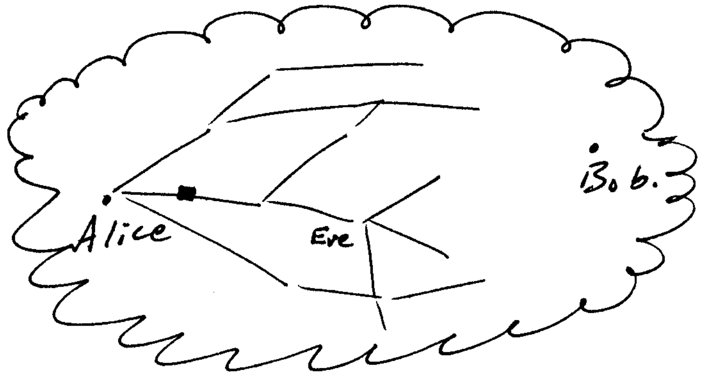
Authentication & Digital Signatures

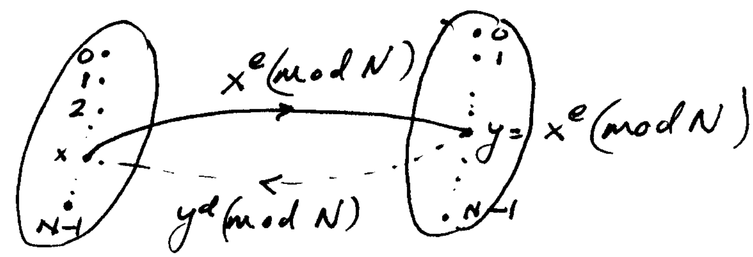


Privacy — Eavesdropper cannot get any information about plaintext message.

Authentication — Message was sent by the person who claims to send it.

<u>Public-key</u>

Recall RSA:



Some Considerations:

1) Choose $e = 3$ so encryption is fast.
2) Computing d from (N, e) is as hard as factoring N.
3) If $x < \sqrt[3]{N}$ decrypting $y = E(x)$ is easy.

The difficulty of decryption relies on x^e wrapping around N a large and unknown number of times.

4) Public key cryptosystems typically much slower than symmetric key schemes. Therefore use public key system to establish a private key.

5) To send a message m, randomize it by appending a random string in the high order bits. $x = r \cdot m$ where $\cdot$ denotes concatenation.

RSA Digital Signature:

A digital signature of a message m by Alice should:

1) Prove that Alice actually signed the message m. i.e. no one else should be able to produce ~~the~~ a signed message of their choice.

2) The adversary should not be able to use the signed message m to produce a signed version of m'.

To sign m using RSA, Alice simply decrypts m using her private key:

$$S(m) = m^d \pmod N.$$

To verify the signature, anyone can look up her public key (N, e) and check that $S(m)^e \bmod N \equiv m$.

In practice, we must prepare m suitably before decrypting it. m is usually restricted to at most half the length of N, and is mapped to a number between 0 and $N-1$ by a redundancy mapping. For example, if $m = m_1 m_2 \cdots m_k$ is a k byte string, then the redundant message might be

$$m_r = m_1\, \pi(m_1)\; m_2\, \pi(m_2)\; \cdots\; m_k\, \pi(m_k)$$

where π is some fixed permutation of 8 bit strings. This prevents the following type of attack:

Choose some $x \pmod N$, apply Alice's public encryption function to obtain $y \equiv x^e \pmod N$. Claim that Alice signed y to obtain x.

y is very unlikely to satisfy the redundancy format.

<u>Certificates</u>:

When Bob is sending a message to Alice, how does he verify that the posted public key (N_A, e_A) is really Alice's key? Certificates provide a way of doing this. Certificates require a trusted party who everyone trusts, and whose public key (N, e) is well known. The trusted party verifies that Alice is who she claims and signs a message that says:

Alice's public key is (N_A, e_A) signed ...

Now Alice can publicly post this certificate and ~~any~~ anyone who wishes can verify the trusted party's signature to reassure themselves about the authenticity of the key.

Message Authentication Codes (MACs):

In the symmetric key setting, authentication is accomplished by a MAC. In this setting Alice and Bob share a secret key K, and are using a secure block cipher (E_K, D_K) on n bit blocks.

If the message m to be authenticated is shorter than n bits, then it can be authenticated by simply sending $(m, E_K(m))$. Since E_K is indistinguishable from a random permutation, an adversary ~~who does not know~~ cannot authenticate any previously unseen messages.

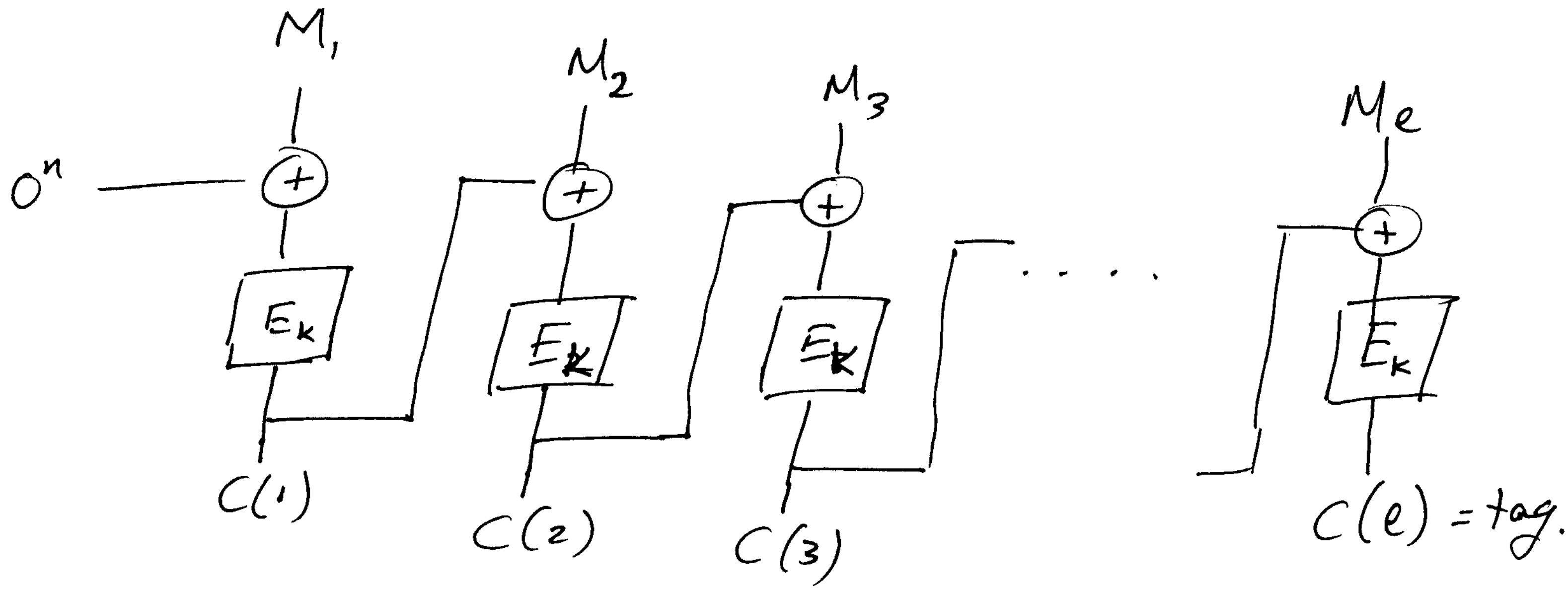
To authenticate a long message: $m = m_1 m_2 \cdots m_\ell$ where each m_i is n bits long, we can use CBC mode to create an n bit tag t. Then the authenticate message is (m, t):

CBC MAC:

$$C(0) = 0^n$$

$$C(i) = E_K(C(i-1) \oplus M_i)$$

$$\text{tag} = C(\ell).$$



If the message length is fixed, i.e. ℓ is not allowed to vary, then the CBC MAC can be proved to be secure.

For variable length messages provable security can be obtained by a simple augmentation of the above procedure. Pick a second block cipher $(E'_{K'}, D'_{K'})$. Let the authentication tag be $E'_{K'}(C(\ell))$.

Formalizing security of MACs:

We allow the adversary, A, access to a box that will authenticate messages m of his choice. Thus the adversary obtains a list $(m_1, t_1), (m_2, t_2), \ldots, (m_j, t_j)$.

The advantage of the adversary

$$Adv(A) = Pr[A \text{ can create } (m', t') \text{ which is a valid authenticated message and } m' \neq m_i \ \forall i \leq j]$$

The formal proof of security of the fixed length (length ℓ) CBC MAC says:

1) If E_K is a random permutation then

$$Adv(A) \leq \frac{1.5 j^2 \ell^2}{2^n}$$

2) Now a simple reduction shows that ~~against~~ ~~a~~ if E_K is a secure block cipher:

$$Adv(A) \leq \varepsilon + \frac{1.5 j^2 \ell^2}{2^n}$$

where ε is an upper bound on the adversary's advantage in distinguishing E_K and a random permutation

Let us examine some examples of insecure MACs:

$$C(i) = E_K(M_i)$$

$$tag = C(1) \oplus \cdots \oplus C(\ell)$$

The following adversary has advantage 1: the adversary makes no queries and creates a tag for a new message $M = x\,x$ i.e. $\ell = 2$ and $M_1 = M_2 = x$. Now ~~tag~~ $tag = E_K(x) \oplus E_K(x) = 0^n$ no matter what E_K is.

We could strengthen this MAC by letting

$$C(i) = E_K(i \cdot M_i)$$

i.e. concatenate i with M_i.

Now the above attack no longer works.
The following adversary has advantage 1:

Query $M_1 = a_1 \cdot a_2$ $\quad M_2 = a_1 \cdot b_2$ $\quad M_3 = b_1 \cdot a_2$

$\quad t_1 \quad t_2 \quad t_3$.

Then the tag for $M = b_1 \cdot b_2$ is $t_1 \oplus t_2 \oplus t_3$.