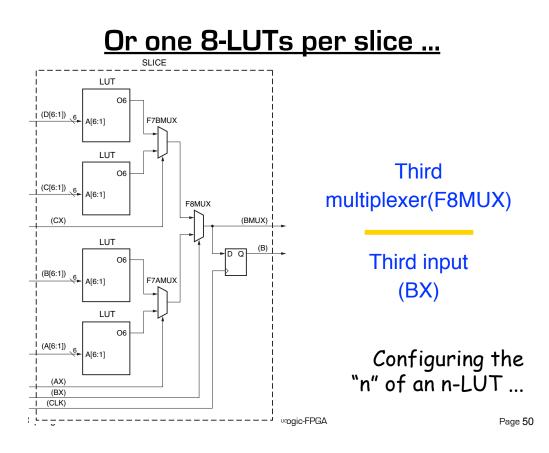
An actual Virtex-5 slice adds many small features to this simplified diagram. We show them one by one ... ECS150-Lec02-logicFPGA Page 48

#### Two 7-LUTs per slice ...

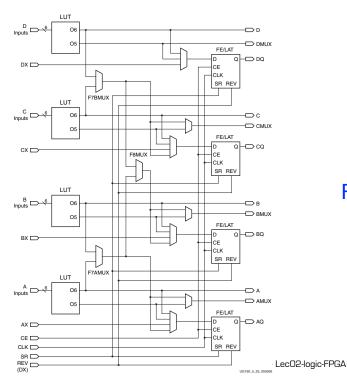


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**Extra** 



## Extra muxes to chose LUT option ...



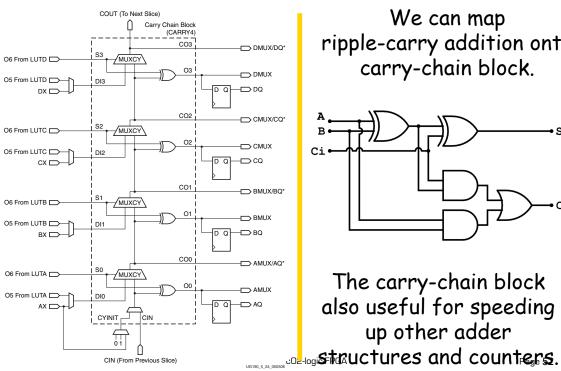
From eight 5-LUTs ... to one 8-LUT.

Combinational or registered outs.

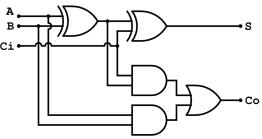
Flip-flops unused by LUTs can be used standalone.

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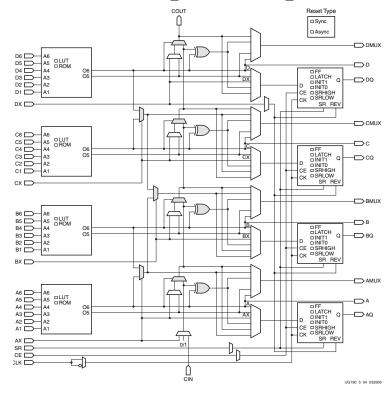
# Virtex 5 Verical Logic



We can map ripple-carry addition onto carry-chain block.



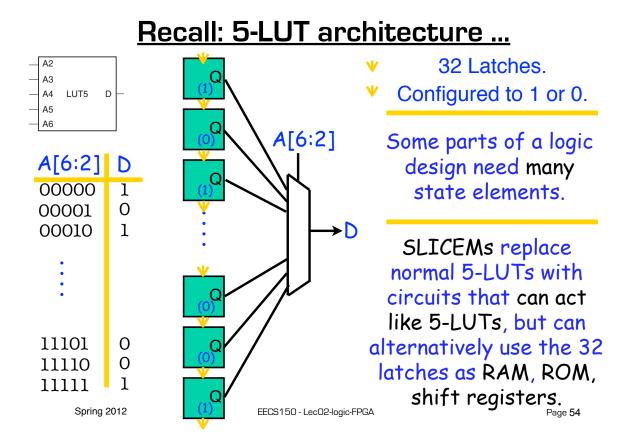
## Putting it all together ... a SLICEL.

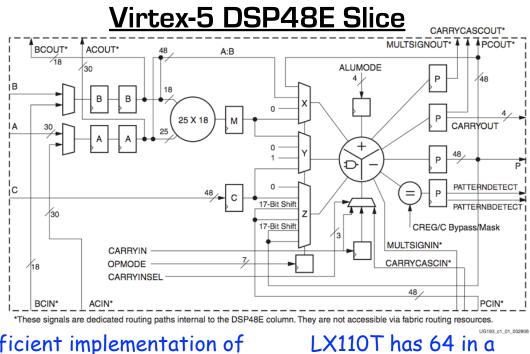


The previous slides explain all SLICEL features.

About 50% of the 17,280 slices in an LX110T are SLICELs.

The other slices are SLICEMs, and have extra features.<sub>Page 53</sub>





# Efficient implementation of LX110T has 64 multiply, add, bit-wise logical. single column.

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Device	Configurable Logic Blocks (CLBs)				Block RAM Blocks			PowerPC	Endpoint		Max RocketlO Transceivers <sup>(6)</sup>		Total	Max	
	Array (Row x Col)	Virtex-5 Slices <sup>(1)</sup>	Max Distributed RAM (Kb)	DSP48E Slices <sup>(2)</sup>	18 Kb <sup>(3)</sup>	36 Kb	Max (Kb)	CMTs <sup>(4)</sup>		Blocks for PCI Express	Ethernet MACs <sup>(5)</sup>	GTP	GTX	I/O Banks <sup>(8)</sup>	User
XC5VLX30	80 x 30	4,800	320	32	64	32	1,152	2	N/A	N/A	N/A	N/A	N/A	13	400
XC5VLX50	120 x 30	7,200	480	48	96	48	1,728	6	N/A	N/A	N/A	N/A	N/A	17	560
XC5VLX85	120 x 54	12,960	840	48	192	96	3,456	6	N/A	N/A	N/A	N/A	N/A	17	560
XC5VLX110	160 x 54	17,280	1,120	64	256	128	4,608	6	N/A	N/A	N/A	N/A	N/A	23	800
XC5VLX155	160 x 76	24,320	1,640	128	384	192	6,912	6	N/A	N/A	N/A	N/A	N/A	23	800
XC5VLX220	160 x 108	34,560	2,280	128	384	192	6,912	6	N/A	N/A	N/A	N/A	N/A	23	800
XC5VLX330	240 x 108	51,840	3,420	192	576	288	10,368	6	N/A	N/A	N/A	N/A	N/A	33	1,200
XC5VLX20T	60 x 26	3,120	210	24	52	26	936	1	N/A	1	2	4	N/A	7	172
XC5VLX30T	80 x 30	4,800	320	32	72	36	1,296	2	N/A	1	4	8	N/A	12	360
XC5VLX50T	120 x 30	7,200	480	48	120	60	2,160	6	N/A	1	4	12	N/A	15	480
XC5VLX85T	120 x 54	12,960	840	48	216	108	3,888	6	N/A	1	4	12	N/A	15	480
XC5VLX110T	160 x 54	17,280	1,120	64	296	148	5,328	6	N/A	1	4	16	N/A	20	680
XC5VLX155T	160 x 76	24,320	1,640	128	424	212	7,632	6	N/A	1	4	16	N/A	20	680
XC5VLX220T	160 x 108	34,560	2,280	128	424	212	7,632	6	N/A	1	4	16	N/A	20	680
XC5VLX330T	240 x 108	51,840	3,420	192	648	324	11,664	6	N/A	1	4	24	N/A	27	960
XC5VSX35T	80 x 34	5,440	520	192	168	84	3,024	2	N/A	1	4	8	N/A	12	360
XC5VSX50T	120 x 34	8,160	780	288	264	132	4,752	6	N/A	1	4	12	N/A	15	480
XC5VSX95T	160 x 46	14,720	1,520	640	488	244	8,784	6	N/A	1	4	16	N/A	19	640
XC5VSX240T	240 x 78	37,440	4,200	1,056	1,032	516	18,576	6	N/A	1	4	24	N/A	27	960
XC5VTX150T	200 x 58	23,200	1,500	80	456	228	8,208	6	N/A	1	4	N/A	40	20	680
XC5VTX240T	240 x 78	37,440	2,400	96	648	324	11,664	6	N/A	1	4	N/A	48	20	680
XC5VFX30T	80 x 38	5,120	380	64	136	68	2,448	2	1	1	4	N/A	8	12	360
XC5VFX70T	160 x 38	11,200	820	128	296	148	5,328	6	1	3	4	N/A	16	19	640
XC5VFX100T	160 x 56	16,000	1,240	256	456	228	8,208	6	2	3	4	N/A	16	20	680
XC5VFX130T	200 x 56	20,480	1,580	320	596	298	10,728	6	2	3	6	N/A	20	24	840
XC5VFX200T	240 x 68	30,720	2,280	384	912	456	16,416	6	2	4	8	N/A	24	27	960

#### Table 1: Virtex-5 FPGA Family Members

