EECS150 - Digital Design

Lecture 17 - Finite State Machines Revisited

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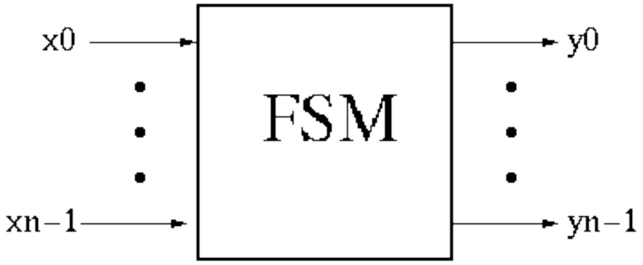
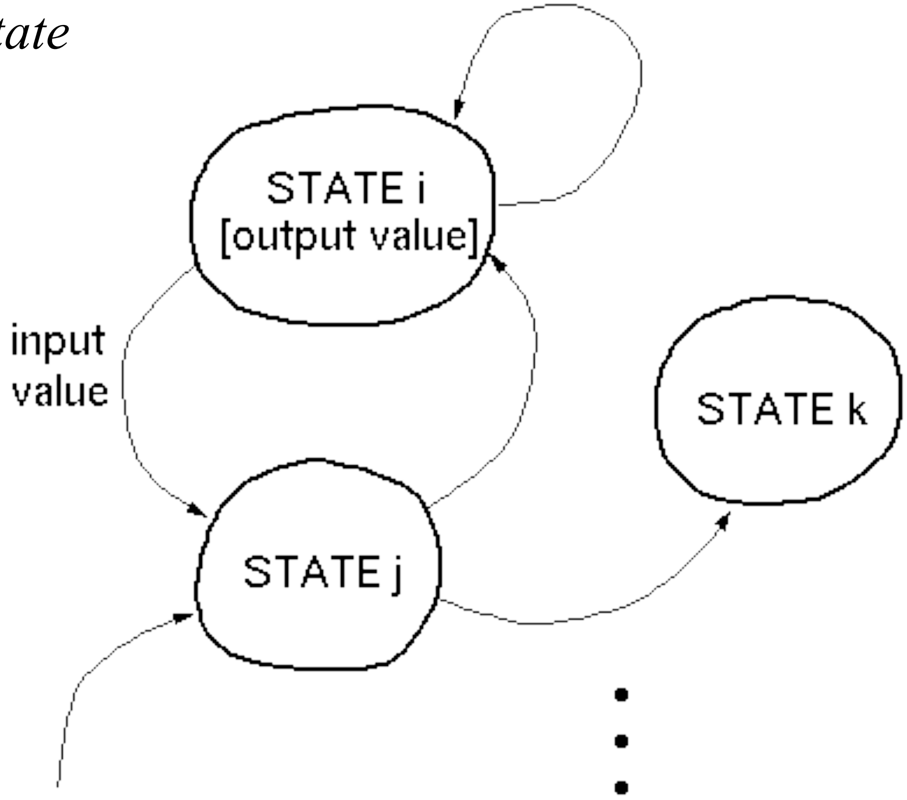
Finite State Machines (FSMs)

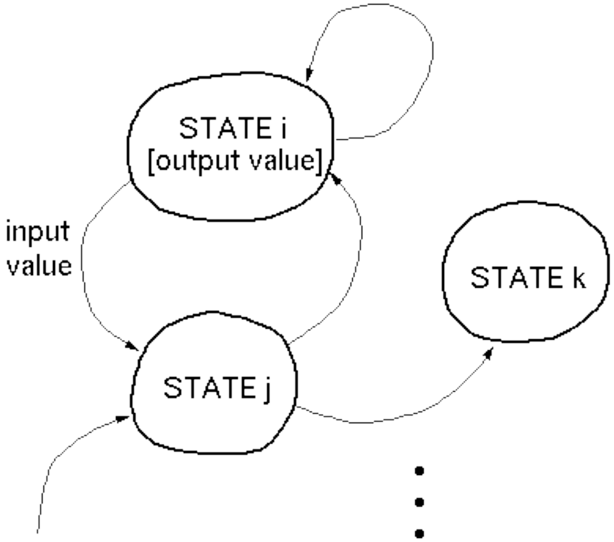
- FSM circuits are a type of *sequential circuit*:
  - output depends on present *and* past inputs
    - effect of past inputs is represented by the current *state*

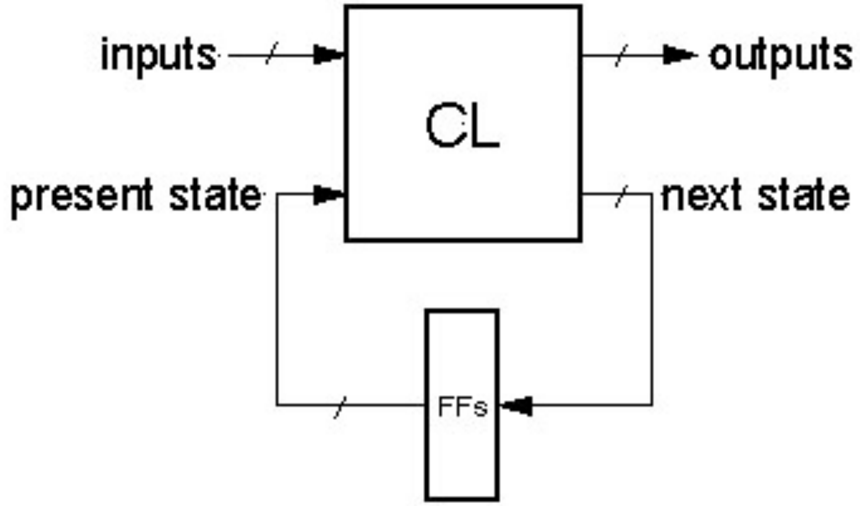


- Behavior is represented by *State Transition Diagram*:
  - traverse one edge per clock cycle.



FSM Implementation





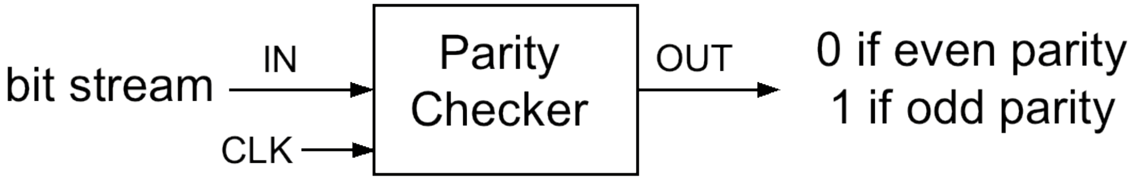
- Flip-flops form *state register*
- number of states ≤ $2^{\text{number of flip-flops}}$
- CL (combinational logic) calculates next state and output
- Remember: The FSM follows exactly one edge per cycle.

So far we have learned how to implement in Verilog. Now we learn how to design "by hand" to the gate level.

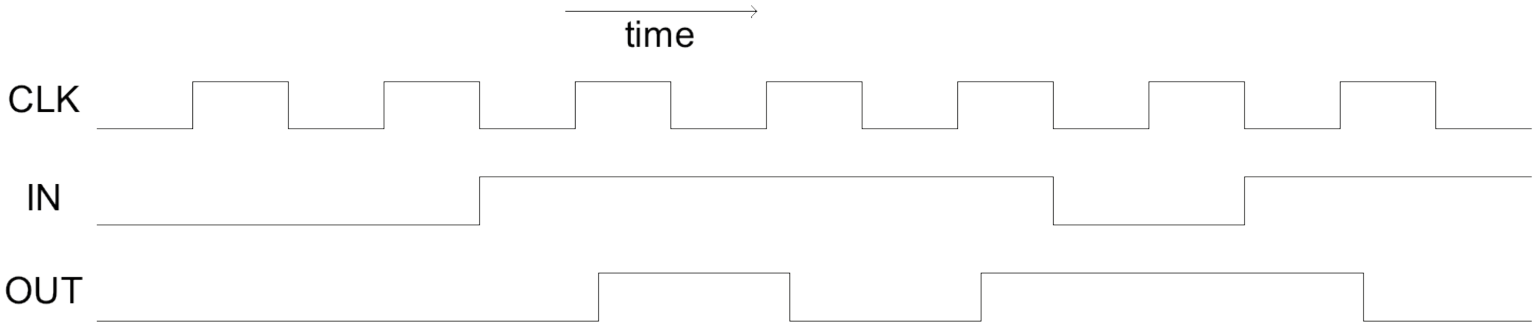
Parity Checker Example

A string of bits has "even parity" if the number of 1's in the string is even.

- Design a circuit that accepts a bit-serial stream of bits and outputs a 0 if the parity thus far is even and outputs a 1 if odd:

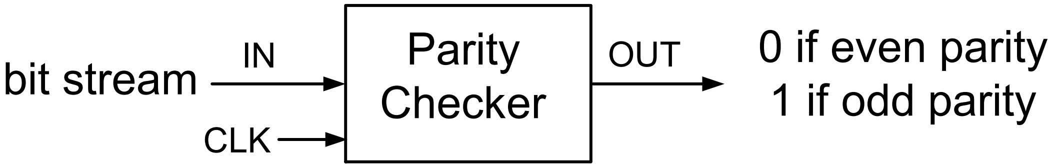


example:	0	0	1	1	1	0	1
	even	even	odd	even	odd	odd	even



Next we take this example through the "formal design process".
But first, can you guess a circuit that performs this function?

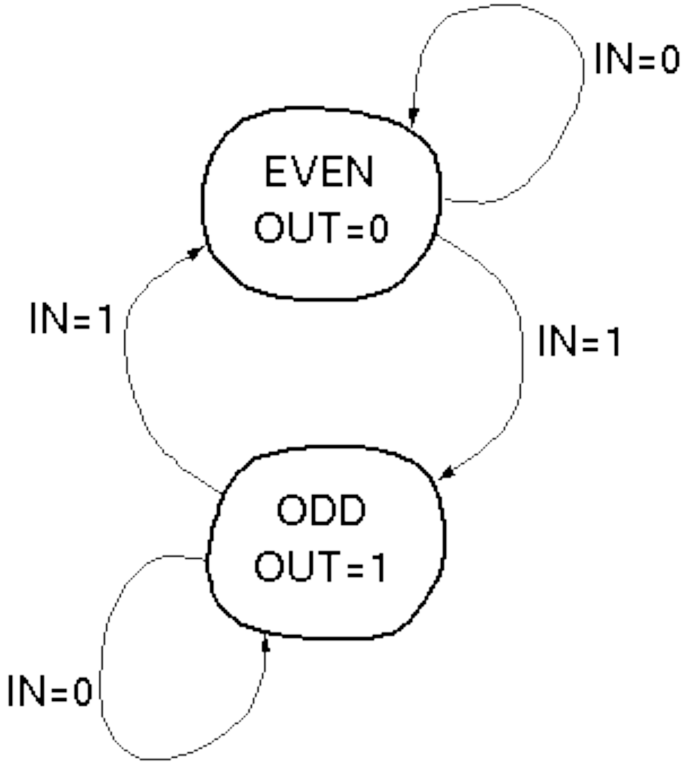
Formal Design Process



0 if even parity
1 if odd parity

example:	0	0	1	1	1	0	1
	even	even	odd	even	odd	odd	even

time →



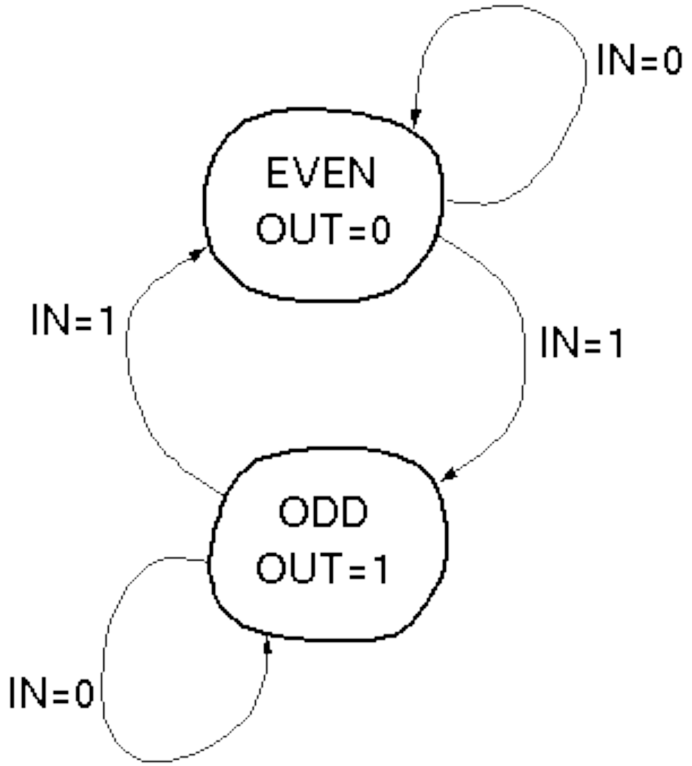
“State Transition Diagram”

- circuit is in one of two “states”.
- transition on each cycle with each new input, over exactly one arc (edge).
- Output depends on which state the circuit is in.

Formal Design Process

State Transition Table:

present state	OUT	IN	next state
EVEN	0	0	EVEN
EVEN	0	1	ODD
ODD	1	0	ODD
ODD	1	1	EVEN



Invent a code to represent states:

Let 0 = EVEN state, 1 = ODD state

present state (ps)	OUT	IN	next state (ns)
0	0	0	0
0	0	1	1
1	1	0	1
1	1	1	0

Derive logic equations from table (how?):

OUT = PS

NS = PS xor IN

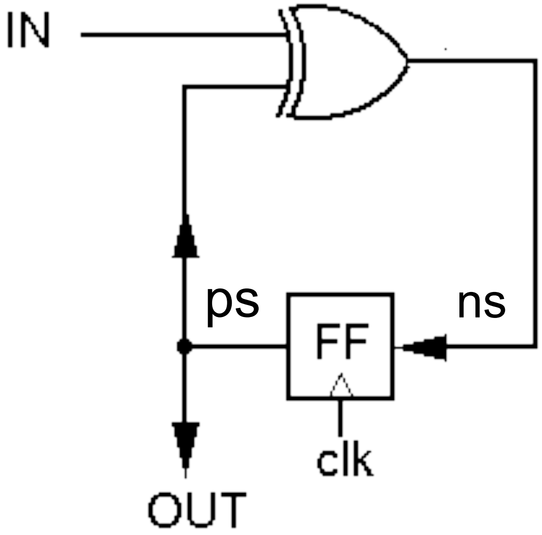
Formal Design Process

Logic equations from table:

OUT = PS

NS = PS xor IN

- Circuit Diagram:
  - XOR gate for NS calculation
  - DFF to hold present state
  - no logic needed for output in this example.

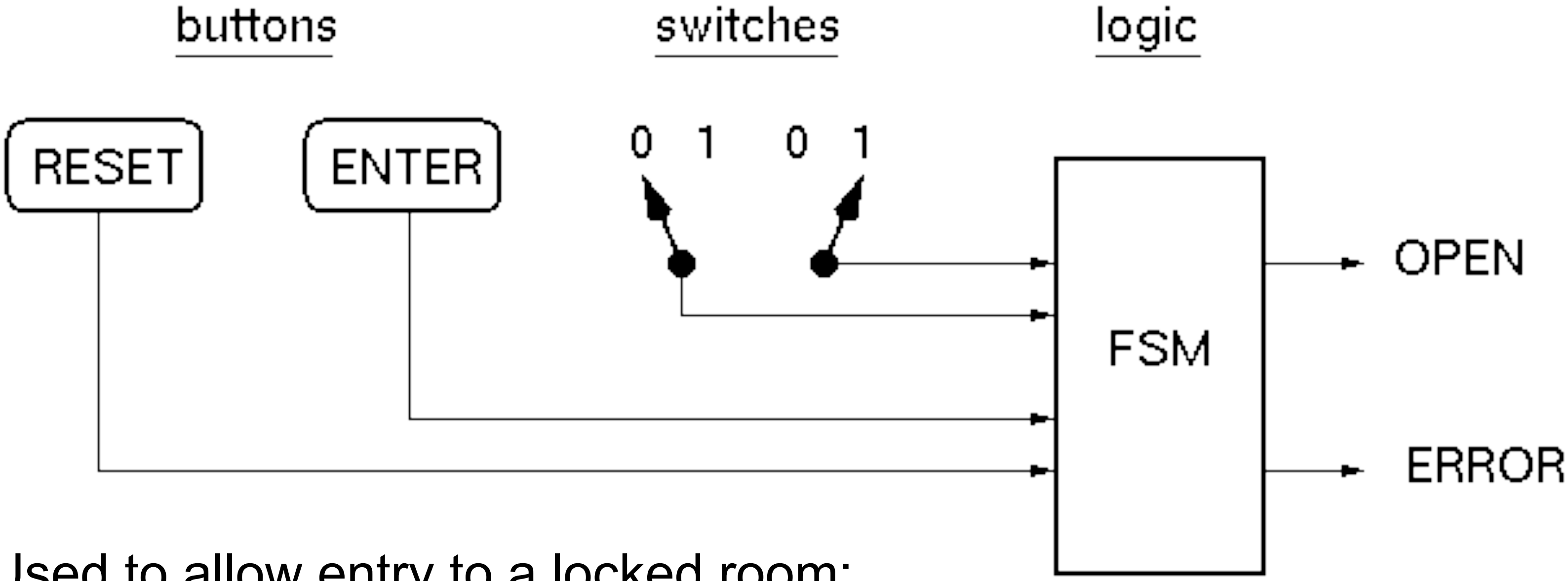


Formal Design Process

Review of Design Steps:

1. Specify **circuit function** (English)
2. Draw **state transition diagram**
3. Write down **symbolic state transition table**
4. Write down **encoded state transition table**
5. Derive **logic equations**
6. Derive **circuit diagram**
    - Register to hold state
    - Combinational Logic for Next State and Outputs

Combination Lock Example



- Used to allow entry to a locked room:

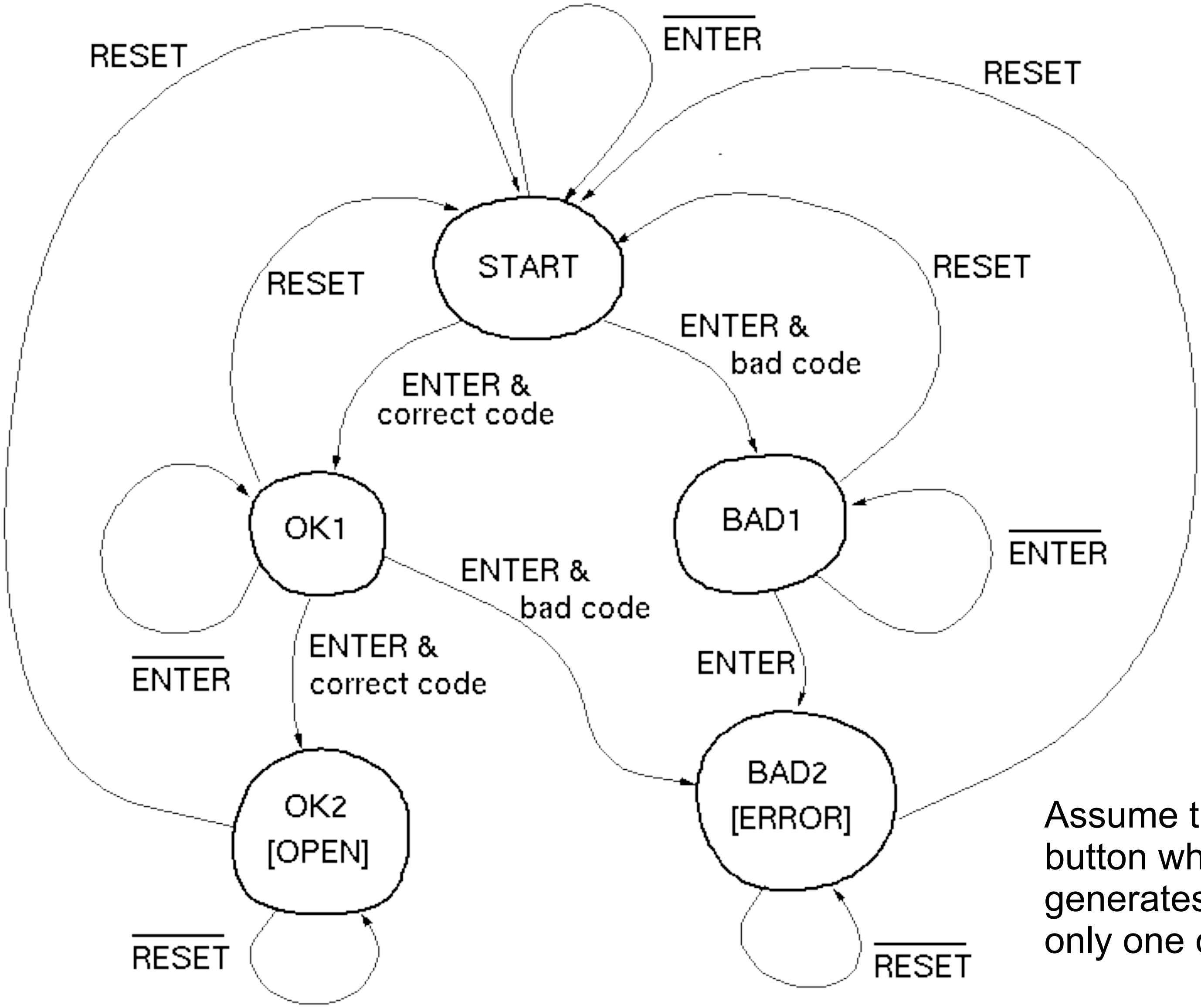
  2-bit serial combination. Example 01,11:

  1. Set switches to 01, press ENTER
  2. Set switches to 11, press ENTER
  3. OPEN is asserted (OPEN=1).

     If wrong code, ERROR is asserted (after second combo word entry).

     Press Reset at anytime to try again.

Combinational Lock STD

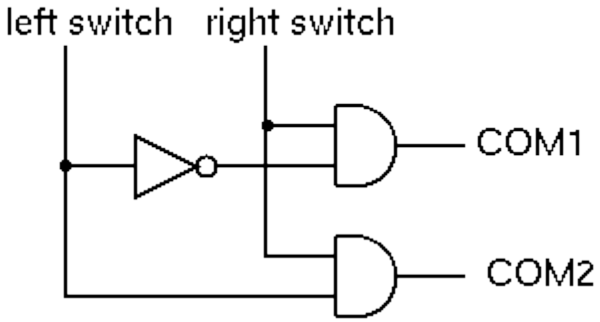


Assume the ENTER button when pressed generates a pulse for only one clock cycle.

Symbolic State Transition Table

RESET	ENTER	COM1	COM2	Preset State	Next State	OPEN	ERROR
0	0	*	*	START	START	0	0
0	1	0	*	START	BAD1	0	0
0	1	1	*	START	OK1	0	0
0	0	*	*	OK1	OK1	0	0
0	1	*	0	OK1	BAD2	0	0
0	1	*	1	OK1	OK2	0	0
0	*	*	*	OK2	OK2	1	0
0	0	*	*	BAD1	BAD1	0	0
0	1	*	*	BAD1	BAD2	0	0
0	*	*	*	BAD2	BAD2	0	1
1	*	*	*	*	START	0	0

Decoder logic for checking combination (01,11):



Encoded ST Table

ENTER	COM1	COM2	PS2	PS1	PS0	NS2	NS1	NS0
0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0
0	1	1	0	0	0	0	0	0
1	0	0	0	0	0	1	0	0
1	0	1	0	0	0	1	0	0
1	1	0	0	0	0	0	0	1
1	1	1	0	0	0	0	0	1
0	0	0	0	0	1	0	0	1
0	0	1	0	0	1	0	0	1
0	1	0	0	0	1	0	0	1
0	1	1	0	0	1	0	0	1
1	0	0	0	0	1	1	0	1
1	1	0	0	0	1	1	0	1
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1	0	1	0	1	1	0	1	1
1	1	0	0	1	1	0	1	1
1	1	1	0	1	1	0	1	1
0	0	0	1	0	0	1	0	0
0	0	1	1	0	0	1	0	0
0	1	0	1	0	0	1	0	0
0	1	1	1	0	0	1	0	0
1	0	0	1	0	0	1	0	1
1	0	1	1	0	0	1	0	1
1	1	0	1	0	0	1	0	1
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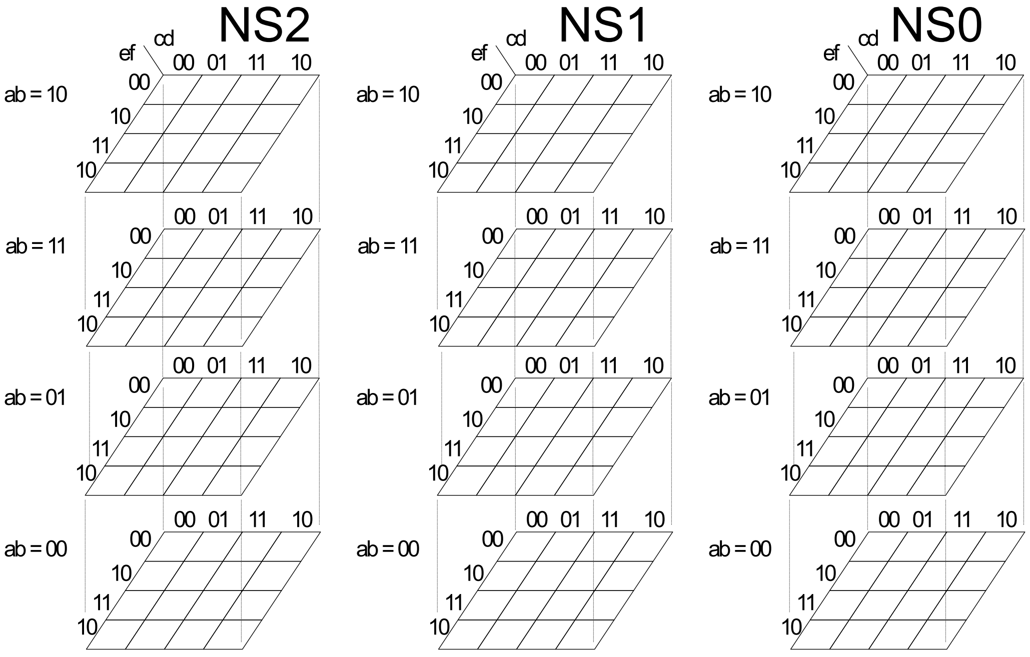
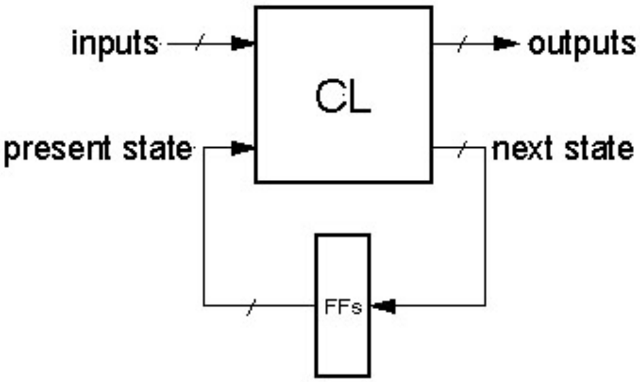
0	1	0	1	0	1	1	0	1
0	1	1	1	0	1	1	0	1
1	0	0	1	0	1	1	0	1
1	0	1	1	0	1	1	0	1
1	1	0	1	0	1	1	0	1
1	1	1	1	0	1	1	0	1

- Assign states:
  START=000, OK1=001, OK2=011
  BAD1=100, BAD2=101
- Omit reset. Assume that primitive flip-flops has reset input.
- Rows not shown have *don't cares* in output. Correspond to invalid PS values.



- What are the output functions for OPEN and ERROR?

State Encoding



- In general:

  # of possible FSM state = $2^{\text{\# of Flip-flops}}$

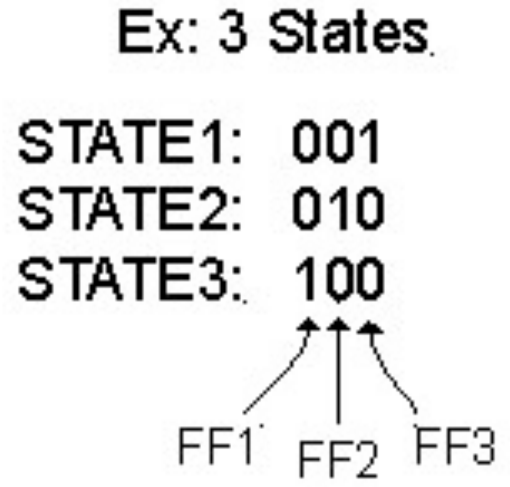
  Example:

  state1 = 01, state2 = 11, state3 = 10, state4 = 00

- However, often more than $\log_2$(# of states) FFs are used, to simplify logic at the cost of more FFs.
- Extreme example is one-hot state encoding.

State Encoding

- **One-hot encoding of states.**
- One FF per state.



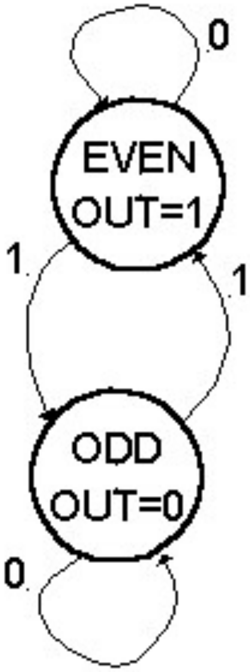
Ex: 3 States

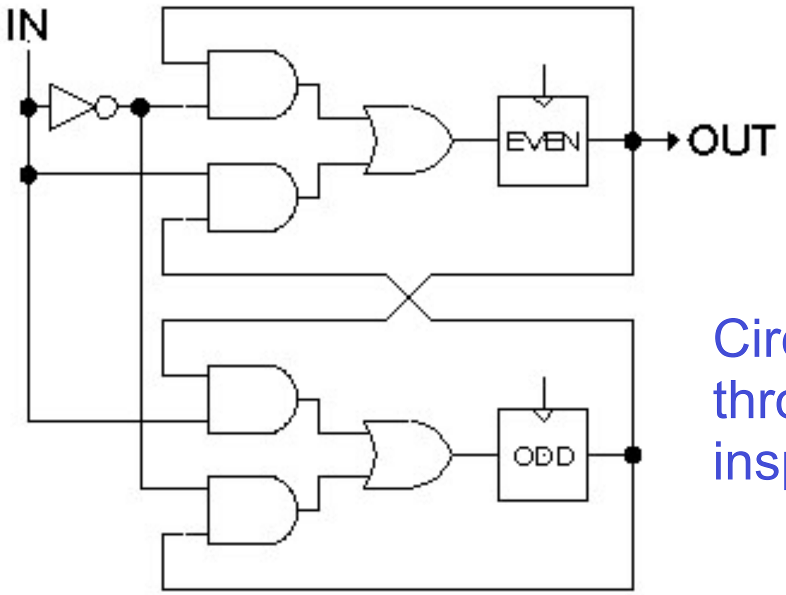
STATE1: 001
STATE2: 010
STATE3: 100

- Why one-hot encoding?
  - Simple design procedure.
    - Circuit matches state transition diagram (example next page).
  - Often can lead to simpler and faster “next state” and output logic.
- Why not do this?
  - Can be costly in terms of Flip-flops for FSMs with large number of states.
- FPGAs are “Flip-flop rich”, therefore one-hot state machine encoding is often a good approach.

One-hot encoded FSM

- Even Parity Checker Circuit:

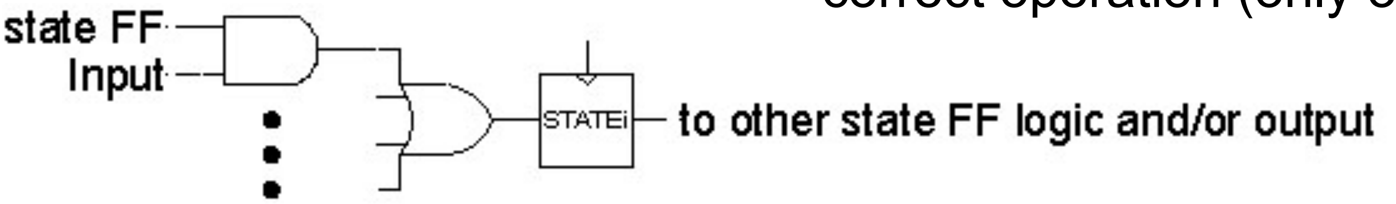




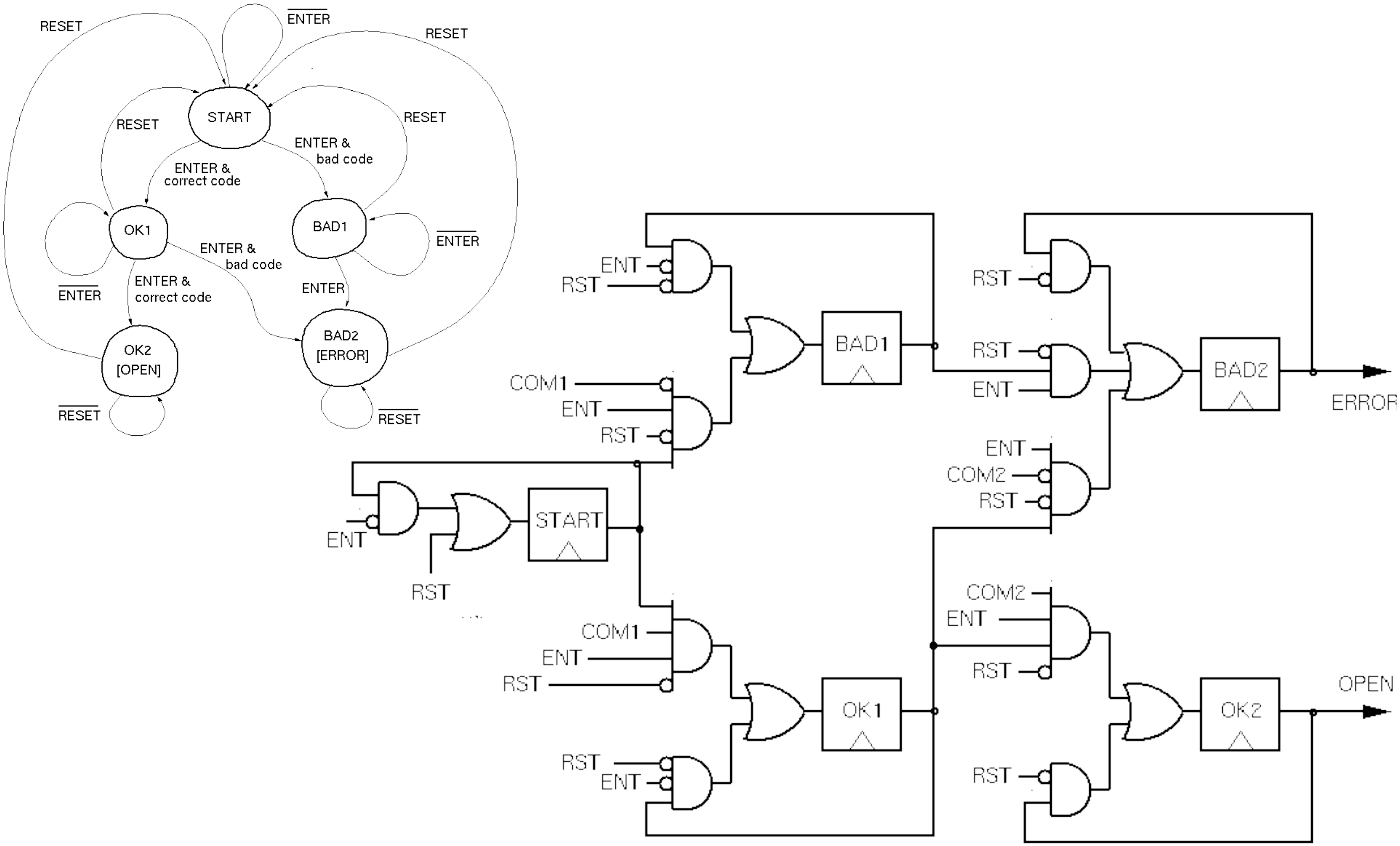
Circuit generated through direct inspection of the STD.

- In General:

- FFs must be initialized for correct operation (only one 1)

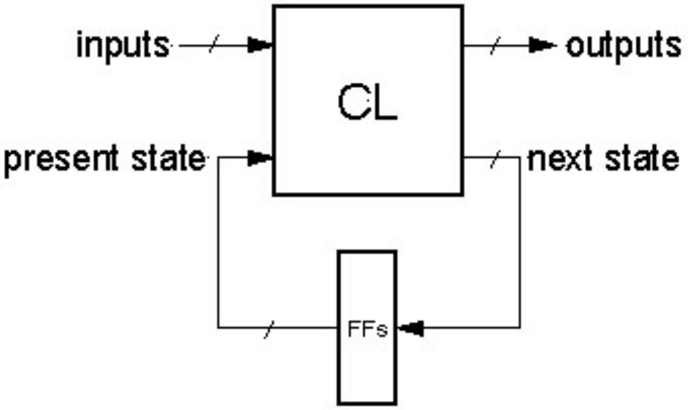


One-hot encoded combination lock

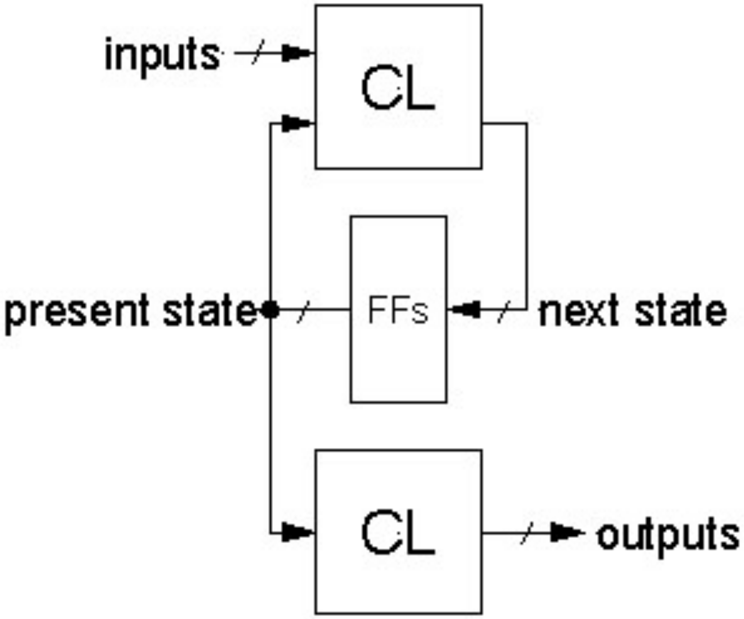


FSM Implementation Notes

- General FSM form:



- All examples so far generate output based only on the present state:



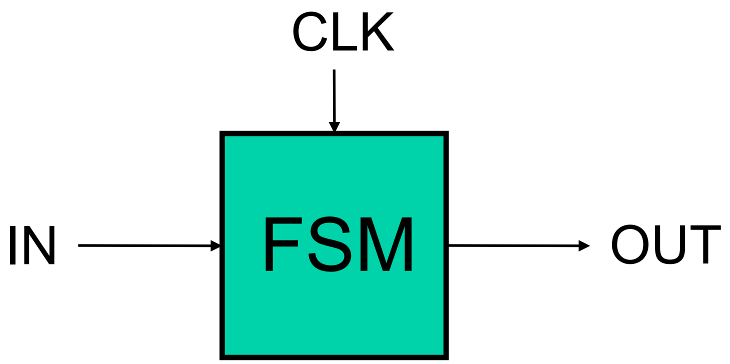
- Commonly name ***Moore Machine***

  (If output functions include both present state and input then called a *Mealy Machine*)

Finite State Machines

- **Example: Edge Detector**

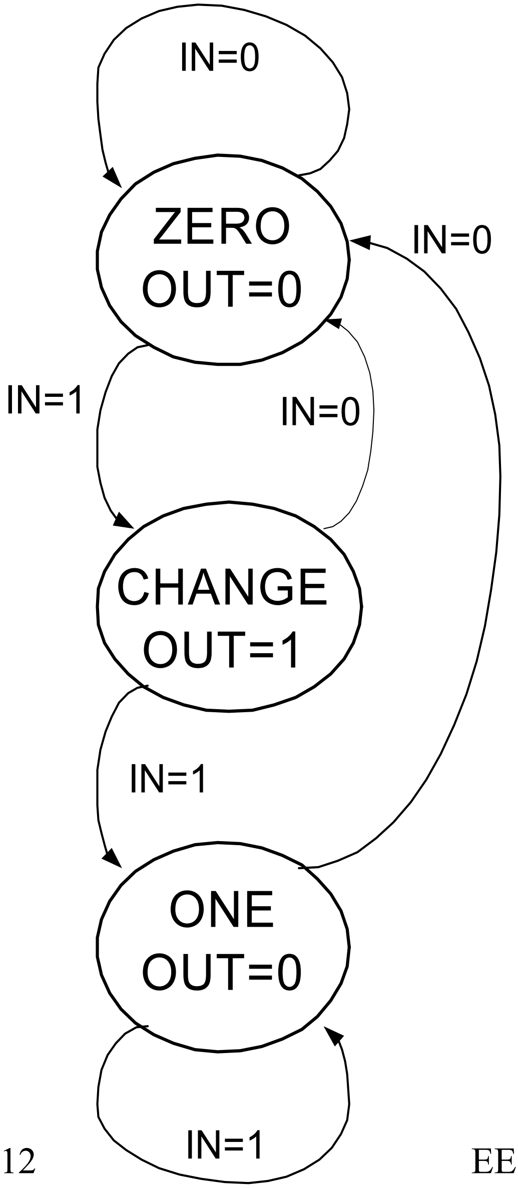
Bit are received one at a time (one per cycle),
such as: 000111010 ⟶ *time*



Design a circuit that asserts its output for one cycle when the input bit stream changes from 0 to 1.

Try two different solutions.

State Transition Diagram Solution A



	IN	PS	NS	OUT
ZERO	0	00	00	0
	1	00	01	0
CHANGE	0	01	00	1
	1	01	11	1
ONE	0	11	00	0
	1	11	11	0

Solution A, circuit derivation

	IN	PS	NS	OUT
ZERO	0	00	00	0
	1	00	01	0
CHANGE	0	01	00	1
	1	01	11	1
ONE	0	11	00	0
	1	11	11	0

IN \ PS	00	01	11	10
0	0	0	0	-
1	0	1	1	-

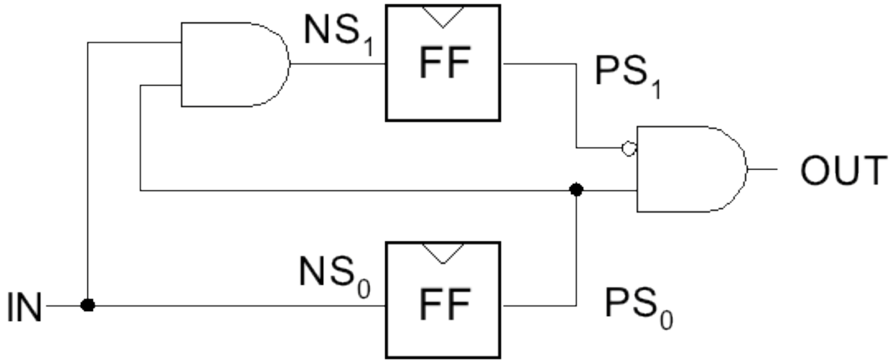
$NS_1 = IN\ PS_0$

IN \ PS	00	01	11	10
0	0	0	0	-
1	1	1	1	-

$NS_0 = IN$

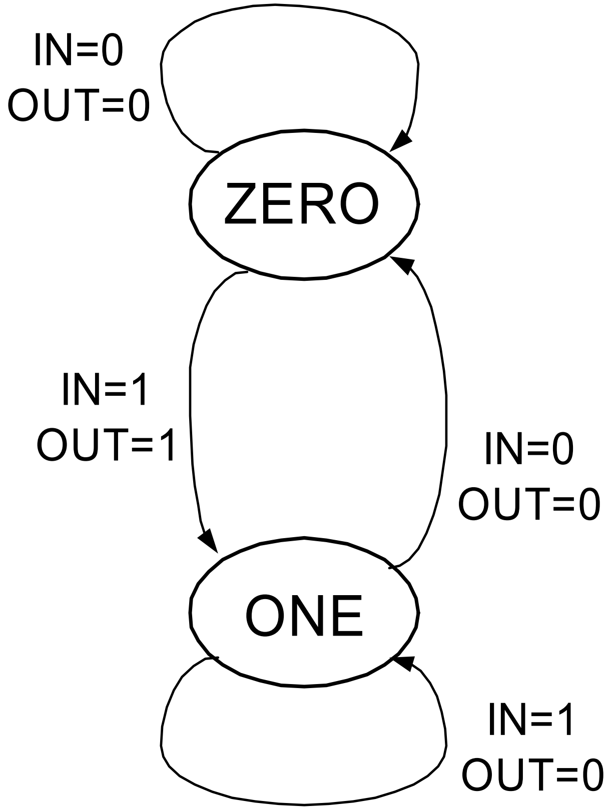
IN \ PS	00	01	11	10
0	0	1	0	-
1	0	1	0	-

$OUT = \overline{PS_1}\ PS_0$



Solution B

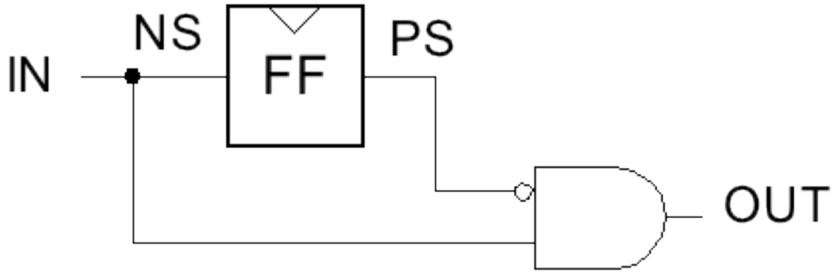
Output depends not only on PS but also on input, IN



Let ZERO=0,
ONE=1

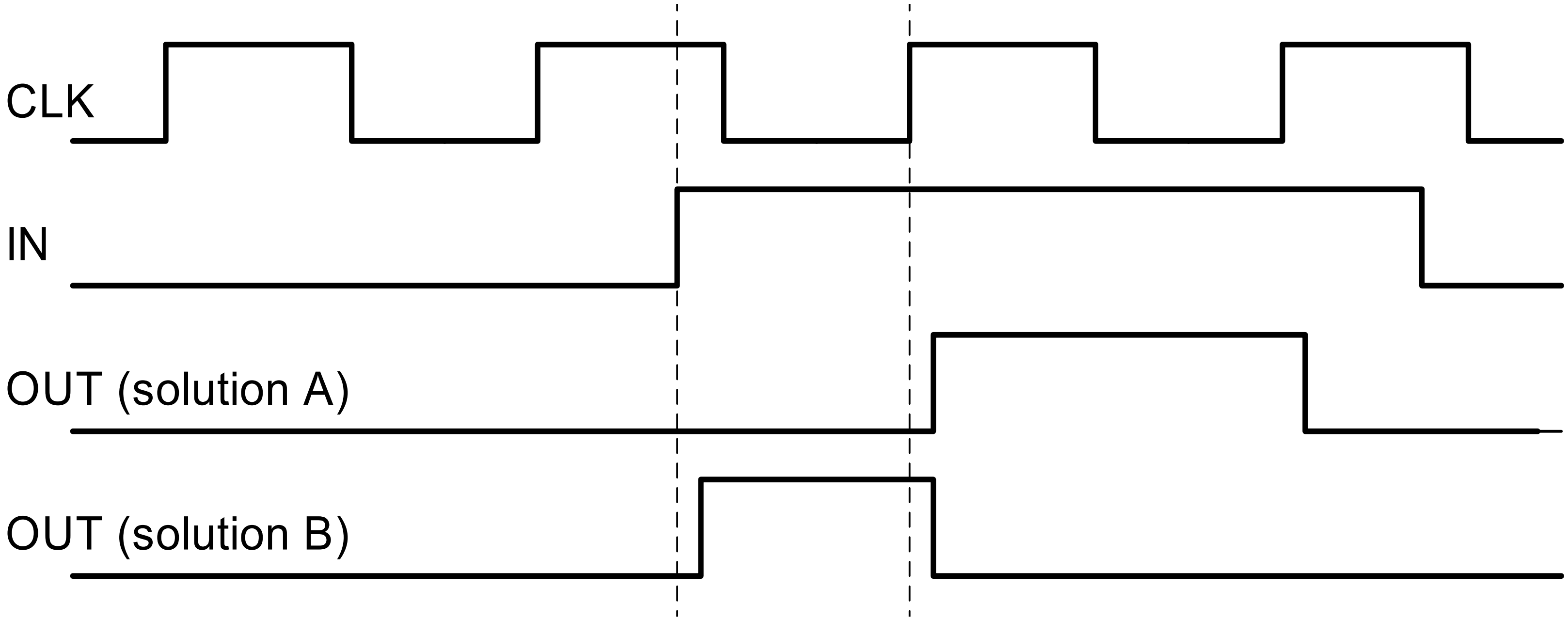
IN	PS	NS	OUT
0	0	0	0
0	1	0	0
1	0	1	1
1	1	1	0

NS = IN, OUT = IN PS’



What’s the *intuition* about this solution?

Edge detector timing diagrams



- Solution A: output follows the clock
- Solution B: output changes with input rising edge and is asynchronous wrt the clock.

FSM Comparison

Solution A

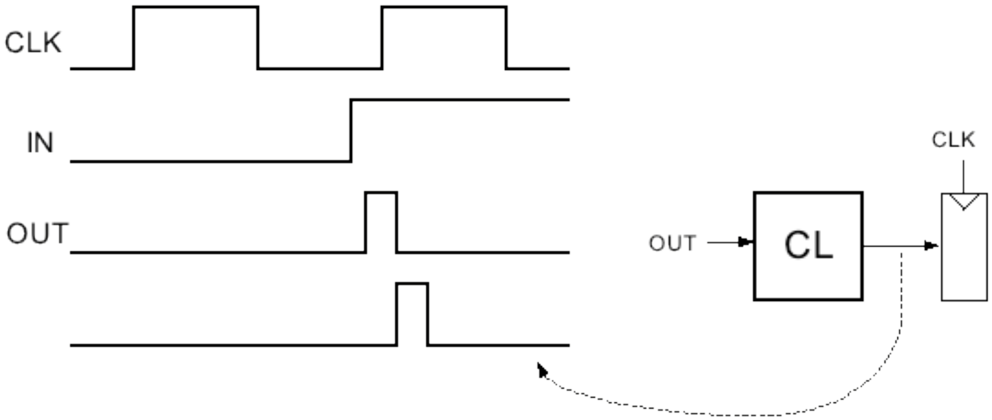
Moore Machine

- output function only of PS
- maybe more states (why?)
- synchronous outputs
  - no glitches
  - one cycle "delay"
  - full cycle of stable output

Solution B

Mealy Machine

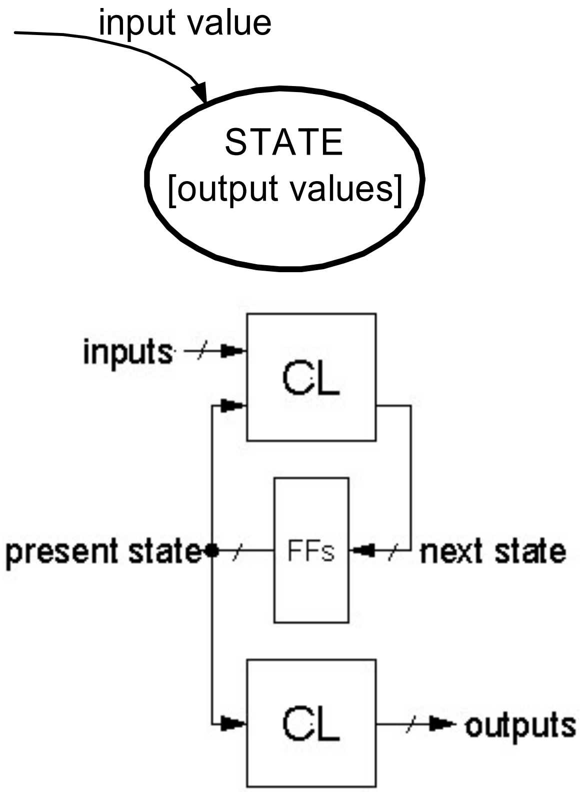
- output function of both PS & input
- maybe fewer states
- asynchronous outputs
  - if input glitches, so does output
  - output immediately available
  - output may not be stable long enough to be useful (below):



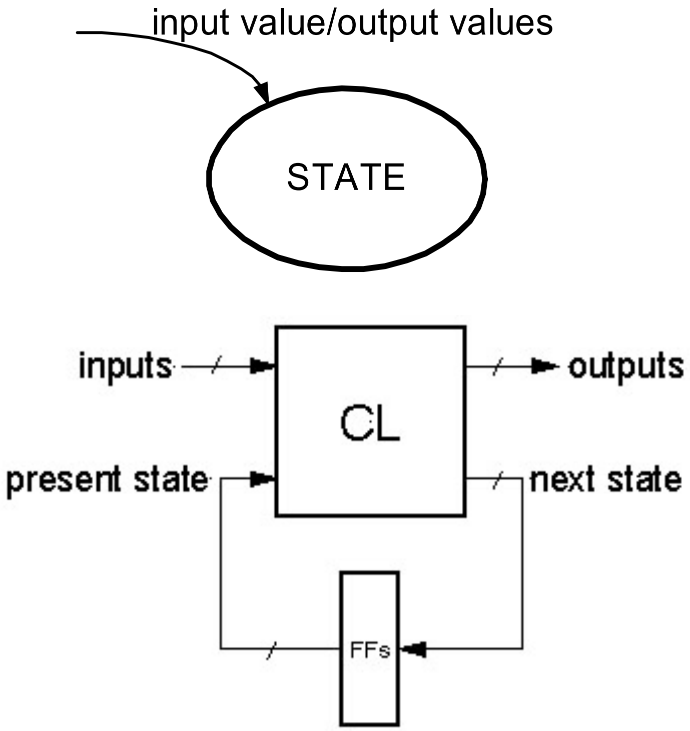
If output of Mealy FSM goes through combinational logic before being registered, the CL might delay the signal and it could be missed by the clock edge.

FSM Recap

Moore Machine



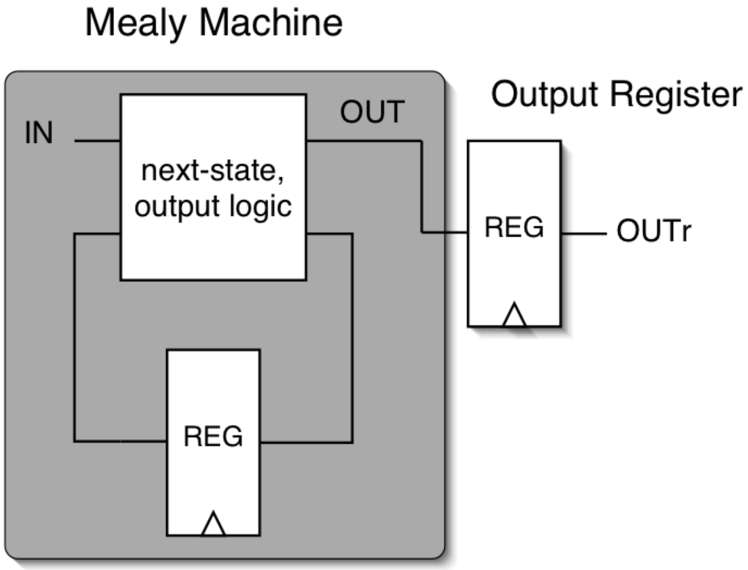
Mealy Machine

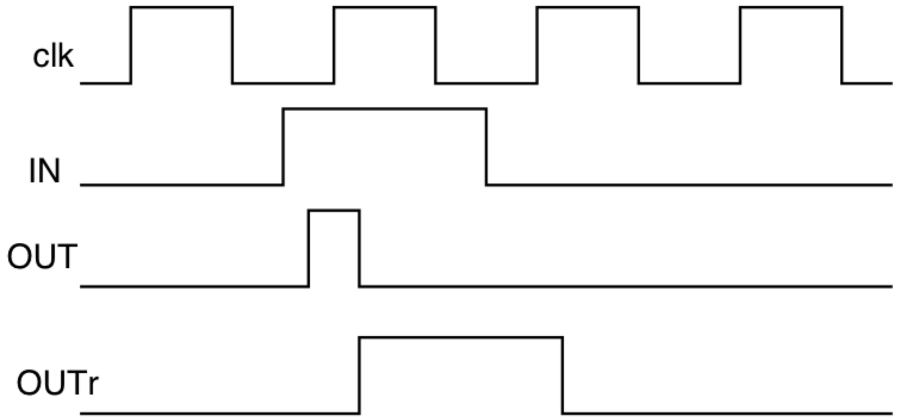


Both machine types allow one-hot *implementations.*

Final Notes on Moore versus Mealy

1. A given state machine *could* have *both* Moore and Mealy style outputs. Nothing wrong with this, but you need to be aware of the timing differences between the two types.
2. The output timing behavior of the Moore machine can be achieved in a Mealy machine by "registering" the Mealy output values:





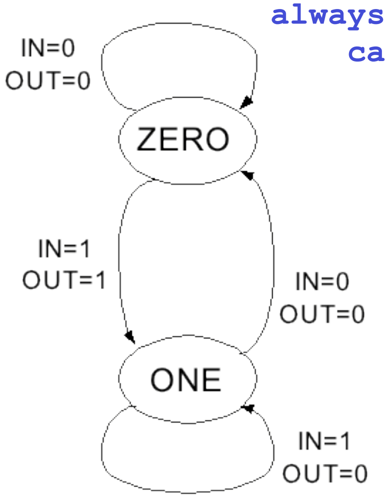
General FSM Design Process with Verilog Implementation

Design Steps:

1. Specify **circuit function** (English)
2. Draw **state transition diagram**
3. Write down **symbolic state transition table**
4. Assign encodings (bit patterns) to symbolic states
5. Code as Verilog behavioral description
   - ✓ Use parameters to represent encoded states.
   - ✓ Use separate always blocks for register assignment and CL logic block.
   - ✓ Use case for CL block. Within each case section assign all outputs and next state value based on inputs. *Note: For Moore style machine make outputs dependent only on state not dependent on inputs.*

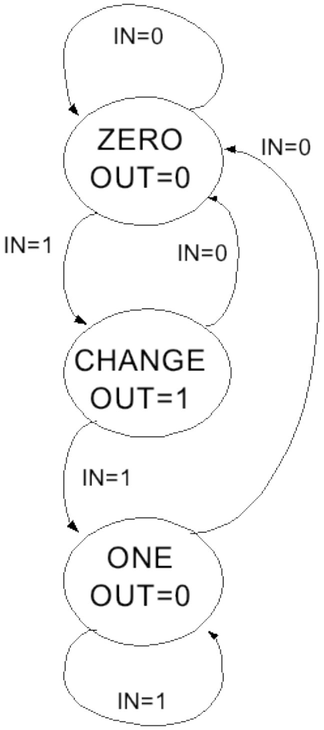
FSMs in Verilog

Mealy Machine



```
always @(posedge clk)
          if (rst) ps <= ZERO;
          else ps <= ns;
always @(ps in)
    case (ps)
      ZERO: if (in) begin
                   out = 1’b1;
                   ns = ONE;
                 end
                 else begin
                   out = 1’b0;
                   ns = ZERO;
                 end
      ONE: if (in) begin
                 out = 1’b0;
                 ns = ONE;
               end
               else begin
                 out = 1’b0;
                 ns = ZERO;
               end
      default: begin
                  out = 1’bx;
                  ns = default;
                 end
```

Moore Machine



```
always @(posedge clk)
          if (rst) ps <= ZERO;
          else ps <= ns;
always @(ps in)
    case (ps)
      ZERO: begin
               out = 1’b0;
               if (in) ns = CHANGE;
                else ns = ZERO;
             end
      CHANGE: begin
                out = 1’b1;
                if (in) ns = ONE;
                else ns = ZERO;
               end
         ONE: begin
                out = 1’b0;
                if (in) ns = ONE;
                else ns = ZERO;
      default: begin
                out = 1’bx;
                ns = default;
               end
```