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Symmetry

Introduction to Encryption

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July 1, 2019

Announcements

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- Homework 1 due in a week (7/8)
- Project 1 due in about a week (7/11)
- Midterm 1 in two weeks (7/15)
 - Attend lecture and discussion sections to learn material to appear on Midterm 1

Premise: Alice wants to ask Bob on a date...

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...but doesn't want anyone else to know or mess up her plans!

Let's help her setup a secure method of communication.

Alice's security specifications

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Introduction

We want CIA. (No, not the Central Intelligence Agency.)

- Confidentiality
 - only Alice and Bob should know the message
- Integrity
 - Alice's message was not modified or tampered with
- Authentication
 - Bob should be able to verify Alice sent the message

Meet Alice's adversaries

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Eve the Eavesdropper



Likes: Reading messages Dislikes: Confidentiality

Mallory the Manipulator



Likes: Altering messages Dislikes: Integrity/Authenticity

For now, we'll focus on giving Eve a hard time.

Achieve confidentiality to upset Eve

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Eve dislikes confidential secrets. How can we ensure that Eve can't read Alice's correspondence? (Rhetorical question)







Let's encrypt Alice's message!

Types of encryption

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Encryption key

Decryption key



- Symmetric key encryption P = P
 - same private key for encryption and decryption
- Asymmetric key encryption $\not P \neq \not P$
 - separate public encryption key and private decryption key

Both types of encryption achieve confidentiality, necessary to annoy Eve. For now, we'll focus on symmetric encryption.

Learning objectives

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- Learn how to make formal security arguments
 - Specifically prove or disprove that a scheme is IND-KPA or IND-CPA secure
- Build intuition from formal security proofs
 - For example, understand what it means to be IND-CPA secure

Symmetric key encryption (think of API or "blueprint")

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Icon Credit: Nikita Golubev

Gen $(1^n) \rightarrow k$: n times

Input: $1^n = 1...1$, allows its runtime to depend on length of the key

|k| = n

Output: secret key k

 $\mathbf{Enc}(k,m) \rightarrow c$:

Input: secret key k, message or

plaintext m

Output: ciphertext c

 $\mathbf{Dec}(k,c) \to m$:

Input: secret key k, ciphertext c

Output: message m

Symmetric key encryption 🕮



Symmetric Key Encryption

How do we know that our encryption works?

$$\forall m, k$$
 $\mathbf{Dec}(k, \mathbf{Enc}(k, m)) = m$

We call this the correctness property!

Is the correctness property enough for encryption to be confidential? No, we need a security guarantee.

How do establish security? Games!

We phrase security of encryption schemes as a game between a challenger and an adversary. If no adversary can win the game with probability greater than random chance, then we consider the scheme secure.

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Photo Credit: lichess

Adversary



Ciphertext Indistinguishability as Games

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- Indistinguishability under Known Plaintext Attack (IND-KPA)
- Indistinguishability under Chosen Plaintext Attack (IND-CPA)
- Indistinguishability under (Non-Adaptive) Chosen Ciphertext Attack (IND-CCA1)
- Indistinguishability under (Adaptive) Chosen Ciphertext Attack (IND-CCA2)

IND-KPA Secure

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Adversary
$${\cal A}$$

Phases Challenger
$$C$$

setup $k \leftarrow \text{Gen}(1^n)$

$$m_0^*, m_1^*$$

$$b \stackrel{\$}{\leftarrow} \{0,1\}$$

$$c^* \leftarrow \operatorname{Enc}(k, m_b^*)$$

$$\xrightarrow{c^*} b'$$

send bit

determine win If
$$b = b'$$
, A wins.

IND-KPA Secure







Adversary
$${\cal A}$$

$$\begin{array}{ll} \textit{Phases} & \textit{Challenger } \mathcal{C} \\ \textit{setup} & \textit{k} \leftarrow \textbf{Gen}(1^n) \end{array}$$

$$m_0^*, m_1^*$$

$$b \xleftarrow{\$} \{0,1\}$$

$$c^* \leftarrow \mathbf{Enc}(k, m_b^*)$$

If
$$b = b'$$
, \mathcal{A} wins.

$$\forall$$
 adversaries A , $\Pr[A \text{ wins game}] \leq \frac{1}{2} + negligible$

Why $\frac{1}{2}$? Some adversary A could guess bit b. They have probability $\frac{1}{2}$ to succeed. We don't consider this an "attack" on our encryption scheme.

IND-KPA Secure

IND-KPA







Adversary
$${\cal A}$$

$$\begin{array}{ll} \textit{Phases} & \textit{Challenger } \mathcal{C} \\ \textit{setup} & \textit{k} \leftarrow \textit{Gen}(1^n) \end{array}$$

$$m_0^*, m_1^*$$

$$b \xleftarrow{\$} \{0,1\}$$

$$c^* \leftarrow \mathbf{Enc}(k, m_b^*)$$

If
$$b = b'$$
, \mathcal{A} wins.

$$\forall$$
 adversaries \mathcal{A} , $\Pr[\mathcal{A} \text{ wins game}] \leq \frac{1}{2} + \textit{negligible}$

What could *negligible* be? Say some adversary A decides to brute force the key k. The attacker has probability $\frac{1}{2^n}$ to succeed. That's a negligible amount.

IND-CPA secure

encrypt plaintext

send bit

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for $i \in poly(n)$

$$b \stackrel{\$}{\leftarrow} \{0,1\}$$

$$c^* \leftarrow \operatorname{Enc}(k, m_b^*) \qquad \longleftarrow$$

$$c_i \leftarrow \operatorname{Enc}(k, m_i') \qquad \longleftarrow$$

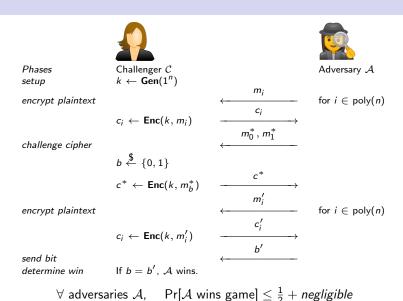
$$\xrightarrow{c'_i} \xrightarrow{b'}$$

 m_i'

determine win If
$$b = b'$$
, A wins.

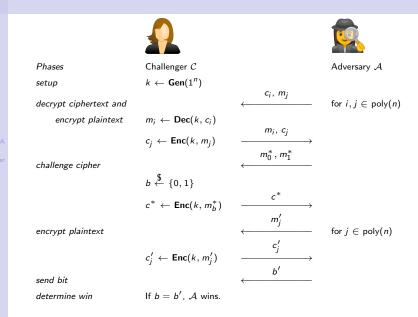
IND-CPA secure

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IND-CCA1 secure

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IND-CCA1 secure

Phases Challenger C $k \leftarrow \operatorname{Gen}(1^n)$ setup c_i, m_i decrypt ciphertext and encrypt plaintext $m_i \leftarrow \mathbf{Dec}(k, c_i)$ m_i, c_i $c_i \leftarrow \operatorname{Enc}(k, m_i)$ IND-CCA1 m_0^*, m_1^* challenge cipher $b \stackrel{\$}{\leftarrow} \{0,1\}$ c* $c^* \leftarrow \operatorname{Enc}(k, m_b^*)$ m_i' encrypt plaintext c_i'



Adversary A

for $i \in poly(n)$

for $i, j \in poly(n)$

 $c_i' \leftarrow \mathbf{Enc}(k, m_i')$ h'

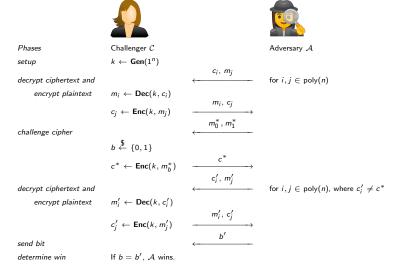
send bit

If b = b'. A wins. determine win

 \forall adversaries \mathcal{A} , $\Pr[\mathcal{A} \text{ wins game}] \leq \frac{1}{2} + negligible$

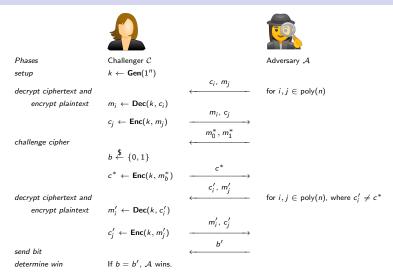
IND-CCA2 secure

IND-CCA2



IND-CCA2 secure

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 \forall adversaries \mathcal{A} , $\Pr[\mathcal{A} \text{ wins game}] \leq \frac{1}{2} + \textit{negligible}$

How are these security definitions related?

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 $\begin{array}{c} \mathsf{IND\text{-}CCA2} \; \mathsf{secure} \; \Longrightarrow \; \mathsf{IND\text{-}CCA1} \; \mathsf{secure} \\ \Longrightarrow \; \mathsf{IND\text{-}CPA} \; \mathsf{secure} \; \Longrightarrow \; \mathsf{IND\text{-}KPA} \; \mathsf{secure} \end{array}$

This means that IND-CCA2 is the stronger security definition. In cryptography we want the **strongest** security to hold.

Example: If your protocol is IND-CCA1 secure, then it is IND-CPA and IND-KPA secure. However, it is not necessarily IND-CCA2 secure.

How are these security definitions related?

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$$\begin{array}{lll} \mathsf{IND\text{-}CCA2} \; \mathsf{insecure} \; & \longleftarrow \; \mathsf{IND\text{-}CCA1} \; \mathsf{insecure} \\ & \longleftarrow \; \mathsf{IND\text{-}CPA} \; \mathsf{insecure} \; & \longleftarrow \; \mathsf{IND\text{-}KPA} \; \mathsf{insecure} \end{array}$$

Example: If your protocol is IND-CCA1 insecure, then it is IND-CCA2 insecure. However, it is not necessarily IND-CPA or IND-KPA insecure.

Note

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In each security definition, we must be secure against all possible adversaries that are allowed to behave within the limits of the definition.

For instance, in IND-KPA the adversary will not receive answers to any encryption queries. (Note that "encrypt plaintext" phase is in IND-CPA, but not in IND-KPA.)

Achieve IND-CPA

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One Time Pad

Let's set a reasonable security goal. For the scope of this class, we want a symmetric encryption scheme that is IND-CPA.

Let's build something! ("move fast and break things", right?)

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We're familiar with XOR.

Α	В	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

The output of two random inputs XORed is 0 w.p. 1/2 and 1 w.p. 1/2. This hides information!

One Time Pad (OTP)

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One Time Pad

Let m be some n bit message or plaintext. OTP will sample a key k as some arbitrary n bit binary string.

$$\mathbf{Gen}(1^n): \quad k \stackrel{\$}{\leftarrow} \{0,1\}^n$$

Then OTP will produce an n bit ciphertext by XORing the message m with the key k.

Enc
$$(k, m)$$
: $m \oplus k$

$$\mathbf{Dec}(k,c): c \oplus k$$

Is OTP correct?

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$$\mathbf{Gen}(1^n): \quad k \xleftarrow{\$} \{0,1\}^n$$

$$\mathbf{Enc}(k,m): m \oplus k$$

$$\mathbf{Dec}(k,c): c \oplus k$$

We have that $\forall m$ messages of length n,

$$\mathbf{Dec}(k, \mathbf{Enc}(k, m)) = \mathbf{Enc}(k, m) \oplus k = (m \oplus k) \oplus k = m \oplus (k \oplus k) = m \oplus 0^n = m.$$

We're done, right? Let's slow down. We should check which security definitions OTP satisfies.

Is OTP IND-CPA? (Break time~)

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Let's slow down, take a break! Pass messages to your neighbor.

One Time Pad
AES Block Cipher
No idea what to talk about? Well, is OTP IND-CPA?

Is OTP IND-CPA?

No. There \exists adversary \mathcal{A} with the following strategy:

Challenger CPhases Adversary A $k \stackrel{\$}{\leftarrow} \{0,1\}^n$ setup n times encrypt plaintext С $c = 0^n \oplus k$ k = c m_0, m_1 One Time Pad challenge cipher Choose $m_0 \neq m_1$ $b \stackrel{\$}{\leftarrow} \{0,1\}$ c^* $c^* = m_b \oplus k$ If $c^* \oplus k = m_0$, let b' = 0. Otherwise, b' = 1. encrypt plaintext *b*′ send bit If $b = b' \cdot A$ wins. determine win

Is OTP IND-CPA?

No. There \exists adversary \mathcal{A} with the following strategy:

Challenger CPhases Adversary A $k \stackrel{\$}{\leftarrow} \{0,1\}^n$ setup n times encrypt plaintext $c = 0^n \oplus k$ k = c m_0, m_1 challenge cipher Choose $m_0 \neq m_1$ $b \stackrel{\$}{\leftarrow} \{0, 1\}$ c^* $c^* = m_b \oplus k$ If $c^* \oplus k = m_0$, let b' = 0. Otherwise, b' = 1. ь′ send hit If b = b'. A wins. determine win

> Since \mathcal{A} has the key k, \mathcal{A} can just decrypt the challenge cipher, winning the game w.p. 1.

One Time Pad

Is OTP "secure"? (No games. Intuition only, please!)

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If you encrypt two messages with the same key, then you leak crucial information.

For example, let m_1 and m_2 be any two messages encrypted with key k. If an adversary got hold of the encryptions $c_1 = m_1 \oplus k$ and $c_2 = m_2 \oplus k$, then they could calculate $c_1 \oplus c_2 = m_1 \oplus k \oplus m_2 \oplus k = m_1 \oplus m_2$.

This leaks information about the messages!

Is OTP "secure"?

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One Time Pad

Historically, OTP was still used. Why?

It's use case is hinted in its name. **One Time** Pad. A key should only be used once. The key should be changed each time a message is encrypted.

If you don't reuse the key for multiple messages, OTP can be considered secure.

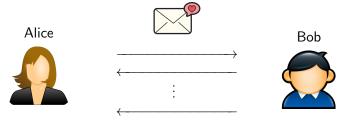
Downsides to OTP

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One Time Pad AES Block Ciphe

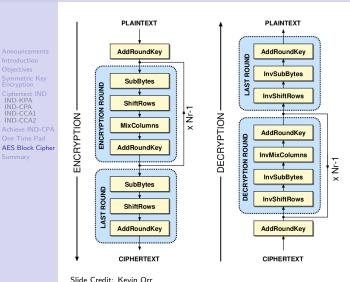
- Not IND-CPA
- No key reuse
 - can't reuse the key for multiple messages
- Key length must be message length
 - lacktriangledown can only encrypt messages of the same length n as the key



Being optimistic that Bob will reply back to Alice, this scheme is rather impractical for our purpose.

For comparison, here's one building block...





...of an encryption construction.

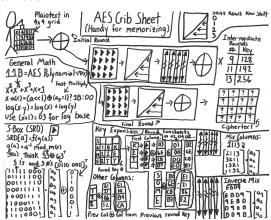
AES Block Cipher

- 128 or 192 or 256 bit key
- up to 14 cycles of repetition

This is complex! We don't expect you to understand how it works.

AES block cipher

If you're curious...



... check out "AES stick figure" online.

Slide Credit: Moserware

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Is AES "secure"?

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Where's the proof?

(Correct response: Which security definition? IND-CPA)

Actually, turns out that AES is not **proven** to be secure.

Fun fact: If we had a real world encryption construction that was provably secure, then $P \neq NP!$

AES in the real world

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Summary

- Two Belgian cryptographers (Vincent Rijmen, Joan Daemen) constructed this scheme which won the AES encryption competition
 - Henceforth to be known as AES
- US Government standard
 - AES is standardized as Federal Information Processing Standard 197 (FIPS 197) by NIST
- Industry
 - SSL/TLS
 - SSH
 - WinZip
 - BitLocker
 - Mozilla Thunderbird
 - Skype

Moral of the day

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"Move fast and break things"

Put well reasoned thought into how you build secure systems. Use established cryptographic protocols and primitives.

Don't build your own cryptography using only knowledge learned from this class and its prerequisite classes!

Let's take our time...

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...next time to look into block ciphers and how we can chain them to achieve IND-CPA.

Alice learned today that ...

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Summary

- confidentiality can be achieved using cryptographic encryption
 - we'll focus on symmetric key
 - we'll see asymmetric key later
- security definitions can be phrased as adversarial games
 - we'll focus on achieving IND-CPA secure encryption
- constructing real-world cryptography relies on these theoretical definitions
 - OTP is not IND-CPA since it's deterministic
 - AES block cipher is quite complicated and is built off of some block cipher definition which we will see tomorrow