Lexical Analysis

Lecture 2-4

Notes by G. Necula, with additions by P. Hilfinger

Administrivia

- Moving to 60 Evans on Wednesday
- HW1 available
- Pyth manual available on line.
- Please log into your account and electronically register today.
- Register your team with “make-team”. See class announcement page. Project #1 available Friday.
- Use “submit hw1” to submit your homework this week.
- Section 101 (9AM) is gone.

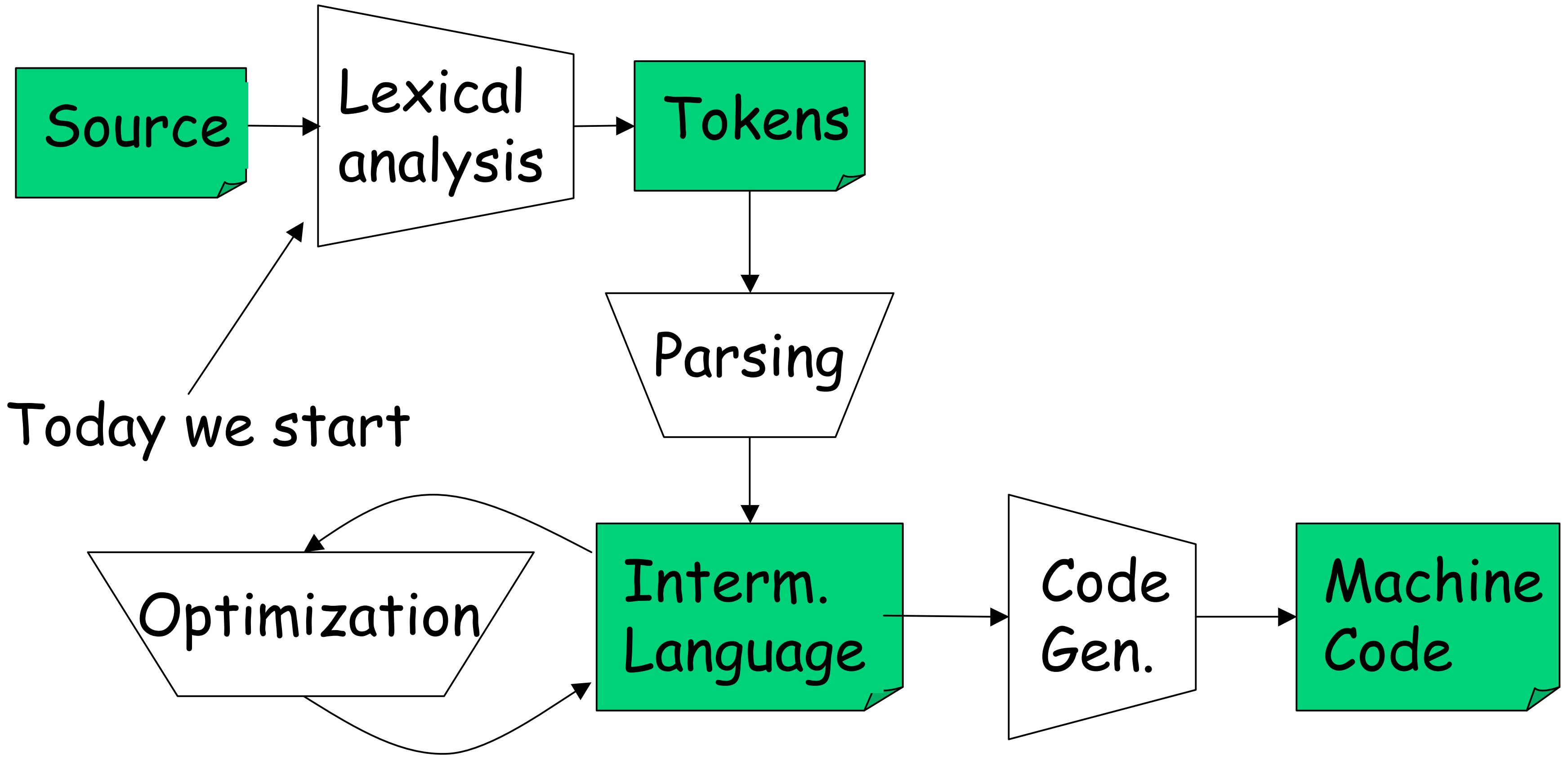
Outline

- Informal sketch of lexical analysis
  - Identifies tokens in input string

- Issues in lexical analysis
  - Lookahead
  - Ambiguities

- Specifying lexers
  - Regular expressions
  - Examples of regular expressions

The Structure of a Compiler



Lexical Analysis

- What do we want to do? Example:

```
if (i == j)
    z = 0;
else
    z = 1;
```

- The input is just a sequence of characters:

```
\tif (i == j)\n\t\tz = 0;\n\telse\n\t\tz = 1;
```

- Goal: Partition input string into substrings
  - And classify them according to their role

What's a Token?

- Output of lexical analysis is a stream of tokens
- A token is a syntactic category
  - In English:

    noun, verb, adjective, …
  - In a programming language:

    Identifier, Integer, Keyword, Whitespace, …
- Parser relies on the token distinctions:
  - E.g., identifiers are treated differently than keywords

Tokens

- Tokens correspond to sets of strings:
  - Identifiers: *strings of letters or digits, starting with a letter*
  - Integers: *non-empty strings of digits*
  - Keywords: *"else" or "if" or "begin" or …*
  - Whitespace: *non-empty sequences of blanks, newlines, and tabs*
  - OpenPars: *left-parentheses*

Lexical Analyzer: Implementation

- An implementation must do two things:

  1. Recognize substrings corresponding to tokens

  2. Return:
     1. The type or *syntactic category* of the token,
     2. the value or *lexeme* of the token (the substring itself).

Example

- Our example again:

  `\tif (i == j)\n\t\tz = 0;\n\telse\n\t\tz = 1;`

- Token-lexeme pairs returned by the lexer:
  - (Whitespace, "\t")
  - (Keyword, "if")
  - (OpenPar, "(")
  - (Identifier, "i")
  - (Relation, "==")
  - (Identifier, "j")
  - …

Lexical Analyzer: Implementation

- The lexer usually discards "uninteresting" tokens that don't contribute to parsing.
- Examples: Whitespace, Comments
- Question: What happens if we remove all whitespace and all comments prior to lexing?

Lookahead.

- Two important points:
  1. The goal is to partition the string. This is implemented by reading left-to-right, recognizing one token at a time

  2. "Lookahead" may be required to decide where one token ends and the next token begins

  – Even our simple example has lookahead issues

    i vs. if

    = vs. ==

Next

- We need
  - A way to describe the lexemes of each token
  - A way to resolve ambiguities
    - Is if two variables i and f?
    - Is == two equal signs = =?

Regular Languages

- There are several formalisms for specifying tokens

- *Regular languages* are the most popular
  - Simple and useful theory
  - Easy to understand
  - Efficient implementations

Languages

Def. Let Σ be a set of characters. *A language over* Σ is a set of strings of characters drawn from Σ

(Σ is called the *alphabet*)

Examples of Languages

- Alphabet = English characters
- Language = English sentences

- Not every string on English characters is an English sentence

- Alphabet = ASCII
- Language = C programs

- Note: ASCII character set is different from English character set

Notation

- Languages are sets of strings.
- Need some notation for specifying which sets we want
- For lexical analysis we care about *regular languages*, which can be described using *regular expressions.*

Regular Expressions and Regular Languages

- Each regular expression is a notation for a regular language (a set of words)

- If A is a regular expression then we write $L(A)$ to refer to the language denoted by A

Atomic Regular Expressions

- Single character: 'c'

  L('c') = { "c" }   (for any $c \in \Sigma$)

- Concatenation: AB (where A and B are reg. exp.)

  $L(AB) = \{ ab \mid a \in L(A) \text{ and } b \in L(B) \}$

- Example: L('i' 'f') = { "if" }

  (we will abbreviate 'i' 'f' as 'if')

Compound Regular Expressions

- Union

  $$L(A \mid B) = L(A) \cup L(B)$$
  $$= \{ s \mid s \in L(A) \text{ or } s \in L(B) \}$$

- Examples:

  'if' | 'then' | 'else' = { "if", "then", "else"}

  '0' | '1' | … | '9' = { "0", "1", …, "9" }

  (note the … are just an abbreviation)

- Another example:

  L(('0' | '1') ('0' | '1')) = { "00", "01", "10", "11" }

More Compound Regular Expressions

- So far we do not have a notation for infinite languages
- Iteration: A^*

  $L(A^*) = \{ \text{""} \} \mid L(A) \mid L(AA) \mid L(AAA) \mid \ldots$
- Examples:

  $\text{'0'}^* = \{ \text{""}, \text{"0"}, \text{"00"}, \text{"000"}, \ldots\}$

  $\text{'1'}\ \text{'0'}^*$ = { strings starting with 1 and followed by 0's }
- Epsilon: ε

  $L(\varepsilon) = \{ \text{""} \}$

Example: Keyword

- Keyword: *"else"* *or* *"if"* *or* *"begin"* *or …*

'else' | *'if'* | *'begin'* | …

('else' abbreviates 'e' 'l' 's' 'e')

Example: Integers

Integer: *a non-empty string of digits*

digit = '0' | '1' | '2' | '3' | '4' | '5' | '6' | '7' | '8' | '9'
number = digit digit^*

Abbreviation: $A^+ = A\ A^*$

Example: Identifier

Identifier: *strings of letters or digits, starting with a letter*

letter = 'A' | ... | 'Z' | 'a' | ... | 'z'

identifier = letter (letter | digit) *

Is (letter* | digit*) the same as (letter | digit) * ?

Example: Whitespace

Whitespace: *a non-empty sequence of blanks, newlines, and tabs*

$(\text{' '} \mid \text{'\textbackslash t'} \mid \text{'\textbackslash n'})^{+}$

(Can you spot a subtle omission?)

Example: Phone Numbers

- Regular expressions are all around you!
- Consider (510) 643-1481

Σ = { 0, 1, 2, 3, …, 9, (,), - }

area = digit^3

exchange = digit^3

phone = digit^4

number = '(' area ')' exchange '-' phone

Example: Email Addresses

- Consider *necula@cs.berkeley.edu*

Σ = letters [{ ., @ }

name = letter$^+$

address = name '@' name ('.' name)*

Summary

- Regular expressions describe many useful languages
- Next: Given a string *s* and a R.E. *R*, is

$$s \in L(R)?$$

- But a yes/no answer is not enough !
- Instead: partition the input into lexemes

- We will adapt regular expressions to this goal

Next: Outline

- Specifying lexical structure using regular expressions

- Finite automata
  - Deterministic Finite Automata (DFAs)
  - Non-deterministic Finite Automata (NFAs)

- Implementation of regular expressions

  RegExp => NFA => DFA => Tables

Regular Expressions => Lexical Spec. (1)

1. Select a set of tokens
   - Number, Keyword, Identifier, ...

2. Write a R.E. for the lexemes of each token
   - Number = digit$^+$
   - Keyword = 'if' | 'else' | ...
   - Identifier = letter (letter | digit)*
   - OpenPar = '('
   - ...

Regular Expressions => Lexical Spec. (2)

3. Construct R, matching all lexemes for all tokens

$$
\begin{aligned}
R &= \text{Keyword} \mid \text{Identifier} \mid \text{Number} \mid \ldots \\
&= R_1 \mid R_2 \mid R_3 \mid \ldots
\end{aligned}
$$

Facts: If $s \in L(R)$ then s is a lexeme

- Furthermore $s \in L(R_i)$ for some "i"
- This "i" determines the token that is reported

Regular Expressions => Lexical Spec. (3)

4. Let the input be $x_1 \ldots x_n$

   ($x_1 \ldots x_n$ are characters in the language alphabet)

   - For $1 \le i \le n$ check

     $x_1 \ldots x_i \in L(R)$?

5. It must be that

   $x_1 \ldots x_i \in L(R_j)$ for some i and j

6. Remove $x_1 \ldots x_i$ from input and go to (4)

Lexing Example

R = Whitespace | Integer | Identifier | '+'

- Parse "f+3 +g"
  - "f" matches R, more precisely Identifier
  - "+" matches R, more precisely '+'
  - …
  - The token-lexeme pairs are
    (Identifier, "f"), ('+', "+"), (Integer, "3")
    (Whitespace, " "), ('+', "+"), (Identifier, "g")
- We would like to drop the Whitespace tokens
  - after matching Whitespace, continue matching

Ambiguities (1)

- There are ambiguities in the algorithm
- Example:

  R = Whitespace | Integer | Identifier | '+'
- Parse "foo+3"
  - "f" matches R, more precisely Identifier
  - But also "fo" matches R, and "foo", but not "foo+"
- How much input is used? What if
    - $x_1 \ldots x_i \in L(R)$ and also $x_1 \ldots x_K \in L(R)$
  - "Maximal munch" rule: *Pick the longest possible substring that matches R*

More Ambiguities

R = Whitespace | 'new' | Integer | Identifier

- Parse "new foo"
  - "new" matches R, more precisely 'new'
  - but also Identifier, which one do we pick?
- In general, if $x_1 \ldots x_i \in L(R_j)$ and $x_1 \ldots x_i \in L(R_k)$
  - Rule: use rule listed first (j if $j < k$)

- We must list 'new' before Identifier

Error Handling

R = Whitespace | Integer | Identifier | '+'

- Parse "=56"
  - No prefix matches R: not "=", nor "=5", nor "=56"
- Problem: Can't just get stuck …
- Solution:
  - Add a rule matching all "bad" strings; and put it last
- Lexer tools allow the writing of:

  $R = R_1 \mid \ldots \mid R_n \mid \text{Error}$
  - Token Error matches if nothing else matches

Summary

- Regular expressions provide a concise notation for string patterns
- Use in lexical analysis requires small extensions
  - To resolve ambiguities
  - To handle errors
- Good algorithms known (next)
  - Require only single pass over the input
  - Few operations per character (table lookup)

Finite Automata

- Regular expressions = specification
- Finite automata = implementation

- A finite automaton consists of
  - An input alphabet Σ
  - A set of states S
  - A start state n
  - A set of accepting states $F \subseteq S$
  - A set of transitions state $\rightarrow^{\text{input}}$ state

Finite Automata

- Transition

$$s_1 \rightarrow^a s_2$$

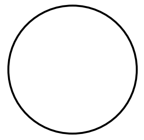
- Is read

  In state s_1 on input “a” go to state s_2

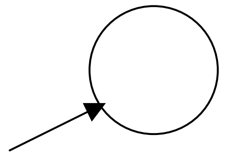
- If end of input
  - If in accepting state => accept, othewise => reject
- If no transition possible => reject

Finite Automata State Graphs

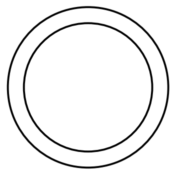
- A state



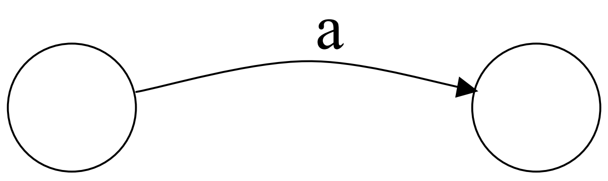
- The start state



- An accepting state

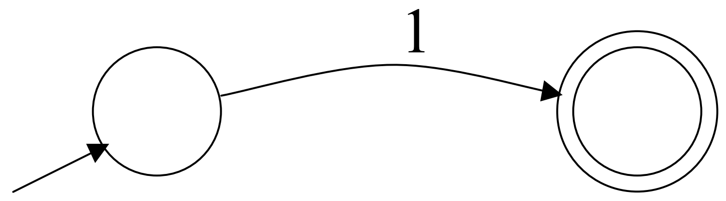


- A transition



A Simple Example

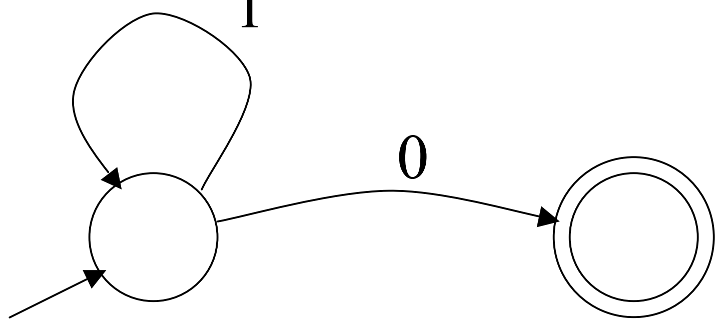
- A finite automaton that accepts only "1"



- A finite automaton accepts a string if we can follow transitions labeled with the characters in the string from the start to some accepting state

Another Simple Example

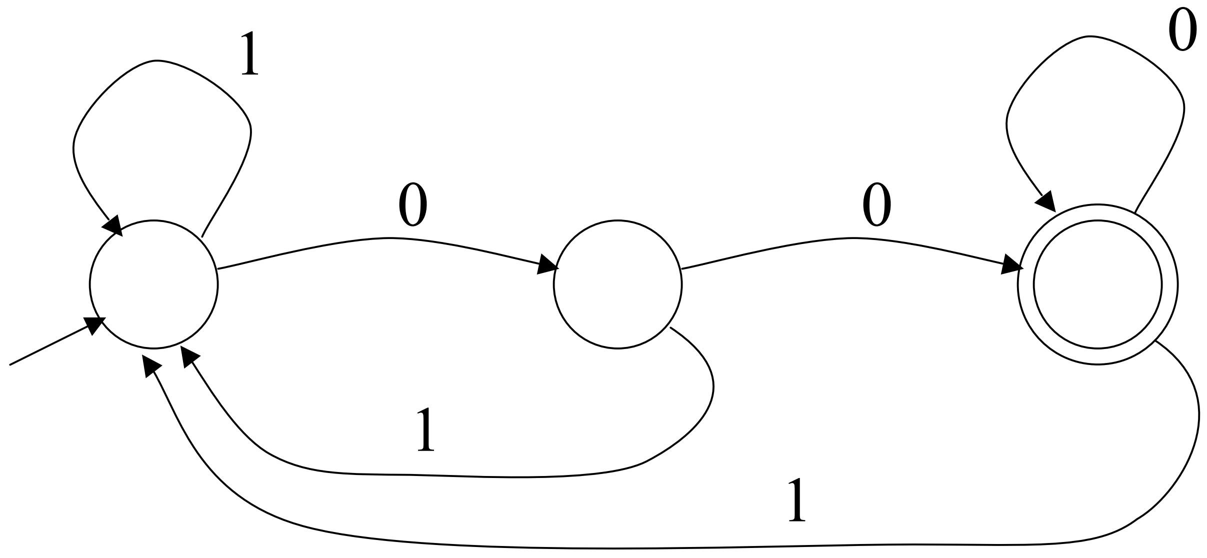
- A finite automaton accepting any number of 1's followed by a single 0
- Alphabet: {0,1}



- Check that "1110" is accepted but "110…" is not

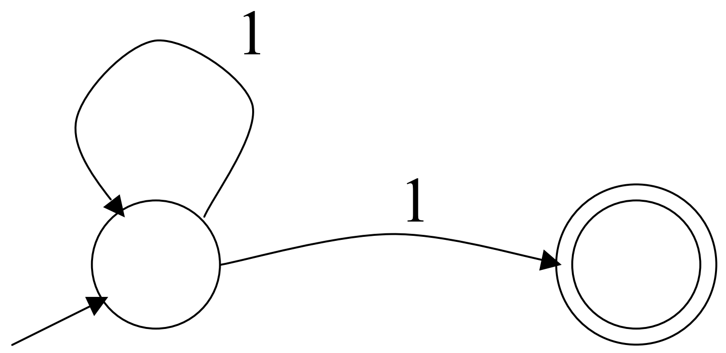
And Another Example

- Alphabet {0,1}
- What language does this recognize?



And Another Example

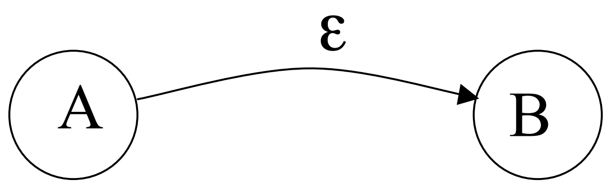
- Alphabet still { 0, 1 }



- The operation of the automaton is not completely defined by the input
  - On input "11" the automaton could be in either state

Epsilon Moves

- Another kind of transition: ε-moves


$$\begin{tikzcd} A \arrow[r, bend left, "\varepsilon"] & B \end{tikzcd}$$

- Machine can move from state A to state B without reading input

Deterministic and Nondeterministic Automata

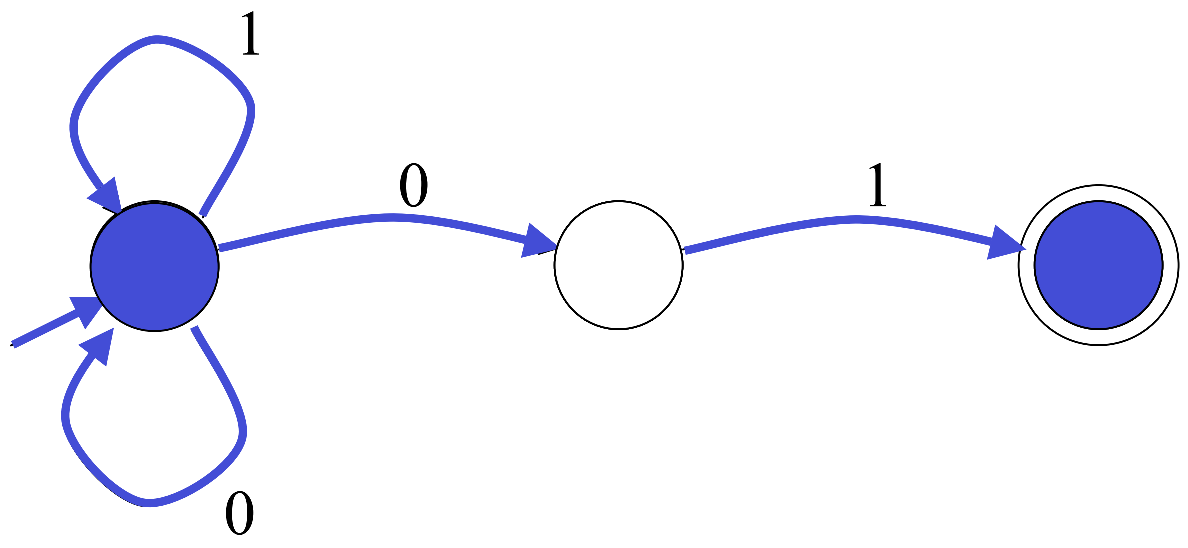
- Deterministic Finite Automata (DFA)
  - One transition per input per state
  - No ε-moves
- Nondeterministic Finite Automata (NFA)
  - Can have multiple transitions for one input in a given state
  - Can have ε-moves
- *Finite* automata have *finite* memory
  - Need only to encode the current state

Execution of Finite Automata

- A DFA can take only one path through the state graph
  - Completely determined by input
- NFAs can choose
  - Whether to make ε-moves
  - Which of multiple transitions for a single input to take

Acceptance of NFAs

- An NFA can get into multiple states



- Input: 1 0 1
- Rule: NFA accepts if it <u>can</u> get in a final state

NFA vs. DFA (1)

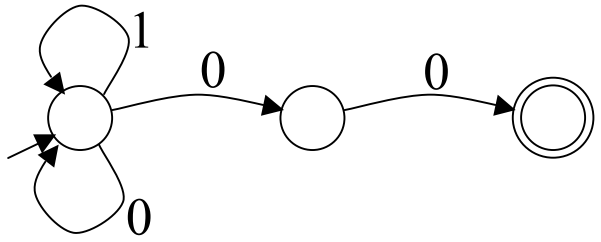
- NFAs and DFAs recognize the same set of languages (regular languages)

- DFAs are easier to implement
  - There are no choices to consider

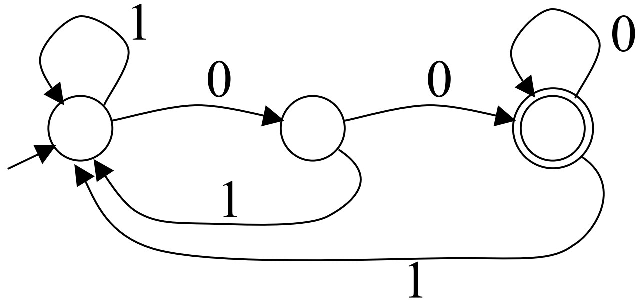
NFA vs. DFA (2)

- For a given language the NFA can be simpler than the DFA

NFA



DFA



- DFA can be exponentially larger than NFA

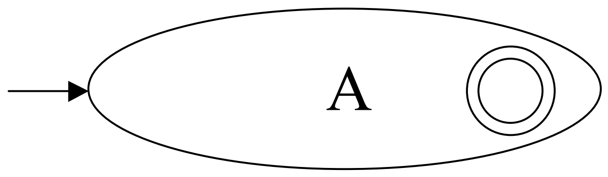
Regular Expressions to Finite Automata

- High-level sketch

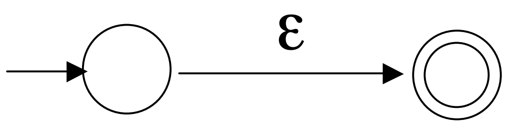
$$\begin{tikzcd}
 & \text{NFA} \arrow[dr] & \\
\text{Regular expressions} \arrow[ur] & & \text{DFA} \arrow[d] \\
\text{Lexical Specification} \arrow[u] & & \text{Table-driven Implementation of DFA}
\end{tikzcd}$$

Regular Expressions to NFA (1)

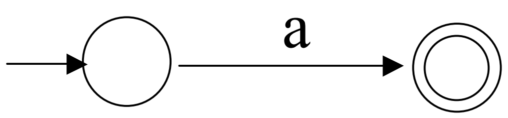
- For each kind of rexp, define an NFA
  - Notation: NFA for rexp A



- For ε

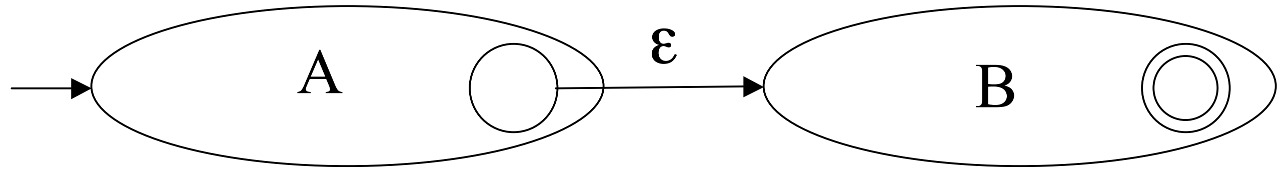


- For input a

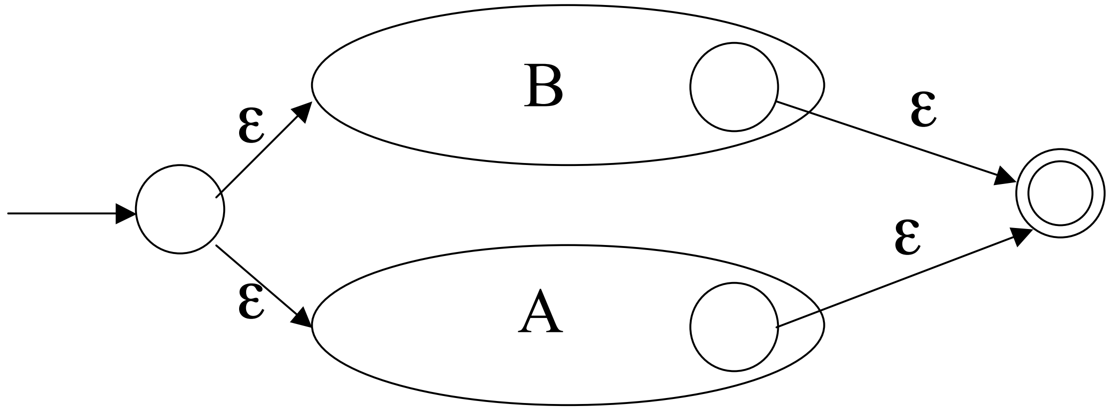


Regular Expressions to NFA (2)

- For *AB*

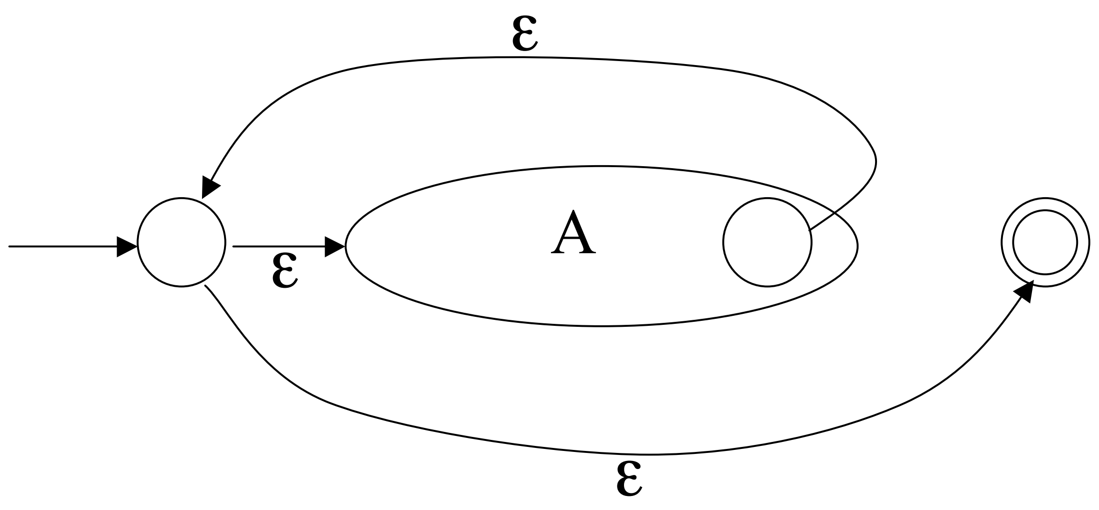


- For *A* | B



Regular Expressions to NFA (3)

- For *A**

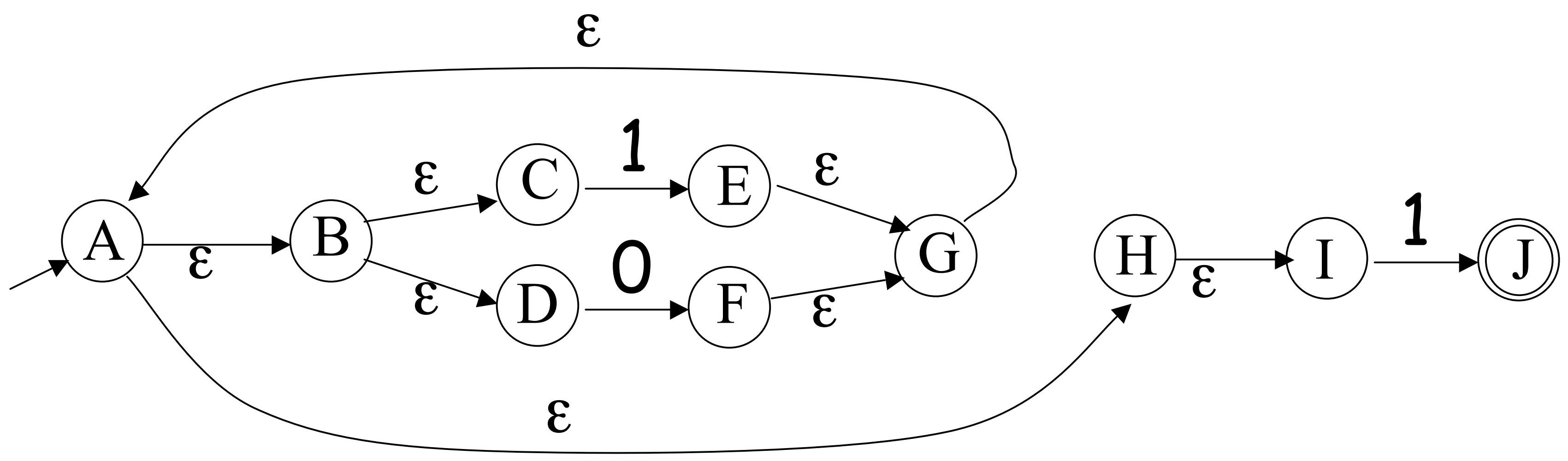


Example of RegExp -> NFA conversion

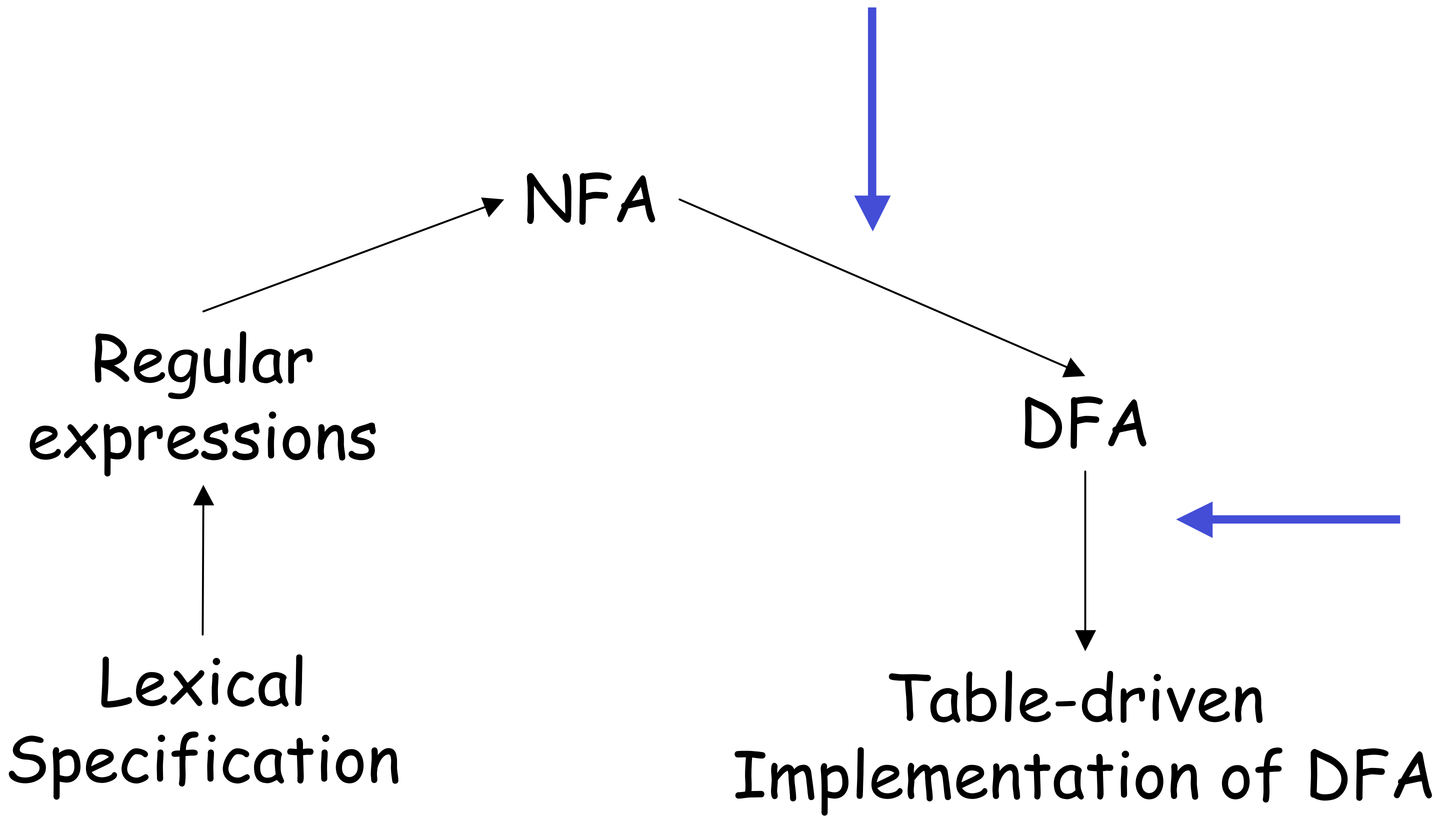
- Consider the regular expression

  (1 | 0)*1

- The NFA is



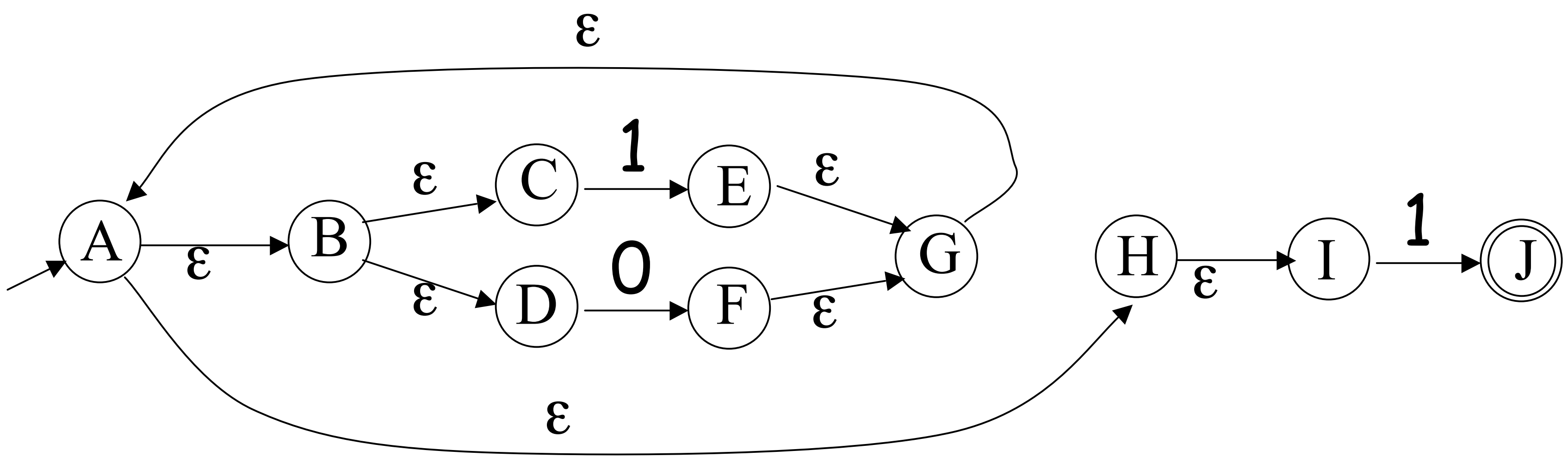
Next

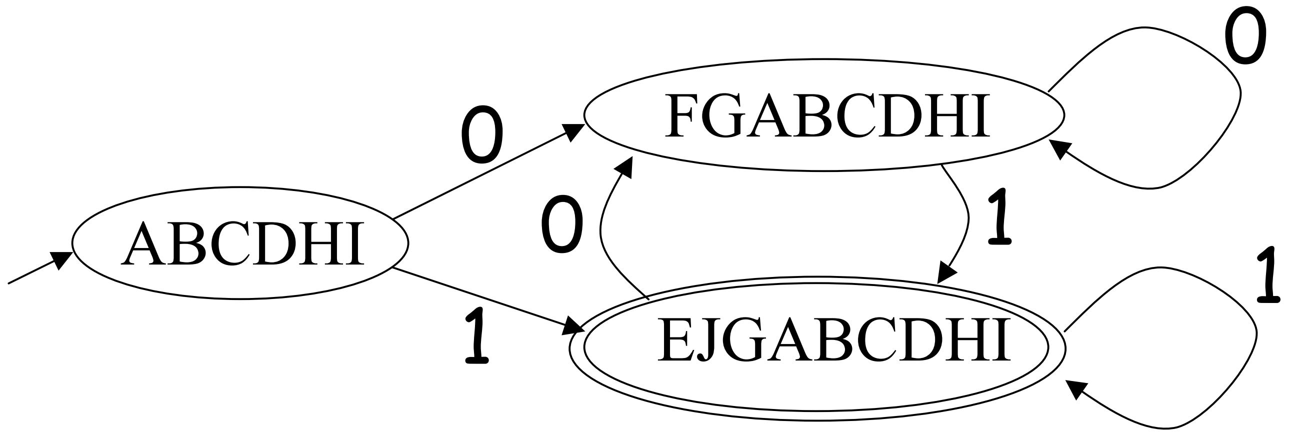

$$\begin{tikzcd}
 & \text{NFA} \arrow[dr] & \\
\text{Regular expressions} \arrow[ur] & & \text{DFA} \arrow[d] \\
\text{Lexical Specification} \arrow[u] & & \text{Table-driven Implementation of DFA}
\end{tikzcd}$$

NFA to DFA. The Trick

- Simulate the NFA
- Each state of resulting DFA
  = a non-empty subset of states of the NFA
- Start state
  = the set of NFA states reachable through ε-moves from NFA start state
- Add a transition $S \rightarrow^{a} S'$ to DFA iff
  - S' is the set of NFA states reachable from the states in S after seeing the input a
    - considering ε-moves as well

NFA -> DFA Example





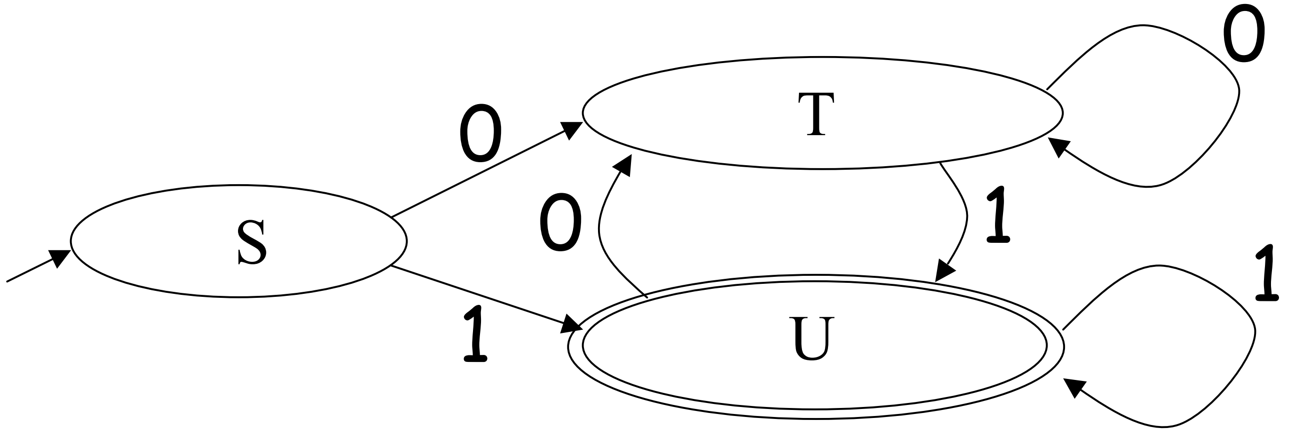
NFA to DFA. Remark

- An NFA may be in many states at any time
- How many different states ?
- If there are N states, the NFA must be in some subset of those N states
- How many non-empty subsets are there?
  - $2^N - 1$ = finitely many, but exponentially many

Implementation

- A DFA can be implemented by a 2D table T
  - One dimension is "states"
  - Other dimension is "input symbols"
  - For every transition $S_i \rightarrow^a S_k$ define T[i,a] = k
- DFA "execution"
  - If in state S_i and input a, read T[i,a] = k and skip to state S_k
  - Very efficient

Table Implementation of a DFA


$$\begin{tikzcd}
 & T \arrow[loop right, "0"] \arrow[d, bend left, "1"] \\
\arrow[r] S \arrow[ur, "0"] \arrow[r, "1"'] & U \arrow[u, bend left, "0"] \arrow[loop right, "1"]
\end{tikzcd}$$

	0	1
S	T	U
T	T	U
U	T	U

Implementation (Cont.)

- NFA -> DFA conversion is at the heart of tools such as flex or jflex
- But, DFAs can be huge
- In practice, flex-like tools trade off speed for space in the choice of NFA and DFA representations

Perl's "Regular Expressions"

- Some kind of pattern-matching feature now common in programming languages.
- Perl's is widely copied (cf. Java, Python).
- *Not* regular expressions, despite name.
  - E.g., pattern /A (\S+) is a $1/ matches "A spade is a spade" and "A deal is a deal", but not "A spade is a shovel"
  - But no regular expression recognizes this language!
  - Capturing substrings with (…) itself is an extension

Implementing Perl Patterns (Sketch)

- Can use NFAs, with some modification
- Implement an NFA as one would a DFA + use backtracking search to deal with states with nondeterministic choices.
- Add extra states (with ε transitions) for parentheses.
  - "(" state records place in input as side effect.
  - ")" state saves string started at matching "("
  - $*n* matches input with stored value.
- Backtracking much slower than DFA implementation.