

# Electronic Voting

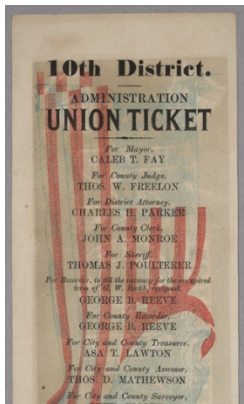
***CS 161: Computer Security***

**Prof. David Wagner**

**April 25, 2013**

# Security Goals for an Election

- Integrity: No election fraud
- Transparency: Everyone – especially the loser – must be able to verify that the election was conducted appropriately
- Privacy: No one learns how the voter has voted
- Secret ballot: Voter cannot prove how she voted



ABSENT VOTER BALLOT  
STUB A No. 7720

PLACE HOLES  
OVER POSTS

INSERT CARD THIS SIDE UP

STUB B No. 7720  
ABSENT VOTER BALLOT

**IMPORTANT  
DO NOT  
DETACH STUB**

.	.	39	58	77	96	115	134	153	172	191	210
1	20	40	.	.	116	.	135	154	173	192	.
2	.	.	59	78	97	.	.	155	174	.	211
.	21	.	.	.	.	117	136	155	174	193	.
3	.	41	60	79	98	.	.	156	175	.	212
.	22	42	.	.	118	137	156	175	194	.	213
4	.	.	61	80	99	.	138	157	176	195	.
.	23	43	.	.	.	119	139	158	177	.	214
5	.	44	62	81	100	.	140	159	178	196	.
.	24	.	63	82	101	.	141	160	179	.	215
6	.	45	.	.	102	121	140	159	178	197	.
.	25	.	64	83	103	.	142	161	180	.	216
7	.	46	.	.	104	122	141	160	179	198	.
.	26	46	65	84	105	.	143	162	181	199	217
8	.	47	.	.	106	123	142	161	180	.	218
.	27	.	66	85	107	.	144	163	182	.	219
9	.	48	.	.	108	124	143	162	181	200	.
.	28	.	67	86	109	.	145	164	183	.	220
10	.	49	.	.	110	125	144	163	182	201	.
.	29	.	68	87	111	.	146	165	184	.	221
11	.	50	.	.	112	126	145	164	183	202	.
.	30	.	69	88	113	.	147	166	185	.	222
12	.	51	.	.	114	127	146	165	184	203	.
.	31	.	70	89	115	.	148	167	186	.	223
13	.	52	.	.	116	128	147	166	185	204	.
.	32	.	71	90	117	.	149	168	187	.	224
14	.	53	.	.	118	129	148	167	186	205	.
.	33	.	72	91	119	.	150	169	188	.	225
15	.	54	.	.	120	130	149	168	187	206	.
.	34	.	73	92	121	.	151	170	189	.	226
16	.	55	.	.	122	131	150	169	188	207	.
.	35	.	74	93	123	.	152	171	190	.	227
17	.	56	.	.	124	132	151	170	189	208	.
.	36	.	75	94	125	.	153	172	191	.	228
18	.	57	.	.	126	133	152	171	190	.	229
.	37	.	76	95	127	.	154	173	192	.	230
19	.	58	.	.	128	134	153	172	191	.	231
.	38	.	77	96	129	.	155	174	193	.	232

TO BE FILLED IN BY ELECTION BOARD ONLY

PRECINCT NO. WRITE-IN NO.

INSERT  
CARD  
HERE



SPECIAL BALLOT  
GENERAL ELECTION  
NOVEMBER 7, 2000

OFFICIAL BALLOT  
GENERAL ELECTION  
NOVEMBER 7, 2000

1 A

FEDERAL	
GEORGE W. BUSH PRESIDENT DICK CHENEY VICE PRESIDENT	3 →
AL GORE PRESIDENT JOE LIEBERMAN VICE PRESIDENT	5 →
HARRY BROWNE PRESIDENT ART OLIVER VICE PRESIDENT	7 →
RALPH NADER PRESIDENT WINDONIA LADUKE VICE PRESIDENT	9 →
JAMES HARRIS PRESIDENT MARGARET TROWE VICE PRESIDENT	11 →
JOHN HAGELIN PRESIDENT NAT. GOLDBERGER VICE PRESIDENT	13 →

FOR PRESIDENT AND VICE PRESIDENT  
OF THE UNITED STATES  
(Vote for ONE GROUP)

1 B

PAT BUCHANAN PRESIDENT EZOLA FOSTER VICE PRESIDENT	← 4
DAVID McREYNOLDS PRESIDENT MARY CAL HOLLIS VICE PRESIDENT	← 6
HOWARD PHILLIPS PRESIDENT J. CURTIS FRAZIER VICE PRESIDENT	← 8
MUNICA MOOREHEAD PRESIDENT GLORIA La RIVA VICE PRESIDENT	← 10
WRITE IN CANDIDATE To vote for a write-in candidate, follow the directions on the long stub of your ballot card.	

g turn back  
page

"Butterfly Ballot"

Palm Beach County



# Confusion at Palm Beach County polls

Some Al Gore supporters may have mistakenly voted for Pat Buchanan because of the ballot's design.

Although the Democrats are listed second in the column on the left, they are the third hole on the ballot.

Punching the second hole casts a vote for the Reform party.

ELECTORS FOR PRESIDENT AND VICE PRESIDENT	
(REPUBLICAN) GEORGE W. BUSH - PRESIDENT DICK CHENEY - VICE PRESIDENT	3 →
(DEMOCRATIC) AL GORE - PRESIDENT JOE LIEBERMAN - VICE PRESIDENT	5 →
(LIBERTARIAN) HARRY BROWNE - PRESIDENT ART OLIVIER - VICE PRESIDENT	7 →
(GREEN) RALPH NADER - PRESIDENT WINONA LaDUKE - VICE PRESIDENT	9 →
(SOCIALIST WORKERS) JAMES HARRIS - PRESIDENT MARGARET TROWE - VICE PRESIDENT	11 →
(NATURAL LAW) JOHN HAGELIN - PRESIDENT NAT GOLDHABER - VICE PRESIDENT	13 →

(REFORM) PAT BUCHANAN - PRESIDENT EZOLA FOSTER - VICE PRESIDENT	← 4
(SOCIALIST) DAVID McREYNOLDS - PRESIDENT MARY CAL HOLLIS - VICE PRESIDENT	← 6
(CONSTITUTION) HOWARD PHILLIPS - PRESIDENT J. CURTIS FRAZIER - VICE PRESIDENT	← 8
(WORKERS WORLD) MONICA MOOREHEAD - PRESIDENT GLORIA La RIVA - VICE PRESIDENT	← 10
WRITE-IN CANDIDATE To vote for a write-in candidate, follow the directions on the long stub of your ballot card.	

(A vote for the candidates will actually be a vote for their electors.)  
(Vote for Group)

# Another anomaly during the 2000 election

From: Lana Hires

Subject: 2000 November Election

I need some answers! Our department is being audited by the County.

I have been waiting for someone to give me an explanation as to why Precinct 216 gave Al Gore a minus 16022 when it was uploaded. Will someone please explain this so that I have the information to give the auditor instead of standing here "looking dumb".





# Summary Ballot Instructions

Press the candidate name or contest title to return to a contest.

Vote button will light up when you may cast your ballot.

Press here to cast your ballot now

## Best Automobile Manufacturer Vote For ONE



FORD

## Best Vocal Artist (Vote for Not More Than TWO)



FA

No selection made.

## Best Ice-Cream Flavor Vote For ONE

No selection made.

## Proposition 1

No selection made.

## Proposition 2

No selection made.



Back

01 / 01



Question: What are the security requirements for electronic voting machines?

1. Machine must allow each authorized voter to vote exactly once; must prevent tampering with votes after they are cast.
2. Machine should be verifiably trustworthy.
3. Machine must randomize the order in which votes were cast.
4. Machine must not give voter a “receipt”.

☞ Security goals for an election:  
Integrity, Transparency, Privacy, Secret ballot

Nov 4, 2002:

State of Georgia votes on Diebold DREs.

March 18, 2003:

Diebold source code leaks.

July 23, 2003:

Tadayoshi Kohno, Adam Stubblefield, Avi Rubin, Dan Wallach, “Analysis of an Electronic Voting System”.

# The voter authorization protocol



QueryStatus



ACTIVE (0x01)



(record vote)

*smartcard*



SetStatus CANCELED (0x08)



Status = CANCELED

Succeeded



# The voter authorization protocol



QueryStatus [Are you a valid card?]

ACTIVE (0x01) [Yup.]

(record vote)

smartcard

[Please cancel yourself.]  
SetStatus CANCELED (0x08)

Status = CANCELED

Succeeded [Ok.]

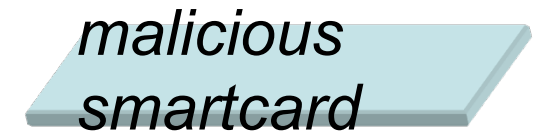
# Attack!

QueryStatus →  
← ACTIVE (0x01)

(record vote)



SetStatus CANCELED (0x08) →  
← Succeeded



QueryStatus →  
← ACTIVE (0x01)

(record another vote)

SetStatus CANCELED (0x08) →  
← Succeeded

# Authenticating election officials

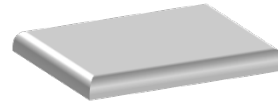
What kind of card are you?

An administrator card.



What's the secret PIN?

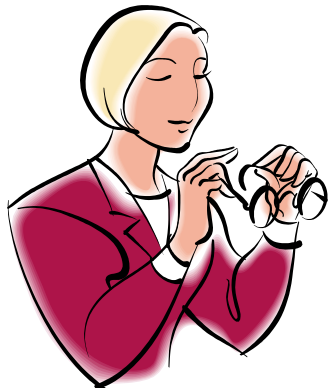
2301



What's the secret PIN?

2301

Ok, you have admin access.



## Source code excerpts

```
#define DESKEY ((des_key*)"F2654hD4")
```

```
DESCBCEncrypt((des_c_block*)tmp,  
(des_c_block*)record.m_Data, totalSize,  
DESKEY, NULL, DES_ENCRYPT);
```



## Source code excerpts

```
// LCG - Linear Congruential Generator -  
// used to generate ballot serial numbers  
// A psuedo-random-sequence generator  
// (per Applied Cryptography, Bruce Schneier)  
  
int lcgGenerator(int lastSN) {  
    return ((lastSN*1366) + 150889)%714025;  
}
```

“Unfortunately, linear congruential  
generators cannot be used for cryptography.”  
— Applied Cryptography, p.369

# **More than 4,500 North Carolina votes lost because of mistake in voting machine capacity**

JACKSONVILLE, N.C. (AP) — More than 4,500 votes have been lost in one North Carolina county because officials believed a computer that stored ballots electronically could hold more data than it did. Scattered other problems may change results in races around the state.

Officials said UniLect Corp., the maker of the county's electronic voting system, told them that each storage unit could handle 10,500 votes, but the limit was actually 3,005 votes.

# Machine error gives Bush 3,893 extra votes in Ohio

By John McCarthy, Associated Press

COLUMBUS, Ohio — An error with an electronic voting system gave President Bush 3,893 extra votes in suburban Columbus, elections officials said.

Franklin County's unofficial results had Bush receiving 4,258 votes to Democrat John Kerry's 260 votes in a precinct in Gahanna. Records show only 638 voters cast ballots in that precinct. Bush's total should have been recorded as 365.

## Fall 2003, Ohio

"I am committed to helping Ohio deliver its electoral votes to the president."

-- Wally O'Dell

CEO of Diebold



<http://denvervoice.org>



# California Top-to-Bottom Review

In 2007, California Secretary of State Debra Bowen commissions a review of California's voting systems.

43 experts (led by David Wagner & Matt Bishop) examine voting systems used nationally.



**THE SECRETARY**  
Bowen opens the public hearing in Sacramento.

# Technical findings of the CA TTBR

All voting systems examined have serious security problems:

- None followed sound engineering principles expected of security-critical systems.
- All were vulnerable to viral attacks: one outsider could subvert all voting machines countywide

## Example flaw (Diebold/Premier system)

Bug: The code that reads data off the memory card has buffer overrun vulnerabilities.

Attack:

1. Attacker writes malicious code onto 1 card
2. When central PC reads votes off card on election night, it gets infected
3. Infected PC writes malicious code onto all cards used in the next election, infecting entire county



## Quotes from the reports

“We found pervasive security weaknesses throughout the Sequoia software. Virtually every important software security mechanism is vulnerable to circumvention.”

“Our study of the Diebold source code found that the system does not meet the requirements for a security-critical system. It is built upon an inherently fragile design and suffers from implementation flaws that can expose the entire voting system to attacks.”

“The Hart software and devices appear to be susceptible to a variety of attacks which would allow an attacker to gain control of some or all of the systems in a county. [...] Many of these attacks can be mounted in a manner that makes them extremely hard to detect and correct. We expect that many of them could be carried out in the field by a single individual, without extensive effort, and without long-term access to the equipment.”



# Trojan Horses and the Insider Threat



Ronald Dale Harris

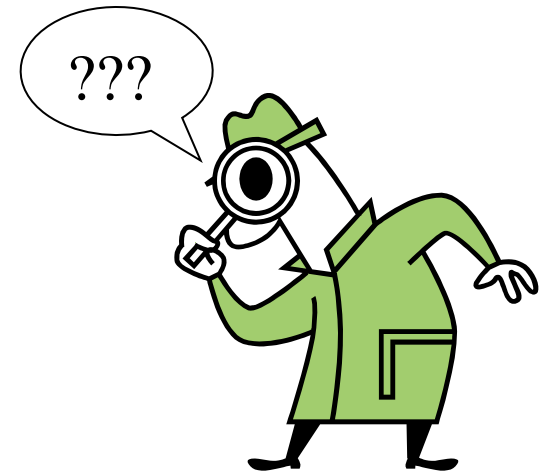
Employee, Gaming Control Board, 1983-1995

Arrested, Jan 15, 1995

Convicted, Sept 23, 1997, for rigging slot machines

# Attempted Trojan Horse in Linux Kernel

```
...
    schedule();
    goto repeat;
}
if ((options == (__WCLONE|__WALL)) && current->uid = 0))
    retval = -EINVAL;
retval = -ECHILD;
end_wait4:
current->state = TASK_RUNNING;
...
```



# Cyberattack on Google Said to Hit Password System

By JOHN MARKOFF

Ever since Google disclosed in January that Internet intruders had stolen information from its computers, the exact nature and extent of the theft has been a closely guarded company secret. But a person with direct knowledge of the investigation now says that the losses included one of Google's crown jewels, a password system that controls access by millions of users worldwide to almost all of the company's Web services, including e-mail and business applications.

The program, code named Gaia for the Greek goddess of the earth, was attacked in a lightning raid taking less than two days last December, the person said. Described publicly only once at a technical conference four years ago, the software is intended to enable users and employees to sign in with their password just once to operate a range of services.

# Trojan Horses and Voting Machines

Malicious logic hidden by an insider might, e.g., record votes incorrectly to favor one candidate. How would we defend a voting system against this kind of insider threat?

Potential solutions:

- Verify that the software is free of Trojans and will work correctly on all future elections.

(beyond the state of the art)

Voting on Satan's computer.

- Assume sw might contain Trojans. Verify that sw worked correctly in this particular election. (voter-verified paper records + random audits)



# SAMPLE BALLOT

<p style="text-align: center;"><b>N.C. STATE SENATE DISTRICT 25</b> You may vote for ONE</p> <p><input type="radio"/> WILLIAM R. (BILL) PURCELL      DEM</p> <p><input type="radio"/> _____</p>	<p style="text-align: center;"><b>DISTRICT COURT JUDGE DISTRICT 20</b> You may vote for ONE</p> <p><input type="radio"/> HUNT GWYN</p>	<p style="text-align: center;"><b>AMENDMENT II</b></p> <p>Constitutional amendment to provide that the General Assembly may place the clear proceeds of civil penalties, civil forfeitures, and civil fines collected by a State agency in a State fund to be used exclusively for maintaining free public schools.</p> <p><input type="radio"/> FOR</p> <p><input type="radio"/> AGAINST</p>
<p style="text-align: center;"><b>N.C. STATE HOUSE DISTRICT 69</b> You may vote for ONE</p> <p><input type="radio"/> PRYOR GIBSON      DEM</p> <p><input type="radio"/> HILDA L. MORTON      REP</p>	<p style="text-align: center;"><b>DISTRICT COURT JUDGE DISTRICT 20</b> You may vote for ONE</p> <p><input type="radio"/> LISA BLUE THACKER</p>	<p style="text-align: center;"><b>AMENDMENT III</b></p> <p>Constitutional amendment to provide for the first term of office for magistrates of the General Court of Justice to be two years and for subsequent terms to be four years.</p> <p><input type="radio"/> FOR</p> <p><input type="radio"/> AGAINST</p>
<p style="text-align: center;"><b>REGISTER OF DEEDS</b> You may vote for ONE</p> <p><input type="radio"/> JOANNE S. HUNTLEY      DEM</p>	<p style="text-align: center;"><b>DISTRICT COURT JUDGE DISTRICT 20</b> You may vote for ONE</p> <p><input type="radio"/> TANYA WALLACE</p>	
<p style="text-align: center;"><b>NON PARTISAN OFFICES</b></p> <p>Non partisan offices are not included in Straight Party voting and must be voted separately to be counted.</p>	<p style="text-align: center;"><b>DISTRICT COURT JUDGE DISTRICT 20</b> You may vote for ONE</p> <p><input type="radio"/> W. DAVID McSHEEHAN</p> <p><input type="radio"/> JOSEPH J. WILLIAMS</p>	
<p style="text-align: center;"><b>ASSOCIATE JUSTICE OF SUPREME COURT</b> You may vote for ONE</p> <p><input type="radio"/> SARAH PARKER</p> <p><input type="radio"/> JOHN M. TYSON</p>	<p style="text-align: center;"><b>NON PARTISAN OFFICES</b></p> <p>Additional instructions to Voter</p> <p>If you wish to write in a name for any of the following offices, write the name in the blank space provided and completely fill in the oval to the left of the name in order for your vote to count.</p>	
<p style="text-align: center;"><b>ASSOCIATE JUSTICE OF SUPREME COURT</b> You may vote for ONE</p> <p><input type="radio"/> RONNIE ANSLEY</p> <p><input type="radio"/> RACHELLEA HUNTER</p> <p><input type="radio"/> HOWARD E. MANNING, JR.</p> <p><input type="radio"/> BETSY McCRODDEN</p> <p><input type="radio"/> FRED MORRISON, JR.</p> <p><input type="radio"/> PAUL MARTIN NEWBY</p> <p><input type="radio"/> MARVIN SCHILLER</p> <p><input type="radio"/> JAMES A. WYNN, JR.</p>	<p style="text-align: center;"><b>BROWN CREEK SOIL AND WATER CONSERVATION DISTRICT SUPERVISOR</b> You may vote for ONE</p> <p><input type="radio"/> JOHN C. SPRINGER</p> <p><input type="radio"/> _____</p>	
<p style="text-align: center;"><b>JUDGE, COURT OF APPEALS</b> You may vote for ONE</p> <p><input type="radio"/> LINDA McGEE</p> <p><input type="radio"/> BILL PARKER</p>	<p style="text-align: center;"><b>STATE OF NORTH CAROLINA CONSTITUTIONAL AMENDMENTS</b></p>	
<p style="text-align: center;"><b>JUDGE, COURT OF APPEALS</b> You may vote for ONE</p> <p><input type="radio"/> WANDA G. BRYANT</p> <p><input type="radio"/> ALICE C. STUBBS</p>	<p style="text-align: center;"><b>AMENDMENT I</b></p> <p>Constitutional amendment to promote local economic and community development projects by (i) permitting the General Assembly to enact general laws giving counties, cities, and towns the power to finance public improvements associated with qualified private economic and community improvements within development districts, as long as the financing is secured by the additional tax revenues resulting from the enhanced property value within the development district and is not secured by a pledge of the local government's faith and credit or general taxing authority, which financing is not subject to a referendum; and (ii) permitting the owners of property in the development district to agree to a minimum tax value for their property, which is binding on future owners as long as the development district is in existence.</p> <p><input type="radio"/> FOR</p> <p><input type="radio"/> AGAINST</p>	
<p style="text-align: center;"><b>JUDGE, COURT OF APPEALS</b> You may vote for ONE</p> <p><input type="radio"/> BARBARA JACKSON</p> <p><input type="radio"/> ALAN THORNBURG</p>		
<p style="text-align: center;"><b>DISTRICT COURT JUDGE DISTRICT 20</b> You may vote for ONE</p> <p><input type="radio"/> CHRIS BRAGG</p>		
<p><b>TURN OVER TO CONTINUE VOTING</b></p>		



**Accu-Vote**

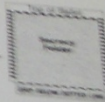


PLACE BALLOT INSIDE TO KEEP VOTING SECRET

**INSTRUCTIONS TO VOTERS**

After Voting is Completed:

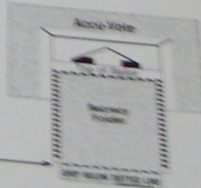
1. Center ballot inside this secrecy folder



2. Take ballot and secrecy folder to Accu-Vote machine



3. Feed ballot straight into machine while holding onto the area below dotted line



When inserting ballot into Accu-Vote Machine,

**GRIP BELOW**



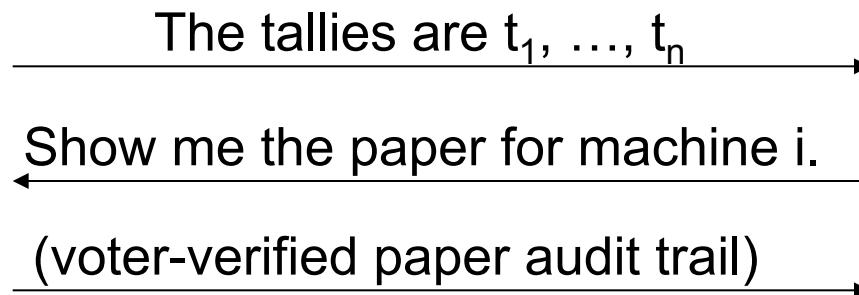
**DOTTED LINE**



# Statistical audit

- After election, randomly choose 1% of machines and manually recount the paper records on those machines. If paper count  $\neq$  electronic count, there was fraud.
- If  $\gg$  100 machines cheat, detection is likely. Consequently: If paper count = electronic count, then no more than  $\sim$ 100 machines cheated.

Prover  
(Elec. Official)



Verifier  
(skeptical voter)

# Conclusions

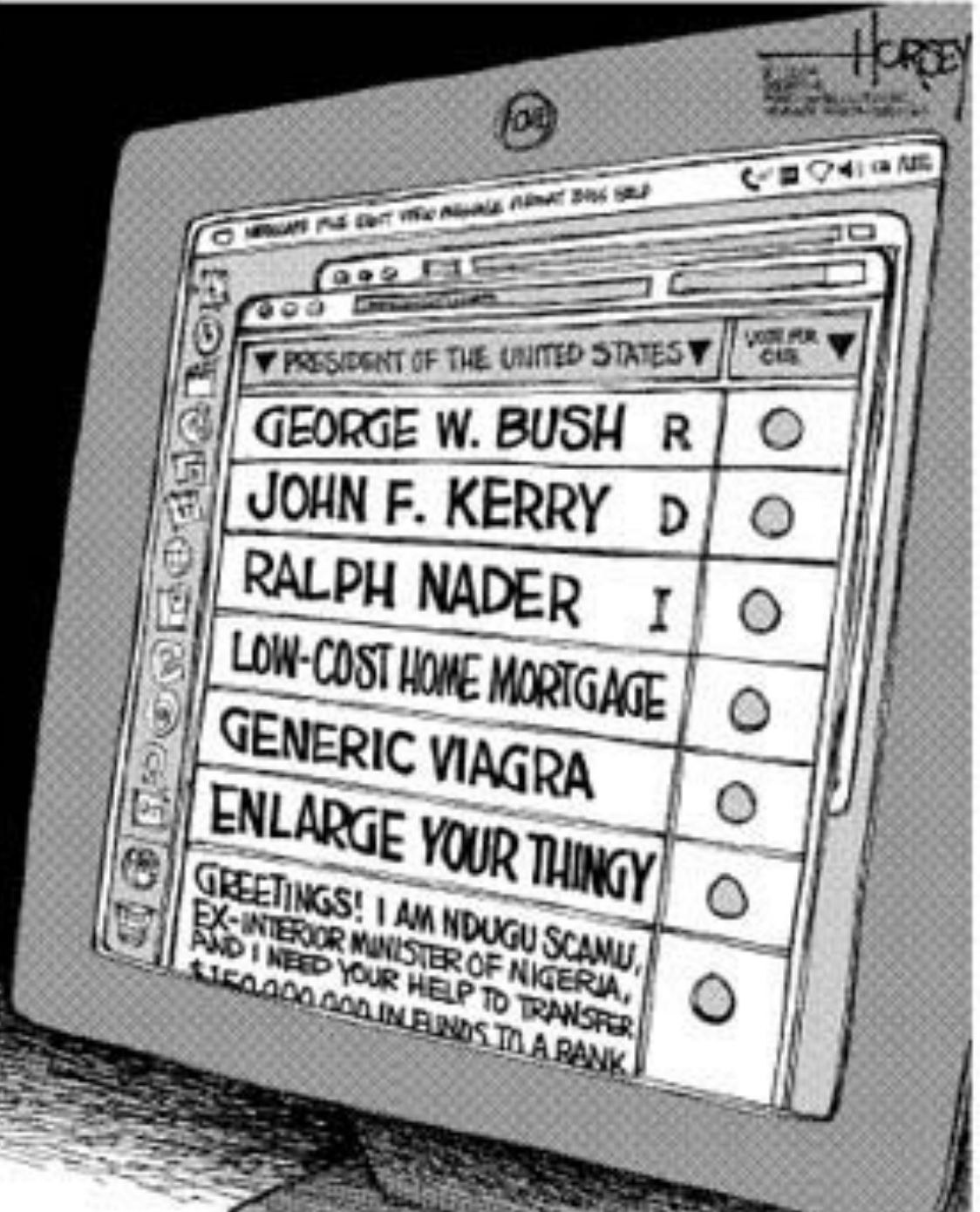
- E-voting security is hard, but...
- E-voting can be made secure and trustworthy, if it can be audited.
  
- Technical principles:
  - Two-person control, separation of duties
  - Statistical audit
  - Security against malicious insiders

# Lessons

- Understand security requirements before you design & deploy an information system.
- Independent review is valuable.
- Sometimes technical threats can be handled through non-technical defenses.
- Seek independent, end-to-end checks that the system is working properly.
- Securing systems against malicious insiders is extremely challenging.
- Business structure determines the technology that is built & deployed. If buyers cannot measure how secure a product is, be prepared for market failures.

**Extra Material**

THE HAZARDS OF ONLINE VOTING...



FORGE

HORSEY

▼ PRESIDENT OF THE UNITED STATES ▼

VOTE PER CAR ▼

GEORGE W. BUSH R

JOHN F. KERRY D

RALPH NADER I

LOW-COST HOME MORTGAGE

GENERIC VIAGRA

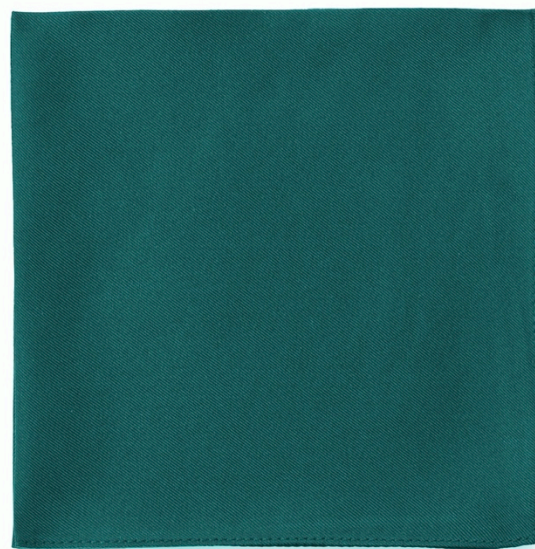
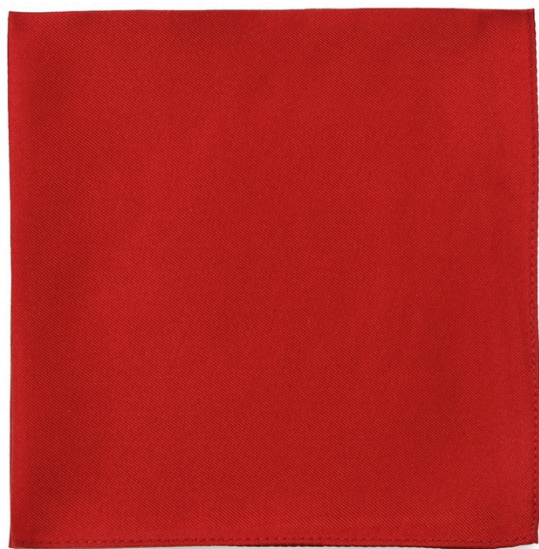
ENLARGE YOUR THINGY

GREETINGS! I AM NDUGU SCAMU,  
EX-INTERIOR MINISTER OF NIGERIA,  
AND I NEED YOUR HELP TO TRANSFER  
\$150,000,000 IN FUNDS TO A BANK.

Can I get a volunteer?

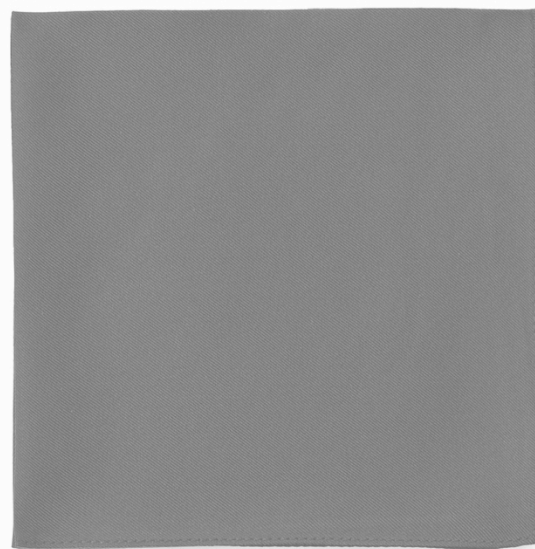
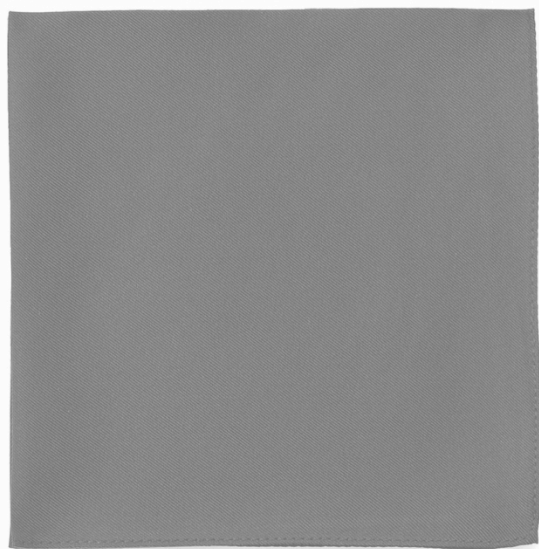


# Interactive proofs



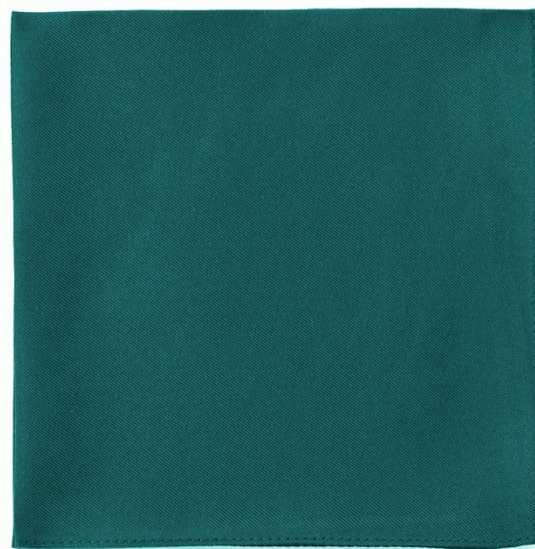
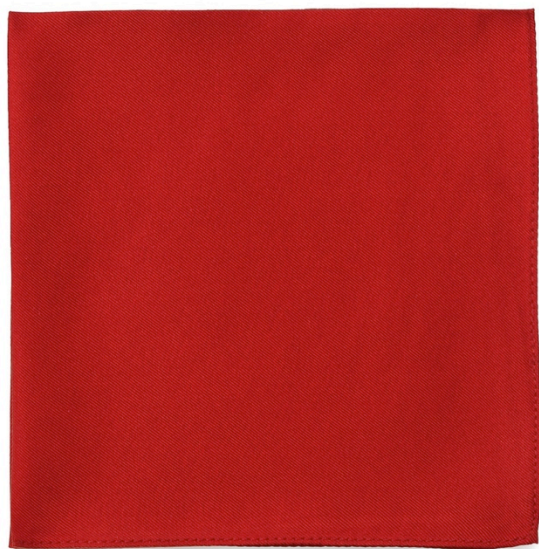
Here are two cloths.

# Interactive proofs



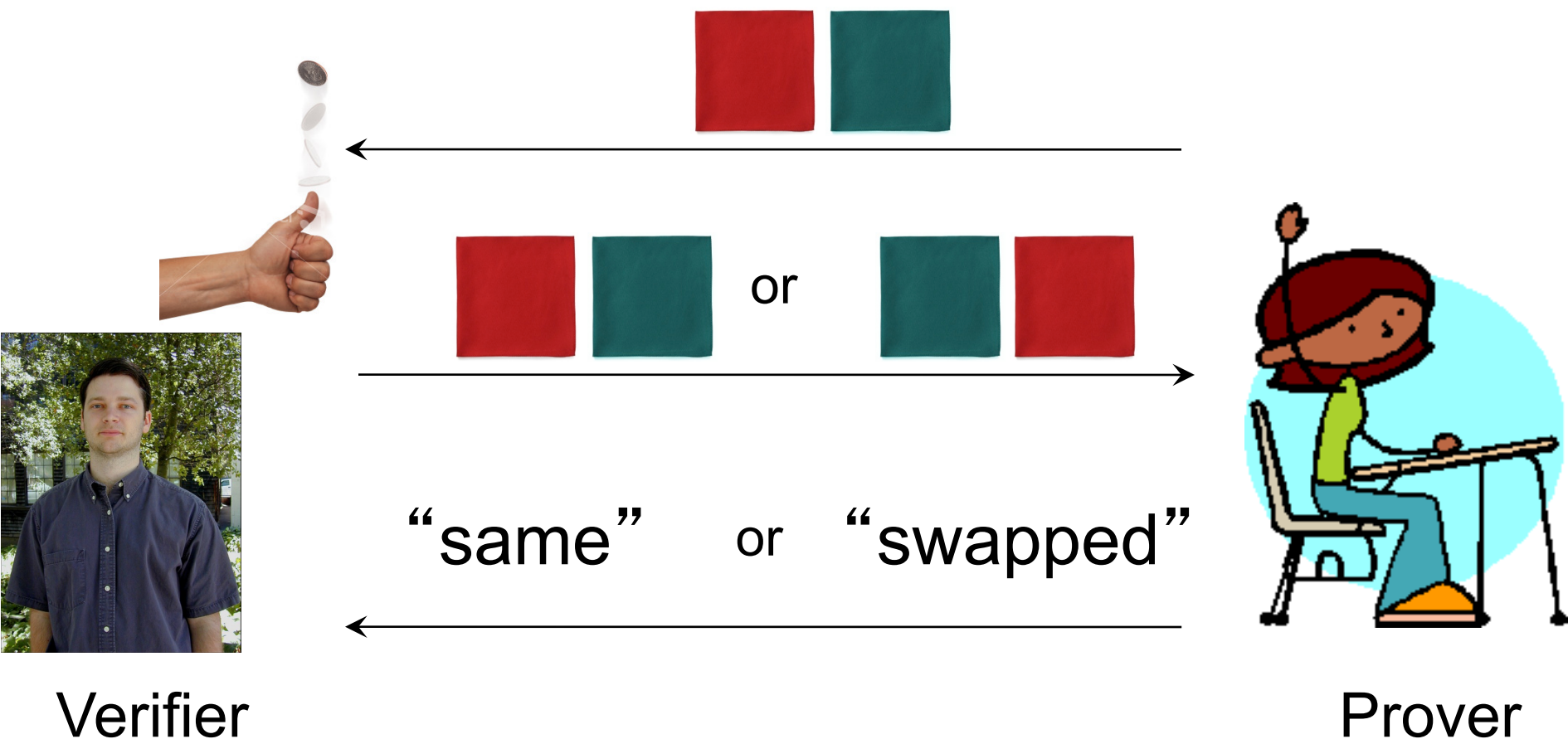
Imagine that I am red-green color-blind...

# Interactive proofs



How could you prove to me that you can distinguish the red cloth from the green cloth, if I am red-green color-blind?

# An interactive proof



# Sudoku

8			4		6			7
						4		
	1					6	5	
5		9		3		7	8	
				7				
	4	8		2		1		3
	5	2					9	
		1						
3			9		2			5

# Sudoku

8	3	5	4	1	6	9	2	7
2	9	6	8	5	7	4	3	1
4	1	7	2	9	3	6	5	8
5	6	9	1	3	4	7	8	2
1	2	3	6	7	8	5	4	9
7	4	8	5	2	9	1	6	3
6	5	2	7	8	1	3	9	4
9	8	1	3	4	5	2	7	6
3	7	4	9	6	2	8	1	5

# Goal: Prove the puzzle is solvable

8		4	6		7
	1			4	5
5	9		3	7	8
	4	8		2	1
	5	2			9
		1			
3		9	2		5

But I haven't learned anything about the solution. Darn.

8	3	5	4	1	6	9	2	7
2	9	6	8	5	7	4	3	1
4	1	7	2	9	3	6	5	8
5	6	9	1	3	4	7	8	2
1	2	3	6	7	8	5	4	9
7	4	8	5	2	9	1	6	3
6	5	2	7	8	1	3	9	4
9	8	1	3	4	5	2	7	6
2	7	4	9	6	2	8	1	5



Verifier



Prover

# You prepare your proof

<b>8</b>	<b>3</b>	<b>5</b>	<b>4</b>	<b>1</b>	<b>6</b>	<b>9</b>	<b>2</b>	<b>7</b>
<b>2</b>	<b>9</b>	<b>6</b>	<b>8</b>	<b>5</b>	<b>7</b>	<b>4</b>	<b>3</b>	<b>1</b>
<b>4</b>	<b>1</b>	<b>7</b>	<b>2</b>	<b>9</b>	<b>3</b>	<b>6</b>	<b>5</b>	<b>8</b>
<b>5</b>	<b>6</b>	<b>9</b>	<b>1</b>	<b>3</b>	<b>4</b>	<b>7</b>	<b>8</b>	<b>2</b>
<b>1</b>	<b>2</b>	<b>3</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>5</b>	<b>4</b>	<b>9</b>
<b>7</b>	<b>4</b>	<b>8</b>	<b>5</b>	<b>2</b>	<b>9</b>	<b>1</b>	<b>6</b>	<b>3</b>
<b>6</b>	<b>5</b>	<b>2</b>	<b>7</b>	<b>8</b>	<b>1</b>	<b>3</b>	<b>9</b>	<b>4</b>
<b>9</b>	<b>8</b>	<b>1</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>2</b>	<b>7</b>	<b>6</b>
<b>3</b>	<b>7</b>	<b>4</b>	<b>9</b>	<b>6</b>	<b>2</b>	<b>8</b>	<b>1</b>	<b>5</b>

- 1 → e
- 2 → h
- 3 → c
- 4 → f
- 5 → i
- 6 → d
- 7 → b
- 8 → a
- 9 → g

8			4	6			7
					4		
	1				6	5	
5		9		3		7	8
				7			
	4	8		2		1	3
	5	2					9
		1					
3			9	2			5



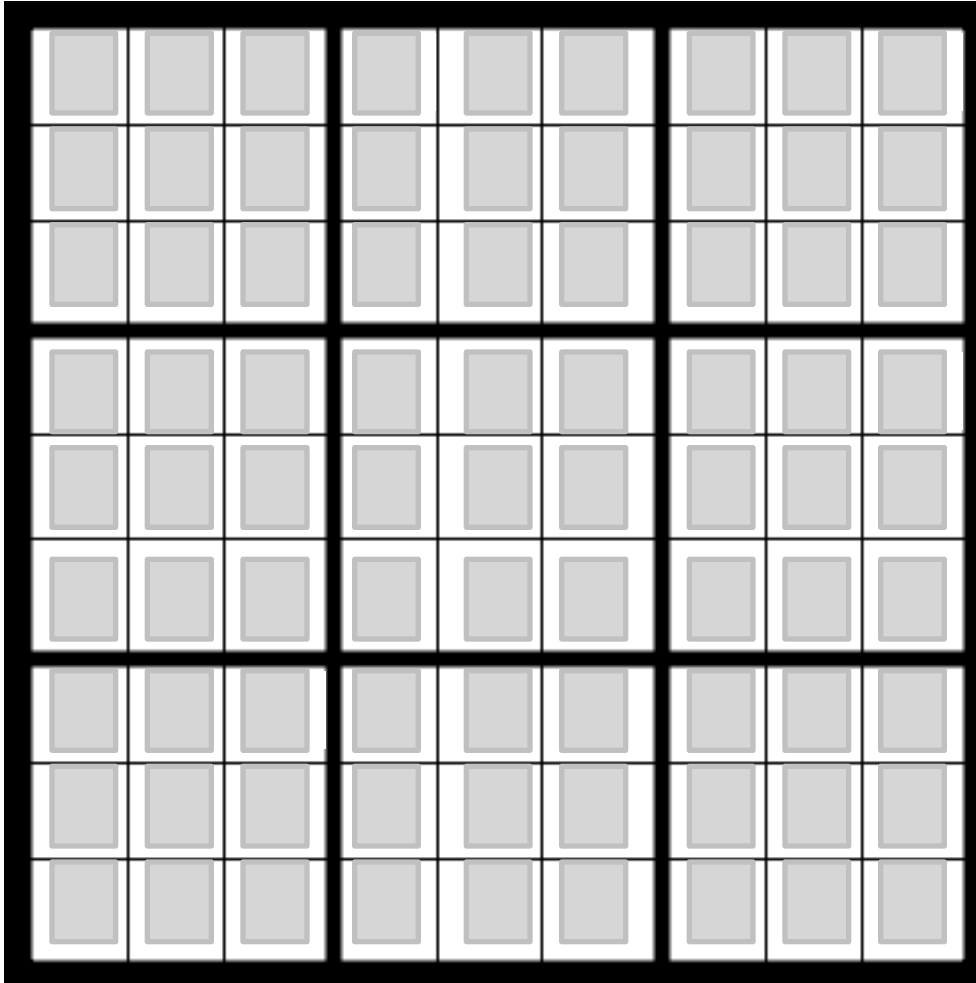
# You prepare your proof

a	c	i	f	e	d	g	h	b
h	g	d	a	i	b	f	c	e
f	e	b	h	g	c	d	i	a
i	d	g	e	c	f	b	a	h
e	h	c	d	b	a	i	f	g
b	f	a	i	h	g	e	d	c
d	i	h	b	a	e	c	g	f
g	a	e	c	f	i	h	b	d
c	b	f	g	d	h	a	e	i

- 1 → e
- 2 → h
- 3 → c
- 4 → f
- 5 → i
- 6 → d
- 7 → b
- 8 → a
- 9 → g

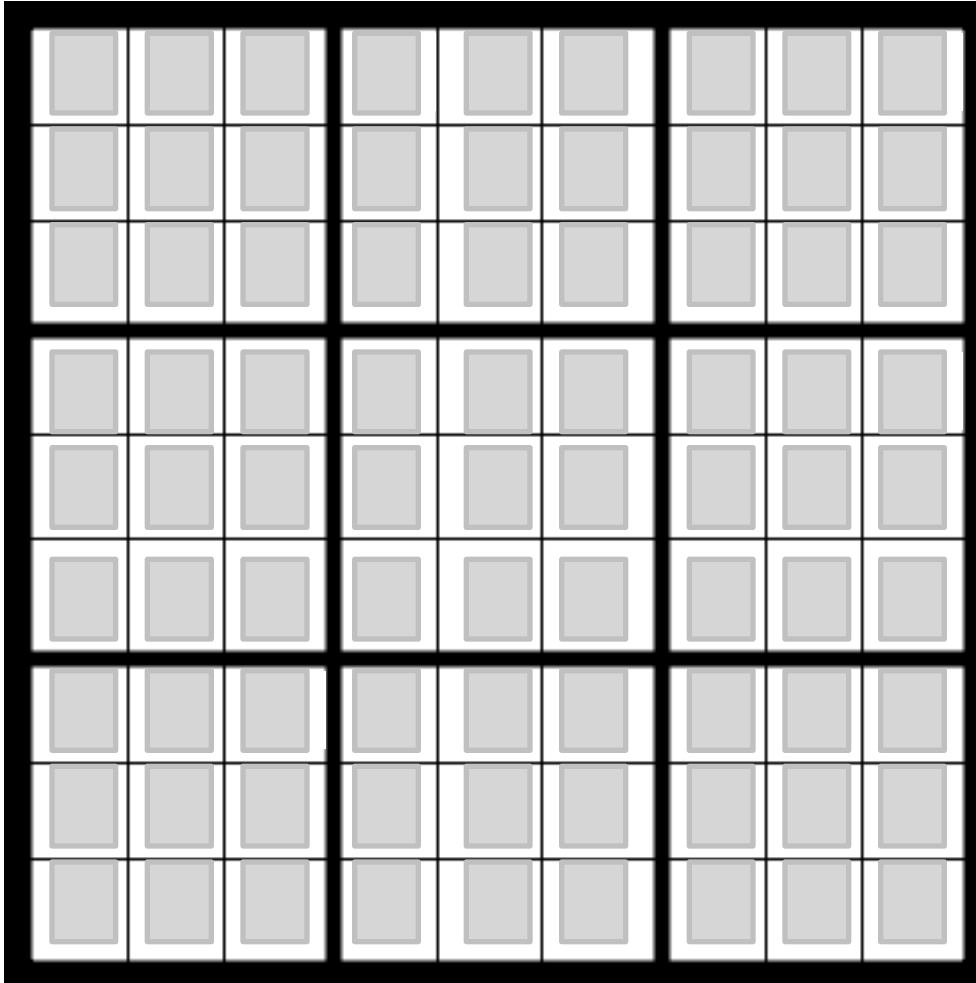
8			4	6			7	
						4		
	1					6	5	
5		9		3		7	8	
				7				
	4	8		2		1		3
	5	2						9
		1						
3			9	2				5

# You prepare your proof



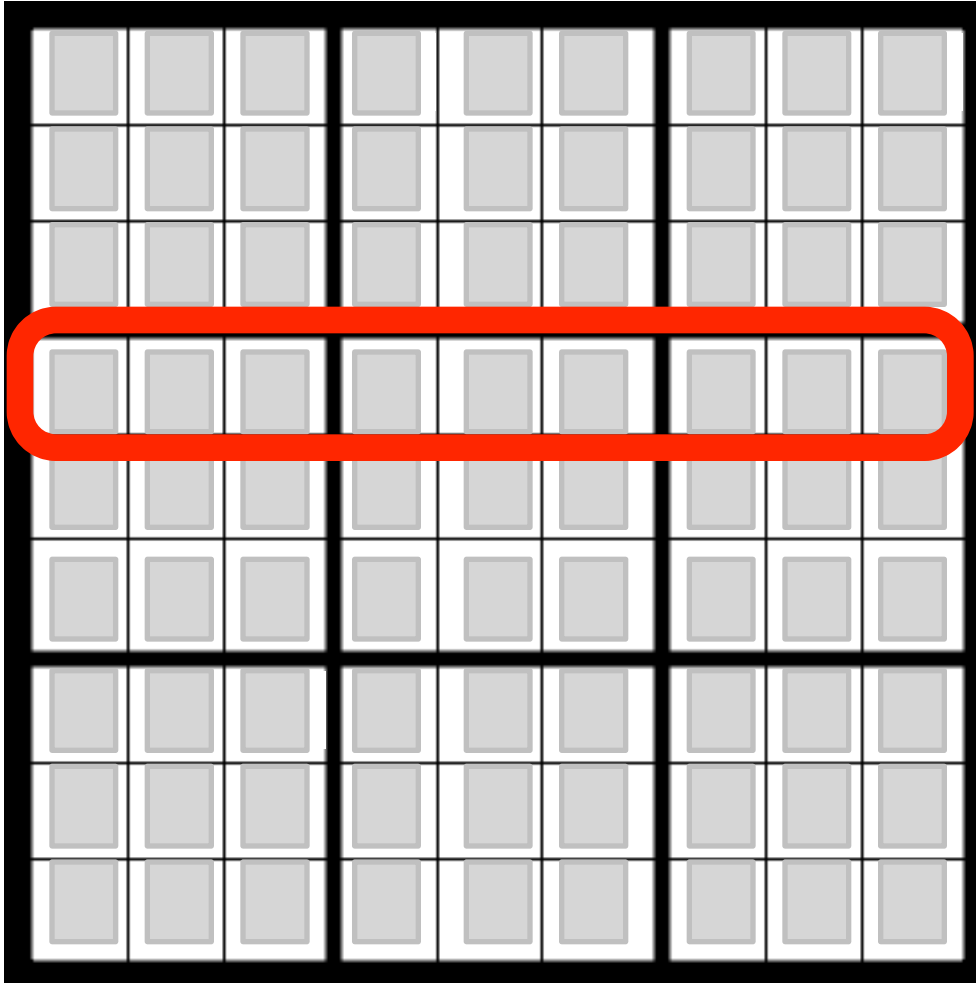
8			4	6			7	
					4			
	1				6	5		
5		9		3		7	8	
				7				
	4	8		2		1		3
	5	2						9
		1						
3			9	2				5

# My turn: I keep you honest



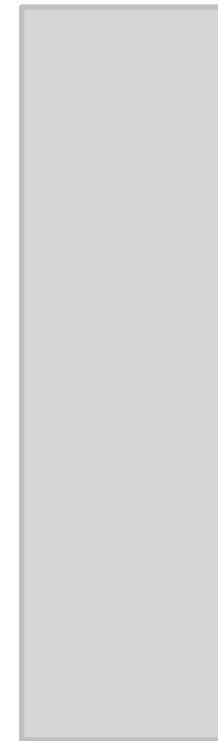
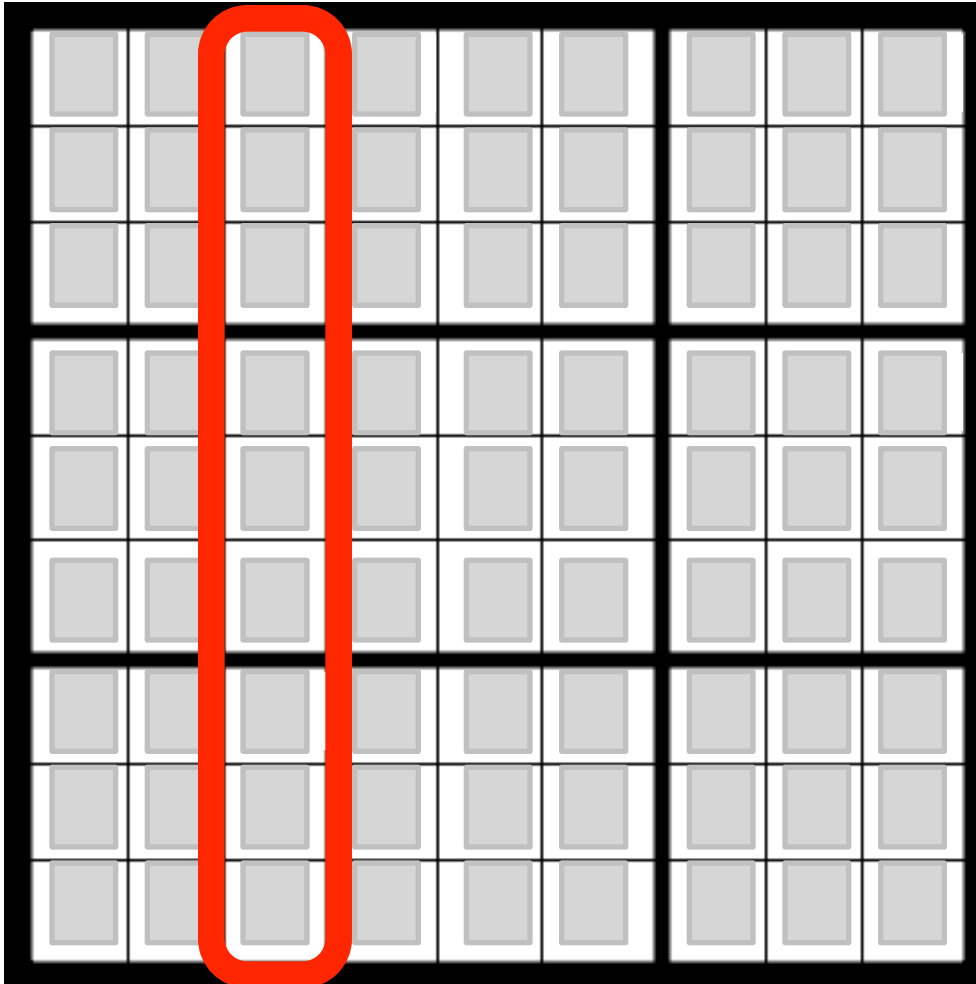
8			4	6			7	
					4			
	1				6	5		
5		9		3		7	8	
				7				
	4	8		2		1		3
	5	2						9
		1						
3			9	2				5

My turn: I keep you honest (option 1)



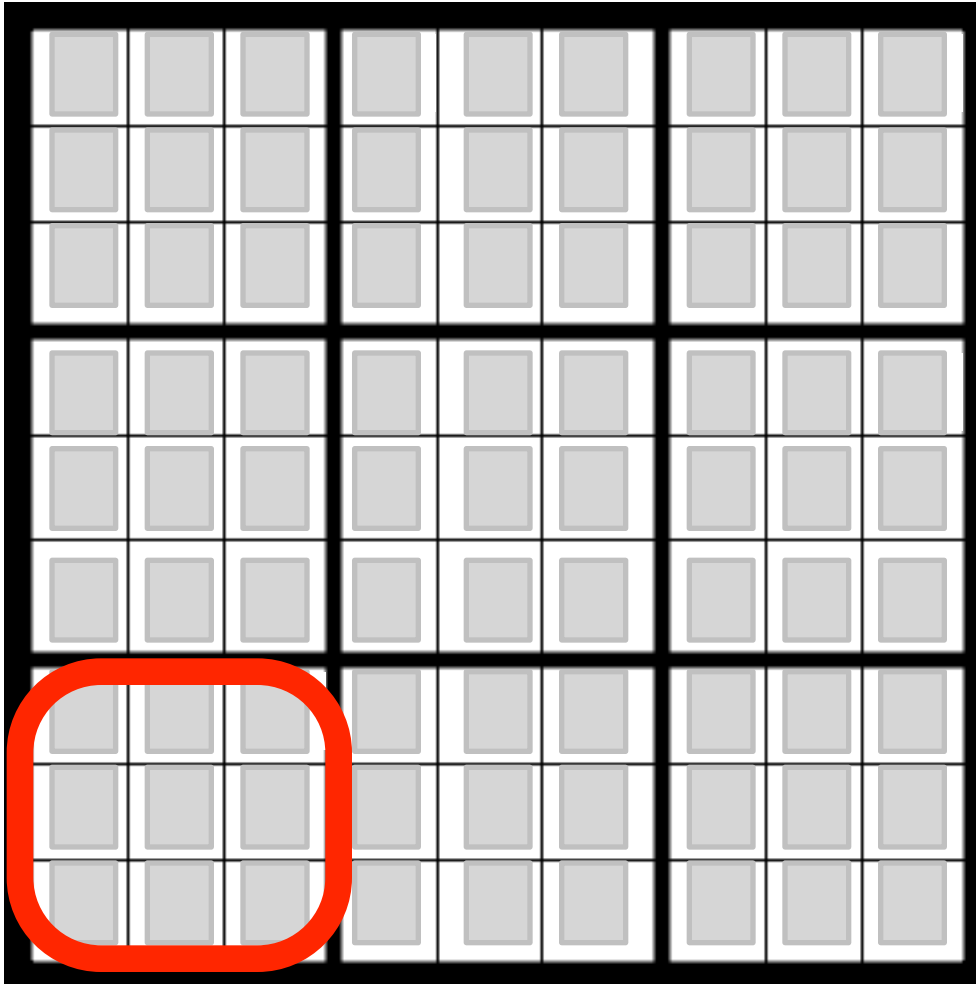
8			4	6			7	
						4		
	1					6	5	
5		9		3		7	8	
				7				
	4	8		2		1		3
	5	2						9
		1						
3			9	2				5

My turn: I keep you honest (option 2)



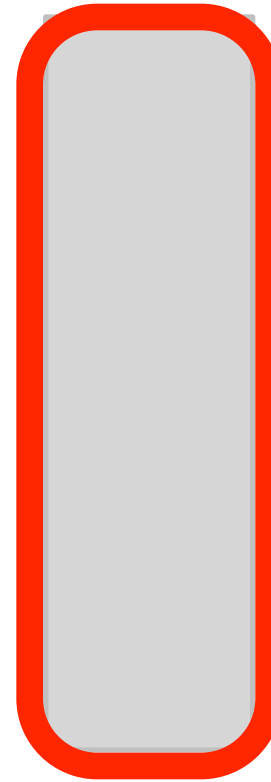
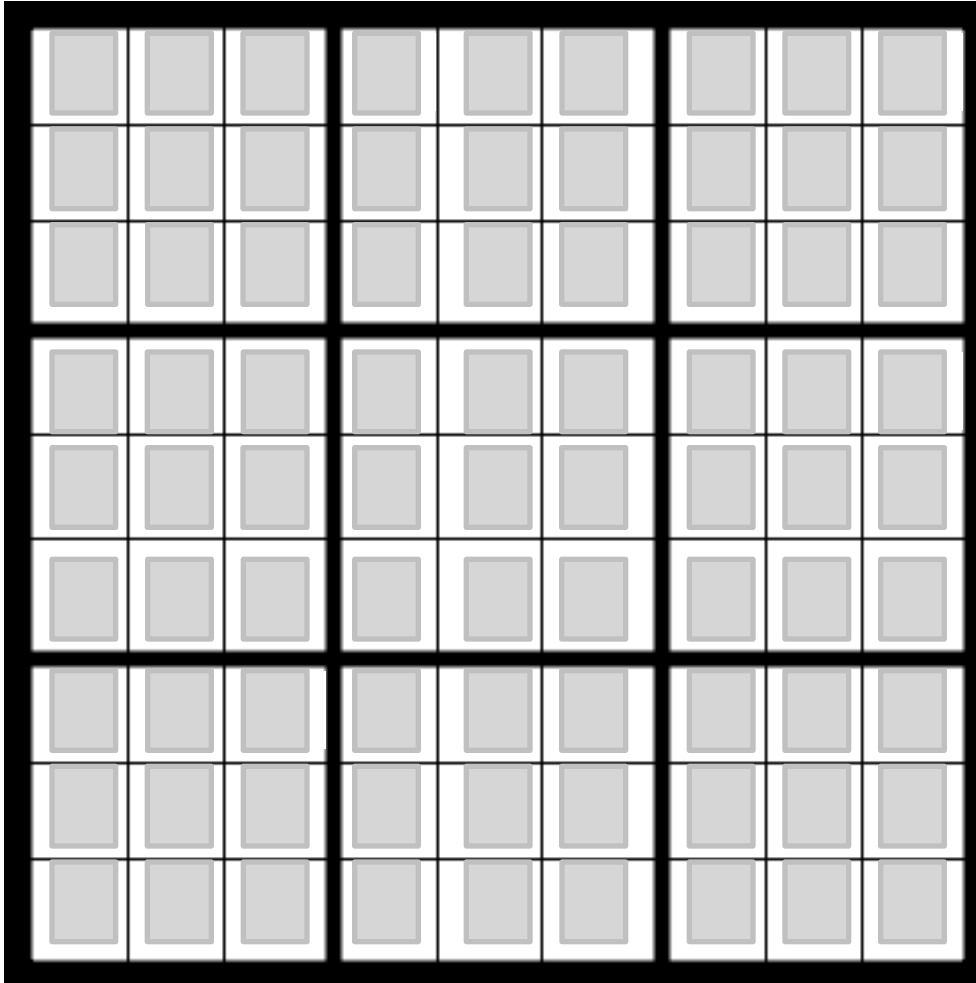
8			4	6			7
					4		
	1				6	5	
5		9		3	7	8	
				7			
	4	8		2	1		3
	5	2					9
		1					
3			9	2			5

My turn: I keep you honest (option 3)



8			4	6			7	
						4		
	1					6	5	
5		9		3		7	8	
				7				
	4	8		2		1		3
	5	2						9
		1						
3			9	2				5

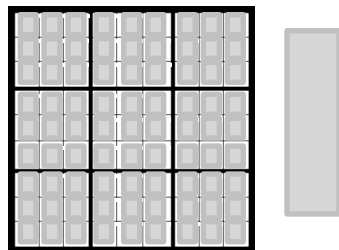
My turn: I keep you honest (option 4)



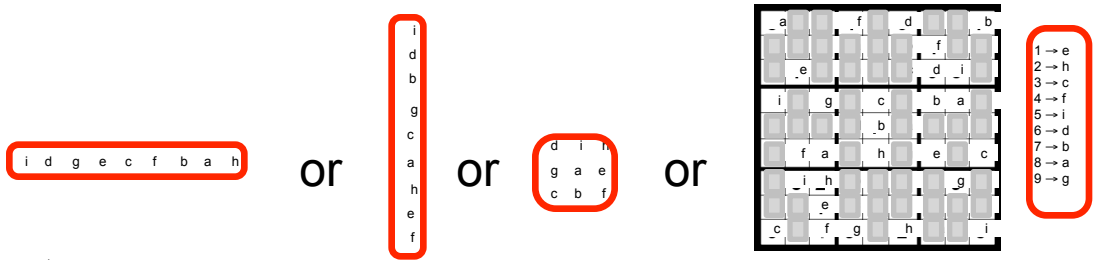
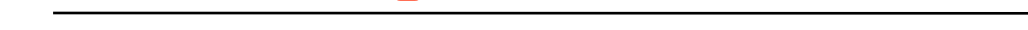
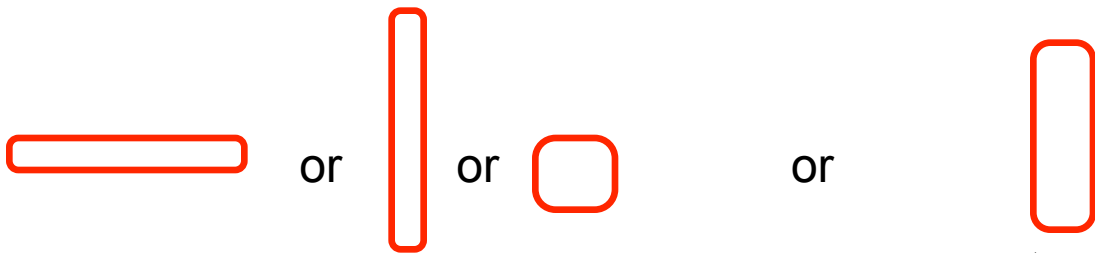
8			4	6			7	
						4		
	1					6	5	
5		9		3		7	8	
				7				
	4	8		2		1		3
	5	2						9
		1						
3			9	2				5

# Zero-knowledge proof: puzzle is solvable

8		4	6		7
1				4	5
5	9	3	7	8	
	4	8	2	1	3
5	2				9
	1				
3		9	2		5



8	3	5	4	1	6	9	2	7
2	9	6	8	5	7	4	3	1
4	1	7	2	9	3	6	5	8
5	6	9	1	3	4	7	8	2
1	2	3	6	7	8	5	4	9
7	4	8	5	2	9	1	6	3
6	5	2	7	8	1	3	9	4
9	8	1	3	4	5	2	7	6
2	7	4	9	6	9	8	1	5



Verifier

Prover

Repeat 1000 times



# Goal: Prove the puzzle is solvable

8		4	6		7
	1			4	
5	9		3	7	8
	4	8		2	
			7		
				1	3
5	2				9
	1				
3		9	2		5

I'm convinced!  
It can be solved!



Verifier

But I haven't learned  
anything about the  
solution. Darn.

8	3	5	4	1	6	9	2	7
2	9	6	8	5	7	4	3	1
4	1	7	2	9	3	6	5	8
5	6	9	1	3	4	7	8	2
1	2	3	6	7	8	5	4	9
7	4	8	5	2	9	1	6	3
6	5	2	7	8	1	3	9	4
9	8	1	3	4	5	2	7	6
3	7	4	9	6	2	8	1	5



Prover

# Summary

Alice can prove to Dave that the Sudoku puzzle has a solution.

Dave gains zero knowledge about the solution.

Sudoku isn't special:

*Theorem.* If I can prove it, I can prove it to you without revealing the proof.

# Summary

*Theorem.* If I can prove it, I can prove it to you without revealing the proof.

# Electronic voting



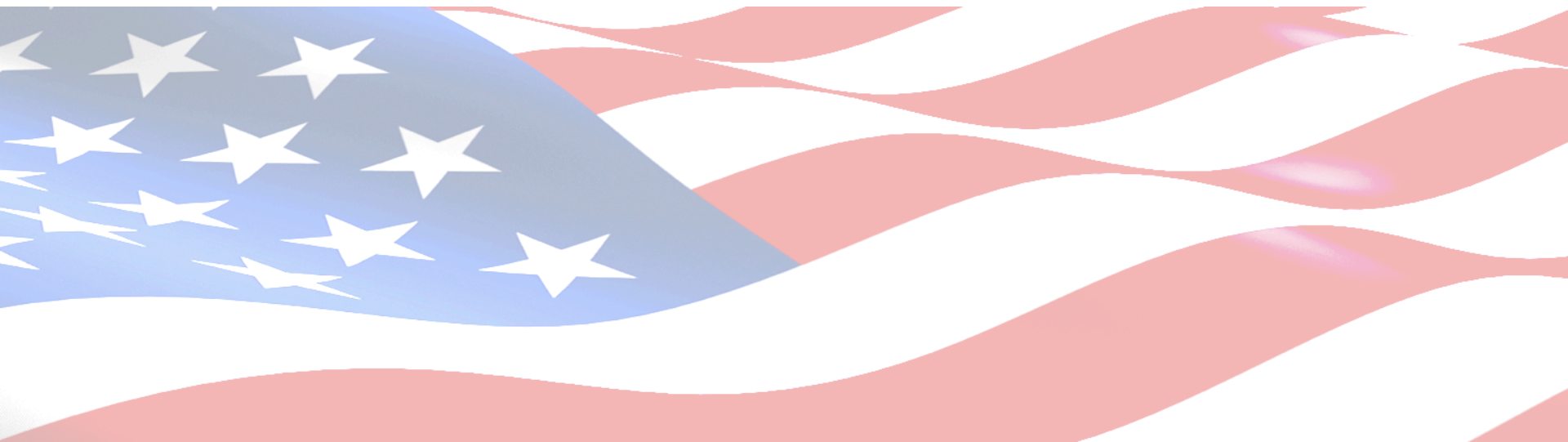
For 25% of overseas and military voters, their vote doesn't count, because the mail is too slow and unreliable.



# Electronic voting

What about voting over the Internet?

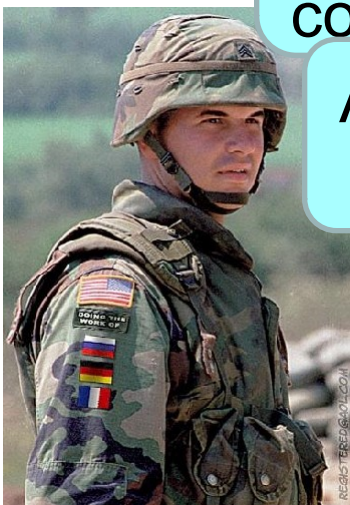
It solves the problem with the mail, but introduces new problems: how do we trust or verify the result?



# Research: Trustworthy Remote Voting

I'm convinced!  
My vote was  
counted accurately!

And no one else can  
learn how I voted.



Voter  
(Verifier)



Server  
(Prover)