# Global Optimization

Lecture 37

(From notes by R. Bodik & G. Necula)

# Lecture Outline

- Global flow analysis
- Global constant propagation
- Liveness analysis

# Local Optimization

Recall the simple basic-block optimizations

- Constant propagation
- Dead code elimination

```
X := 3
Y := Z * W
Q := X + Y
```

→

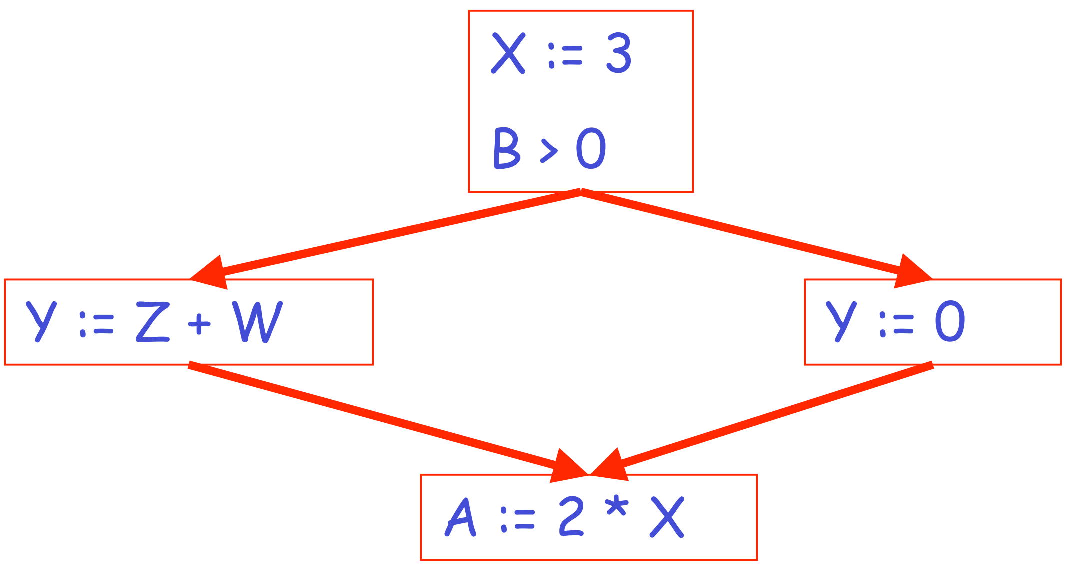
```
X := 3
Y := Z * W
Q := 3 + Y
```

→

```
Y := Z * W
Q := 3 + Y
```

# Global Optimization

These optimizations can be extended to an entire control-flow graph



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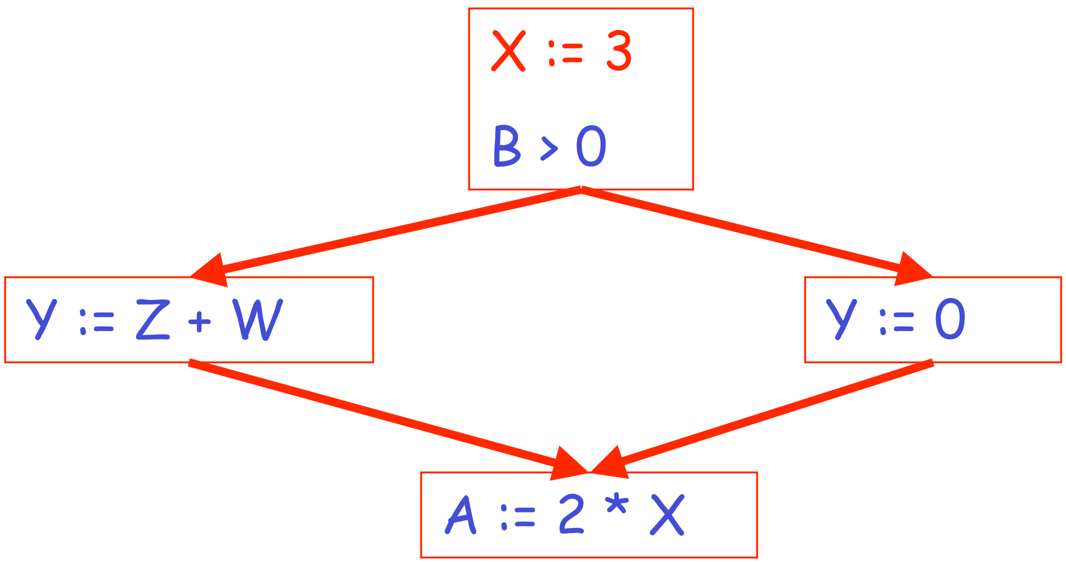

# Global Optimization

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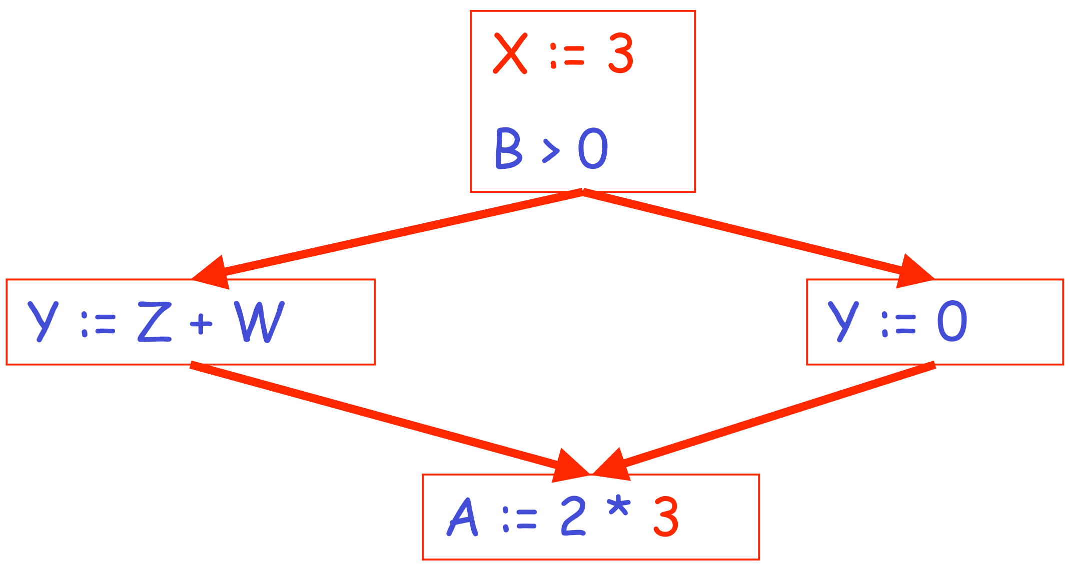
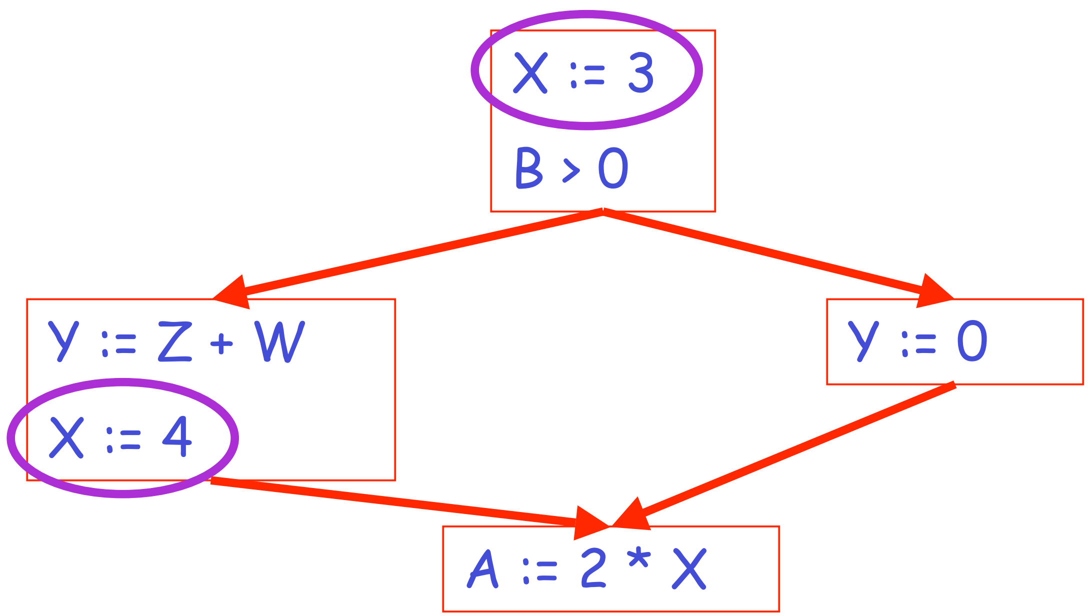


# Correctness

- How do we know it is OK to globally propagate constants?
- There are situations where it is incorrect:

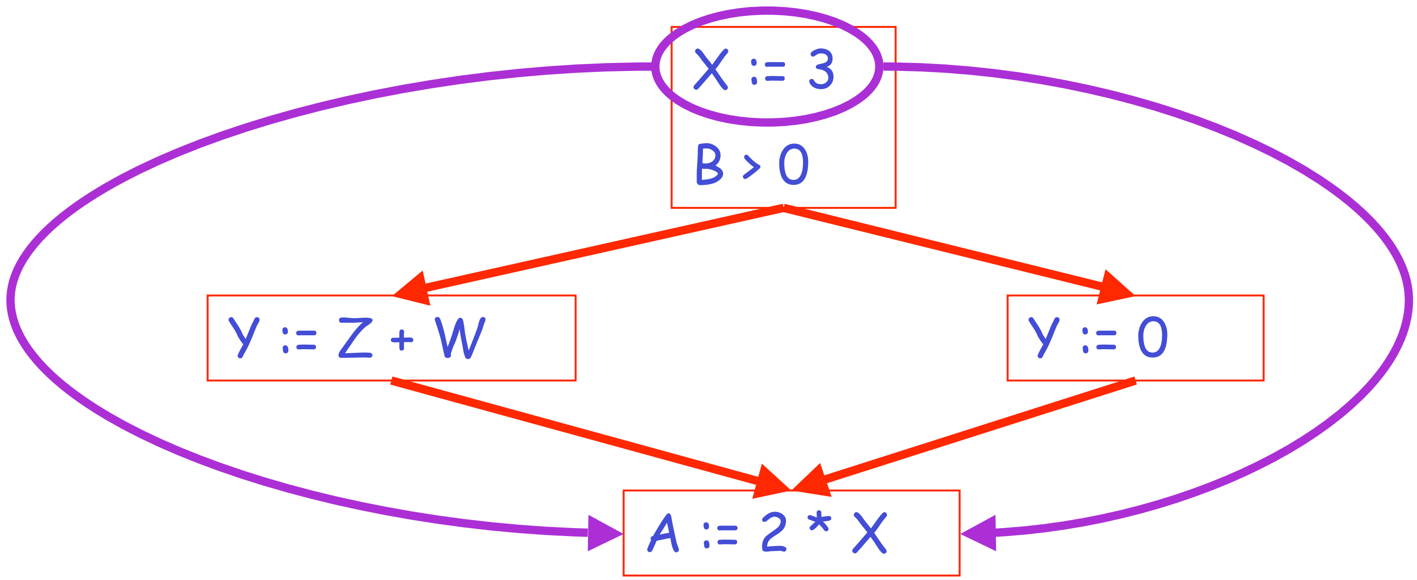


# Correctness (Cont.)

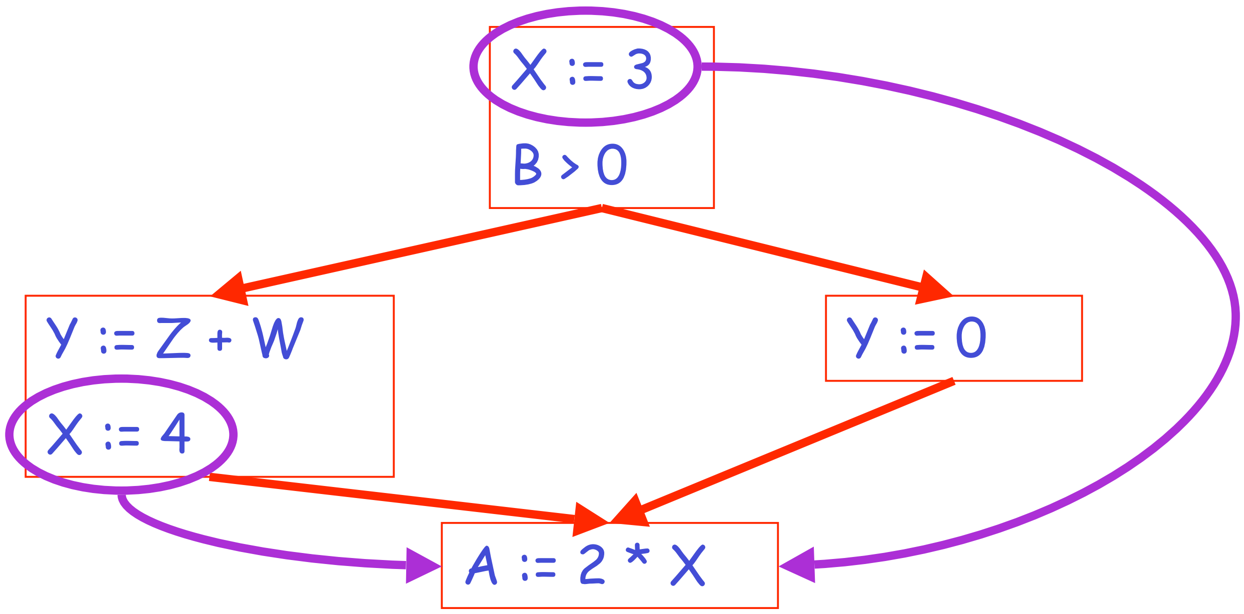
To replace a use of x by a constant k we must know that:

*On every path to the use of x, the last assignment to x is x := k* **

# Example 1 Revisited



# Example 2 Revisited



# Discussion

- The correctness condition is not trivial to check
- "All paths" includes paths around loops and through branches of conditionals
- Checking the condition requires global analysis
  - An analysis of the entire control-flow graph for one method body

## Global Analysis

Global optimization tasks share several traits:

- The optimization depends on knowing a property P at a particular point in program execution
- Proving P at any point requires knowledge of the entire method body

- Property P is typically undecidable !

## Undecidability of Program Properties

- Rice's theorem: Most interesting dynamic properties of a program are undecidable:
  - Does the program halt on all (some) inputs?
    - This is called the halting problem
  - Is the result of a function F always positive?
    - Assume we can answer this question precisely
    - Take function H and find out if it halts by testing function `F(x) { H(x); return 1; }` whether it has positive result
- Syntactic properties are decidable !
  - E.g., How many occurrences of "x" are there?
- Theorem does not apply in absence of loops

## Conservative Program Analyses

- So, we cannot tell for sure that "x" is always 3
  - Then, how can we apply constant propagation?
- It is OK to be *conservative*. If the optimization requires P to be true, then want to know either
  - P is definitely true
  - Don't know if P is true or false
- It is always correct to say "don't know"
  - We try to say don't know as rarely as possible
- All program analyses are conservative

# Global Analysis (Cont.)

- *Global dataflow analysis* is a standard technique for solving problems with these characteristics

- Global constant propagation is one example of an optimization that requires global dataflow analysis

# Global Constant Propagation

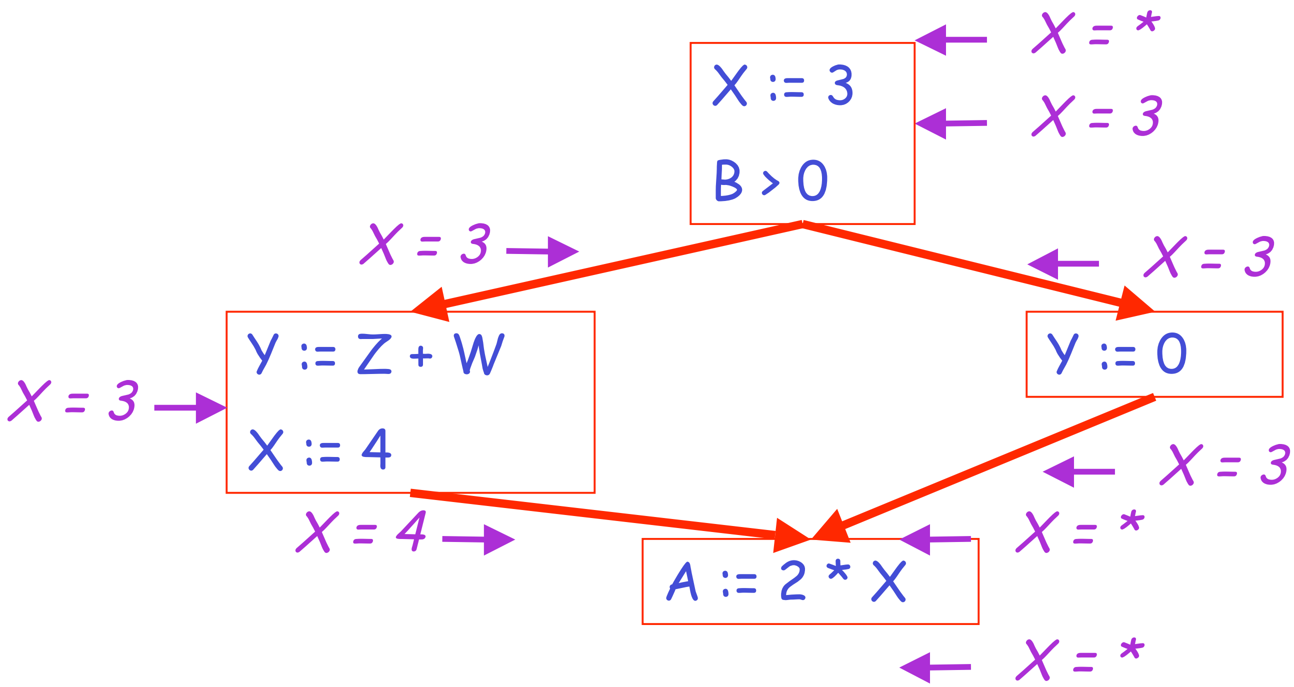
- Global constant propagation can be performed at any point where ** holds
- Consider the case of computing ** for a single variable X at all program points

# Global Constant Propagation (Cont.)

- To make the problem precise, we associate one of the following values with X at every program point

| *value* | *interpretation* |
|---|---|
| # | This statement is not reachable |
| c | X = constant c |
| * | Don't know if X is a constant |

# Example



# Using the Information

- Given global constant information, it is easy to perform the optimization
  - Simply inspect the x = _ associated with a statement using x
  - If x is constant at that point replace that use of x by the constant

- But how do we compute the properties x = _

# The Idea

*The analysis of a complicated program can be expressed as a combination of simple rules relating the change in information between adjacent statements*

# Explanation

- The idea is to “push” or “transfer” information from one statement to the next

- For each statement $s$, we compute information about the value of $x$ immediately before and after $s$

$$C_{in}(x,s) = \text{value of } x \text{ before } s$$

$$C_{out}(x,s) = \text{value of } x \text{ after } s$$

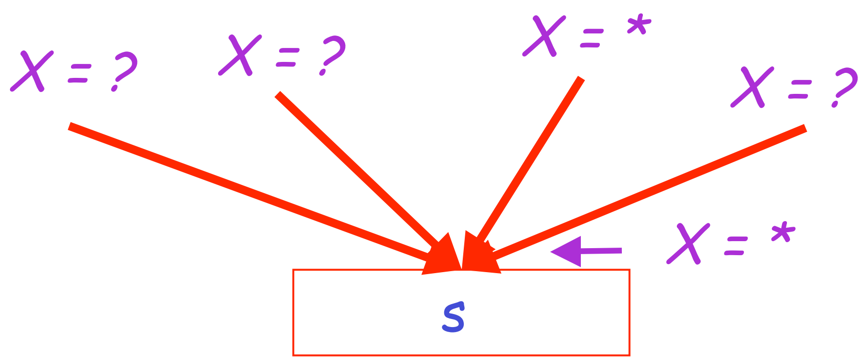
(we care about values #, *, k)

# Transfer Functions

- Define a *transfer function* that transfers information from one statement to another

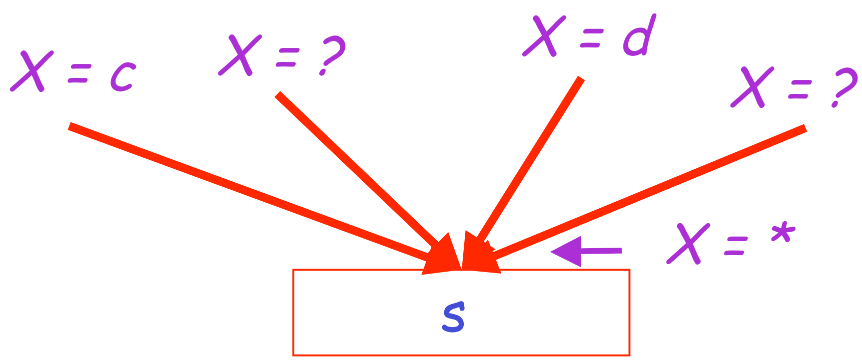
- In the following rules, let statement $s$ have immediate predecessor statements $p_1,\ldots,p_n$

# Rule 1



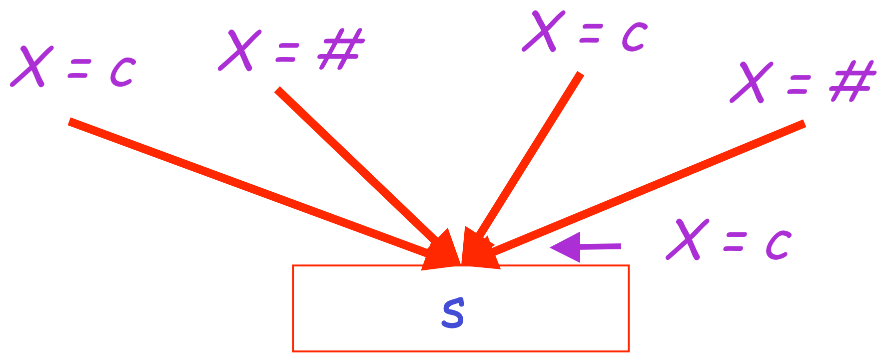

if $C_{out}(x, p_i) = *$ for some $i$, then $C_{in}(x, s) = *$

# Rule 2




If $C_{out}(x, p_i) = c$ and $C_{out}(x, p_j) = d$ and $d \neq c$
then $C_{in}(x, s) = *$

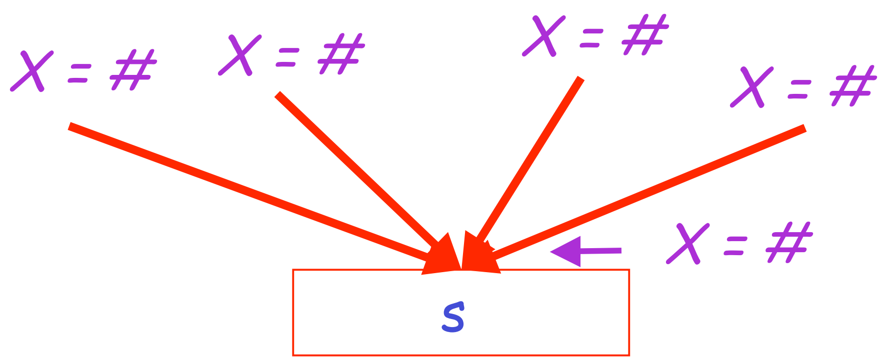
# Rule 3




if $C_{out}(x, p_i)$ = c or # for all $i$,

then $C_{in}(x, s)$ = c

# Rule 4



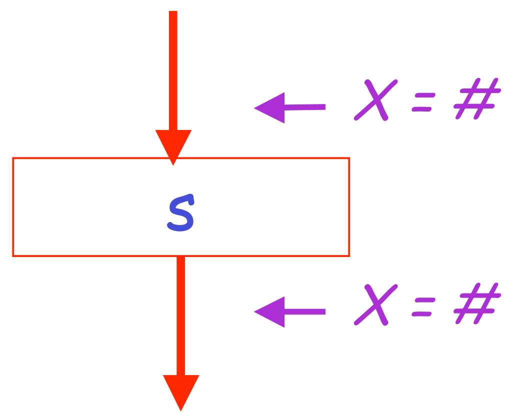

if $C_{out}(x, p_i) = \#$ for all $i$,

then $C_{in}(x, s) = \#$

# The Other Half

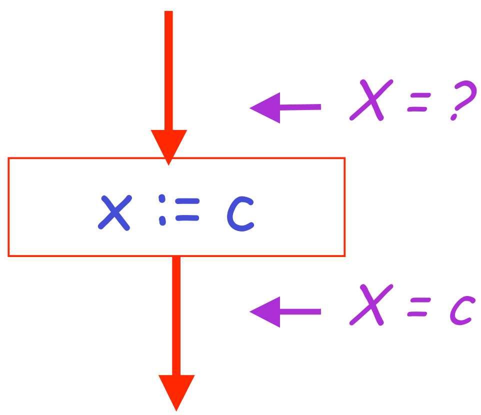
- Rules 1-4 relate the *out* of one statement to the *in* of the successor statement
  - they propagate information *forward* across CFG edges
- Now we need rules relating the *in* of a statement to the *out* of the same statement
  - to propagate information across statements

# Rule 5



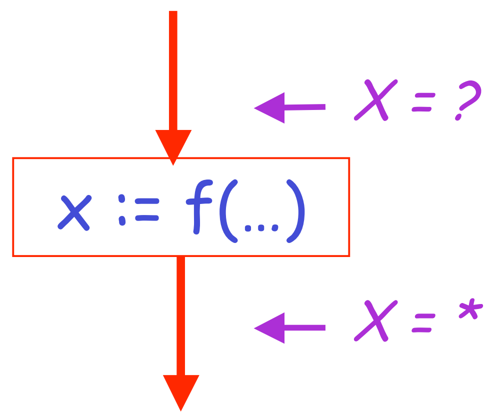

$$C_{out}(x, s) = \# \text{ if } C_{in}(x, s) = \#$$

# Rule 6



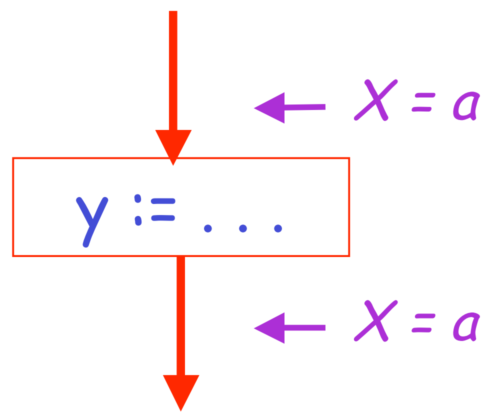

$C_{out}(x, x := c) = c$ if $c$ is a constant

# Rule 7




$$C_{out}(x, x := f(\ldots)) = *$$

# Rule 8




$$C_{out}(x, y := \ldots) = C_{in}(x, y := \ldots) \text{ if } x \neq y$$

## An Algorithm

1. For every entry s to the program, set $C_{in}(x, s) = *$

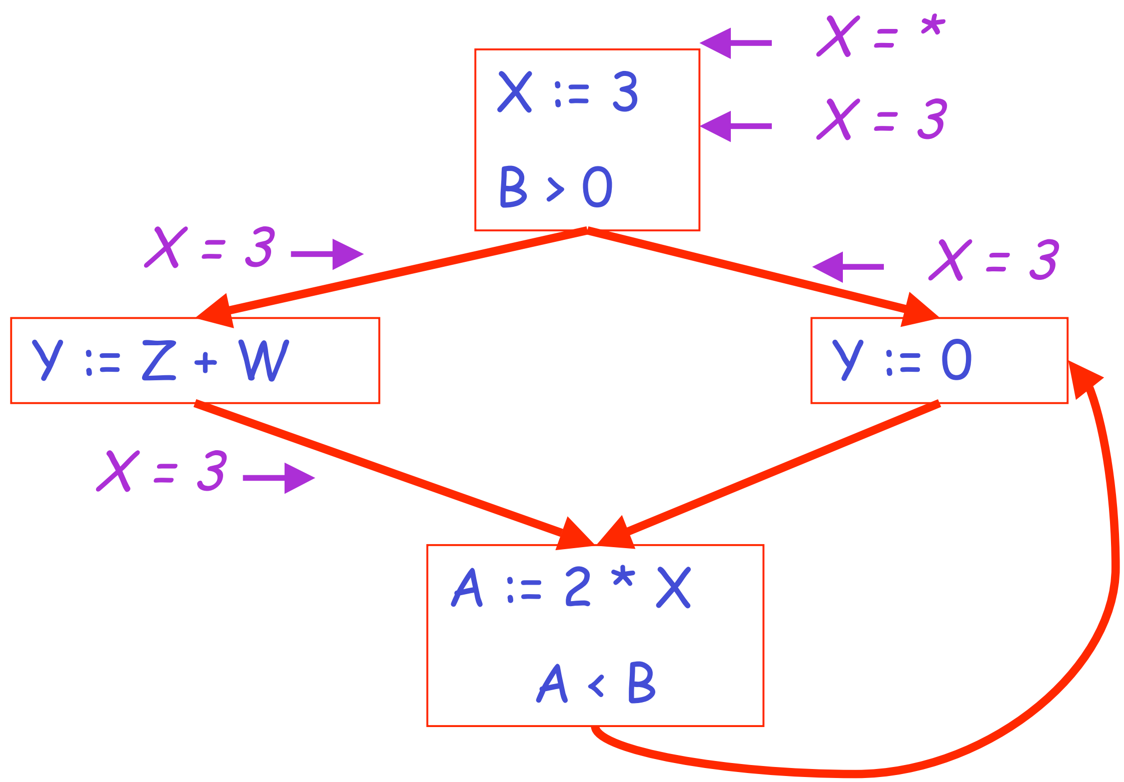
2. Set $C_{in}(x, s) = C_{out}(x, s) = \#$ everywhere else

3. Repeat until all points satisfy 1-8:

   Pick s not satisfying 1-8 and update using the appropriate rule

# The Value #

- To understand why we need #, look at a loop



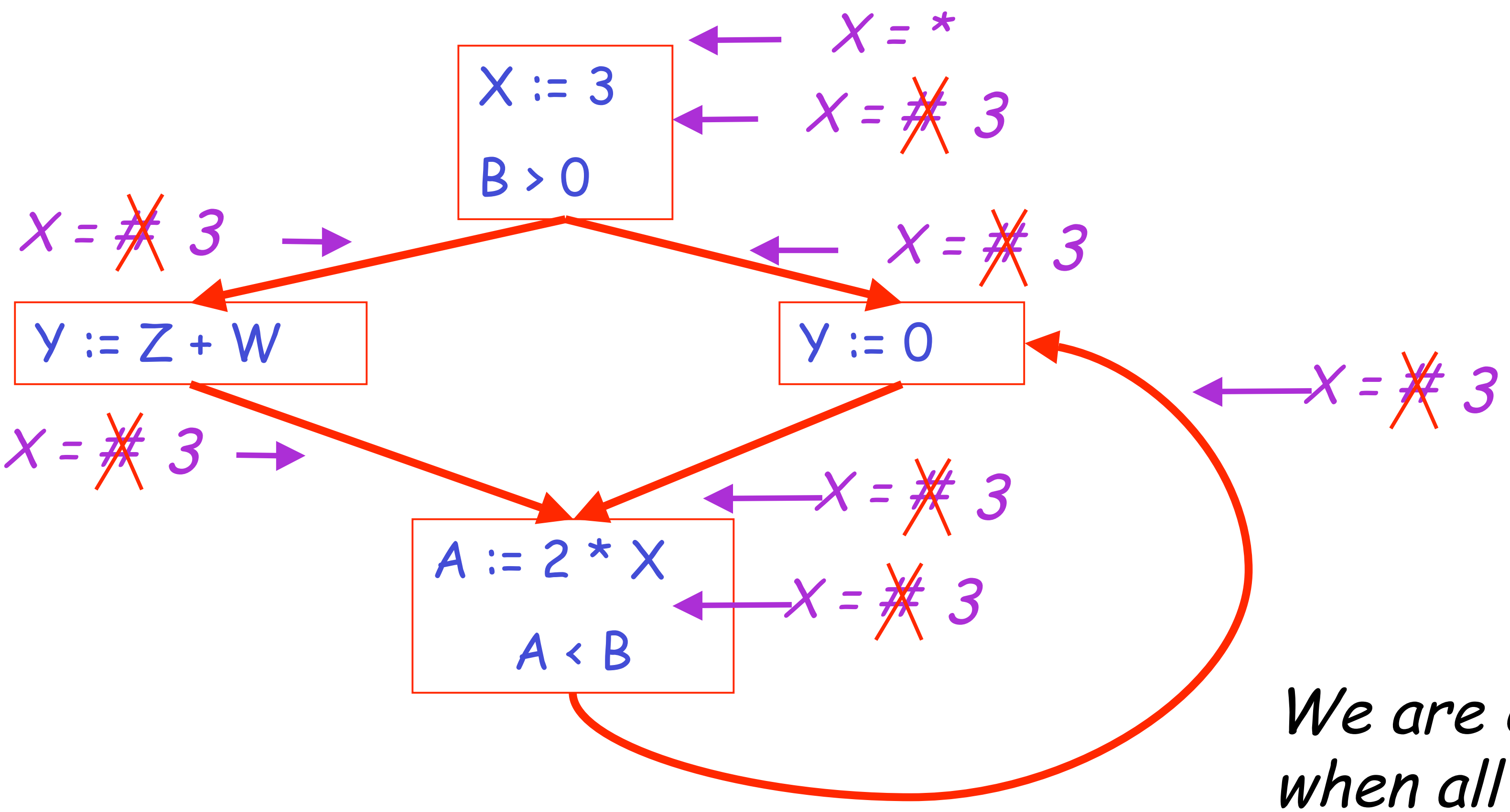
## Discussion

- Consider the statement Y := 0
- To compute whether X is constant at this point, we need to know whether X is constant at the two predecessors
  - X := 3
  - A := 2 * X
- But info for A := 2 * X depends on its predecessors, including Y := 0!

## The Value # (Cont.)

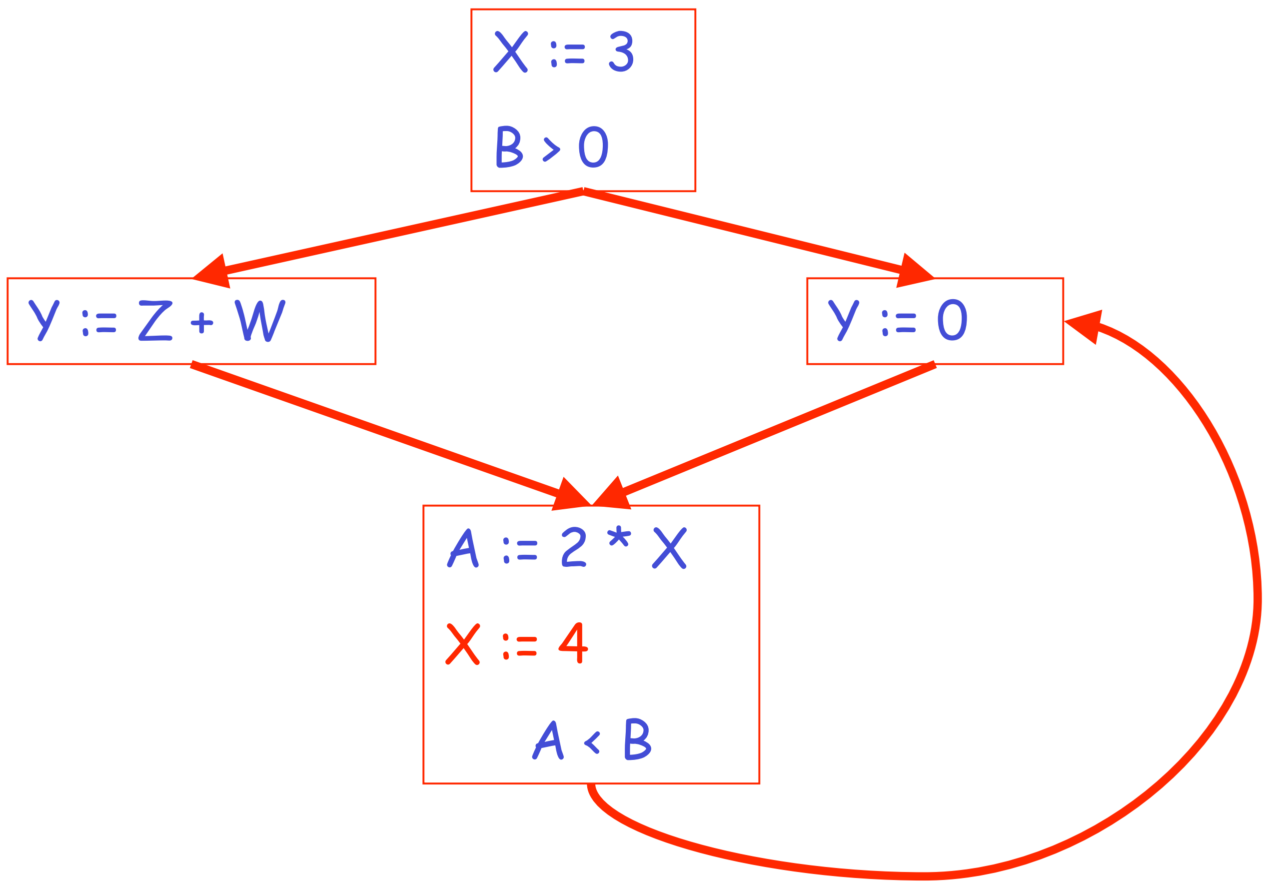
- Because of cycles, all points must have values at all times
- Intuitively, assigning some initial value allows the analysis to break cycles
- The initial value # means "So far as we know, control never reaches this point"

# Example

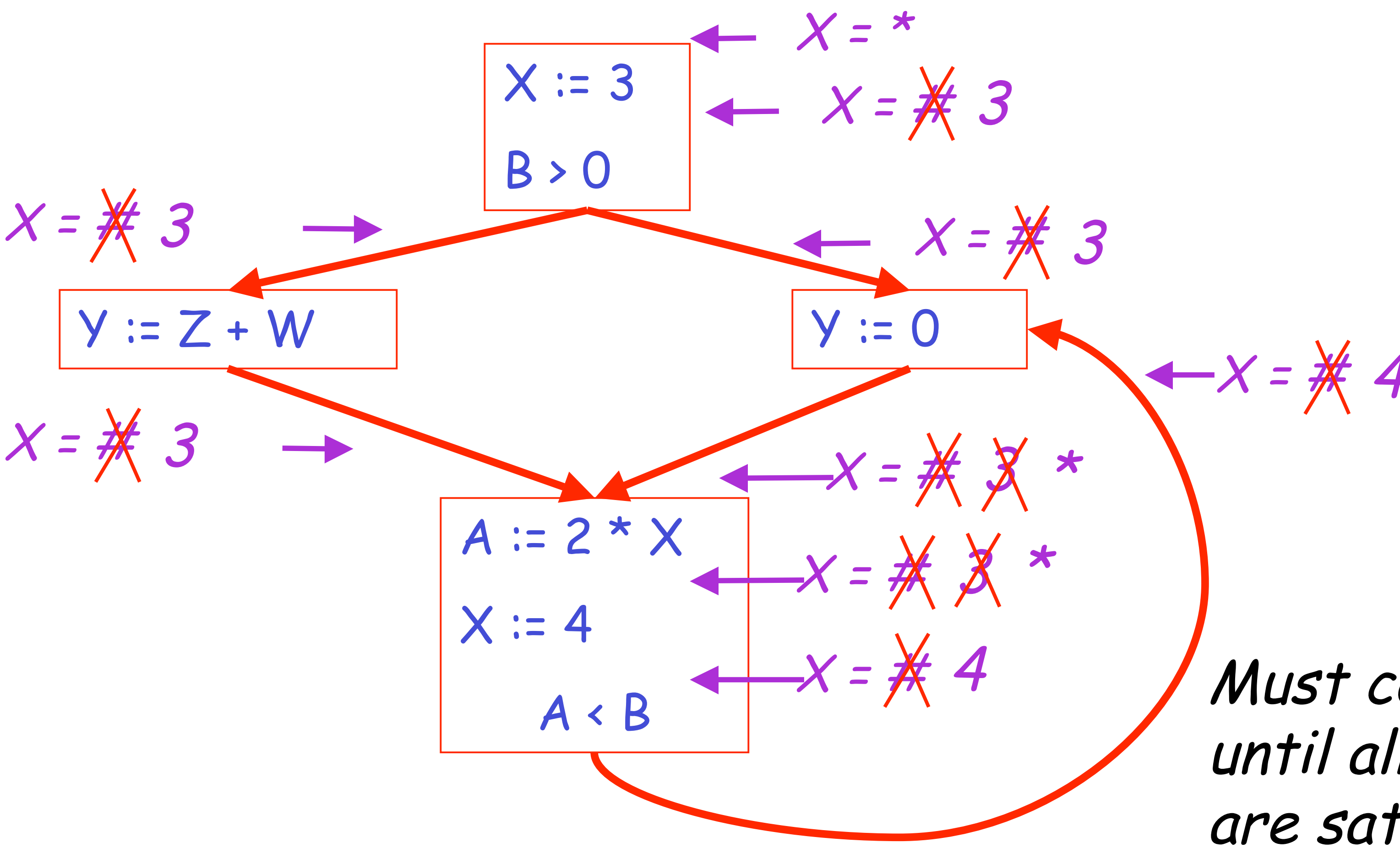



*We are done when all rules are satisfied !*

# Another Example



# Another Example



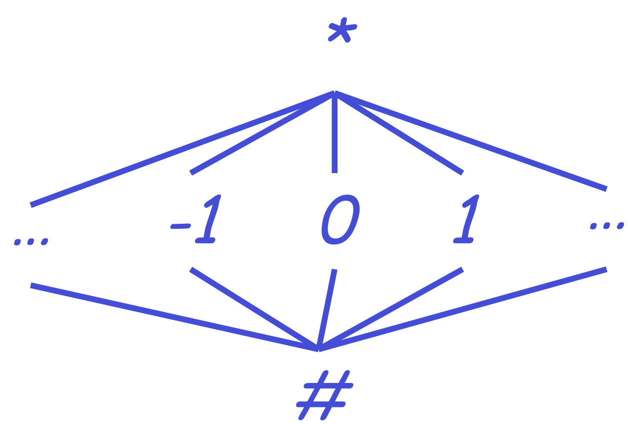

*Must continue until all rules are satisfied !*

## Orderings

- We can simplify the presentation of the analysis by ordering the values

$$\# < c < *$$

- Drawing a picture with "smaller" values drawn lower, we get



# Orderings (Cont.)

- * is the largest value, # is the least
  - All constants are in between and incomparable

- Let *lub* be the least-upper bound in this ordering

- Rules 1-4 can be written using lub:

  $C_{in}(x, s) = \text{lub} \{ C_{out}(x, p) \mid p \text{ is a predecessor of } s \}$

## Termination

- Simply saying "repeat until nothing changes" doesn't guarantee that eventually nothing changes

- The use of lub explains why the algorithm terminates
  - Values start as # and only *increase*
  - # can change to a constant, and a constant to *
  - Thus, C_(x, s) can change at most twice

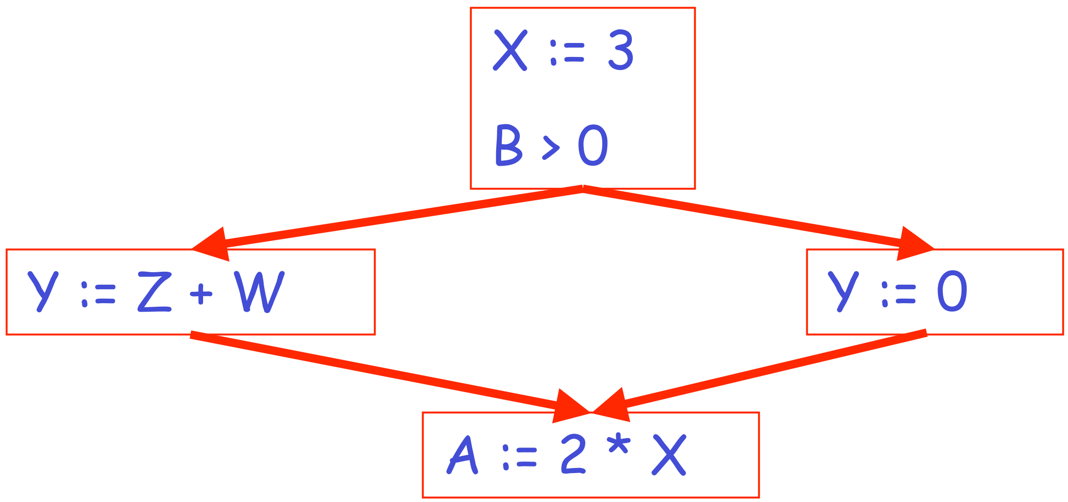
# Termination (Cont.)

Thus the algorithm is linear in program size

Number of steps =
Number of C_(....) values computed * 2 =
Number of program statements * 4

# Liveness Analysis

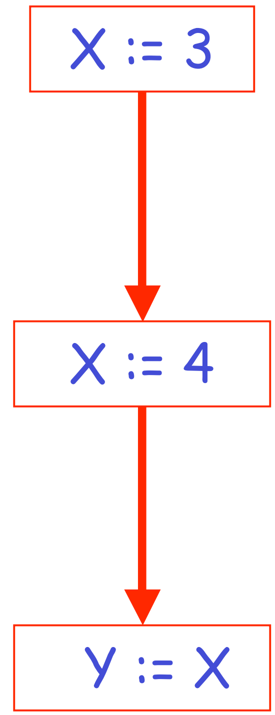
Once constants have been globally propagated, we would like to eliminate dead code




*After constant propagation, X := 3 is dead (assuming this is the entire CFG)*

# Live and Dead

- The first value of x is *dead* (never used)
- The second value of x is *live* (may be used)



# Liveness

A variable x is *live at statement s* if

- There exists a statement s′ that uses x
- There is a path from s to s′
- That path has no intervening assignment to x

# Global Dead Code Elimination

- A statement x := … is dead code if x is dead after the assignment

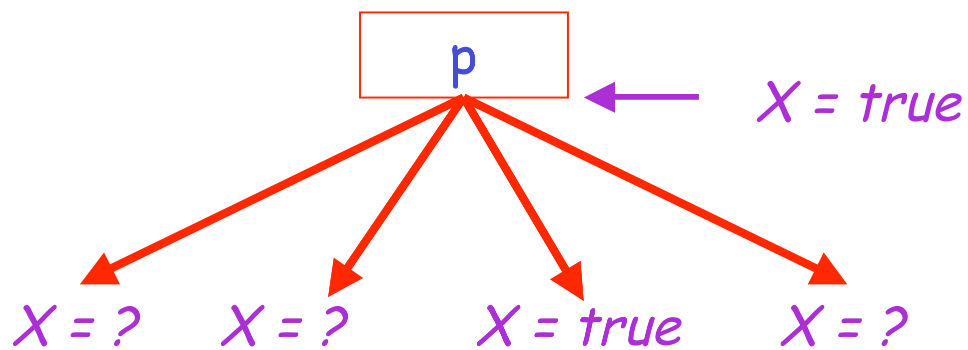
- Dead statements can be deleted from the program

- But we need liveness information first . . .

## Computing Liveness

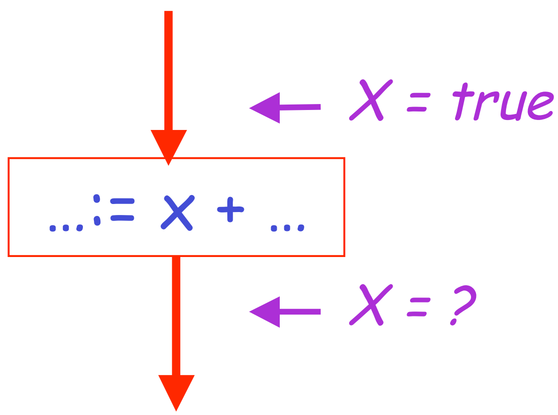
- We can express liveness in terms of information transferred between adjacent statements, just as in copy propagation
- Liveness is simpler than constant propagation, since it is a boolean property (true or false)

# Liveness Rule 1



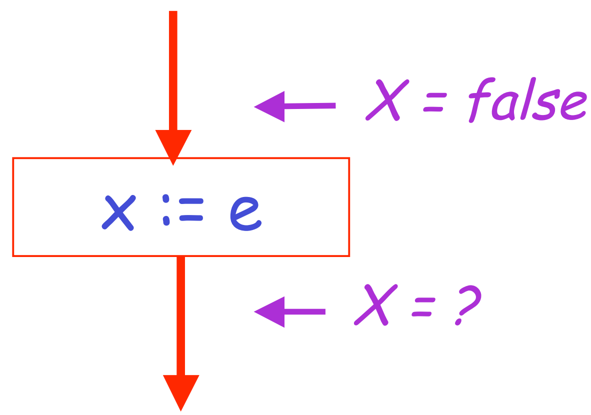

$L_{out}(x, p) = \vee \{ L_{in}(x, s) \mid s$ a successor of $p \}$

# Liveness Rule 2



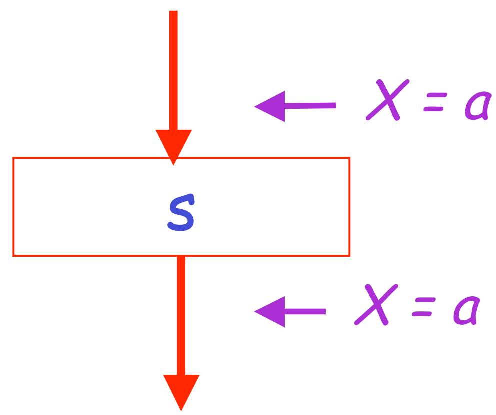

$L_{in}(x, s) = true$ if $s$ refers to $x$ on the rhs

# Liveness Rule 3




$L_{in}(x, x := e) = false$ if $e$ does not refer to $x$

# Liveness Rule 4




$L_{in}(x, s) = L_{out}(x, s)$ if $s$ does not refer to $x$

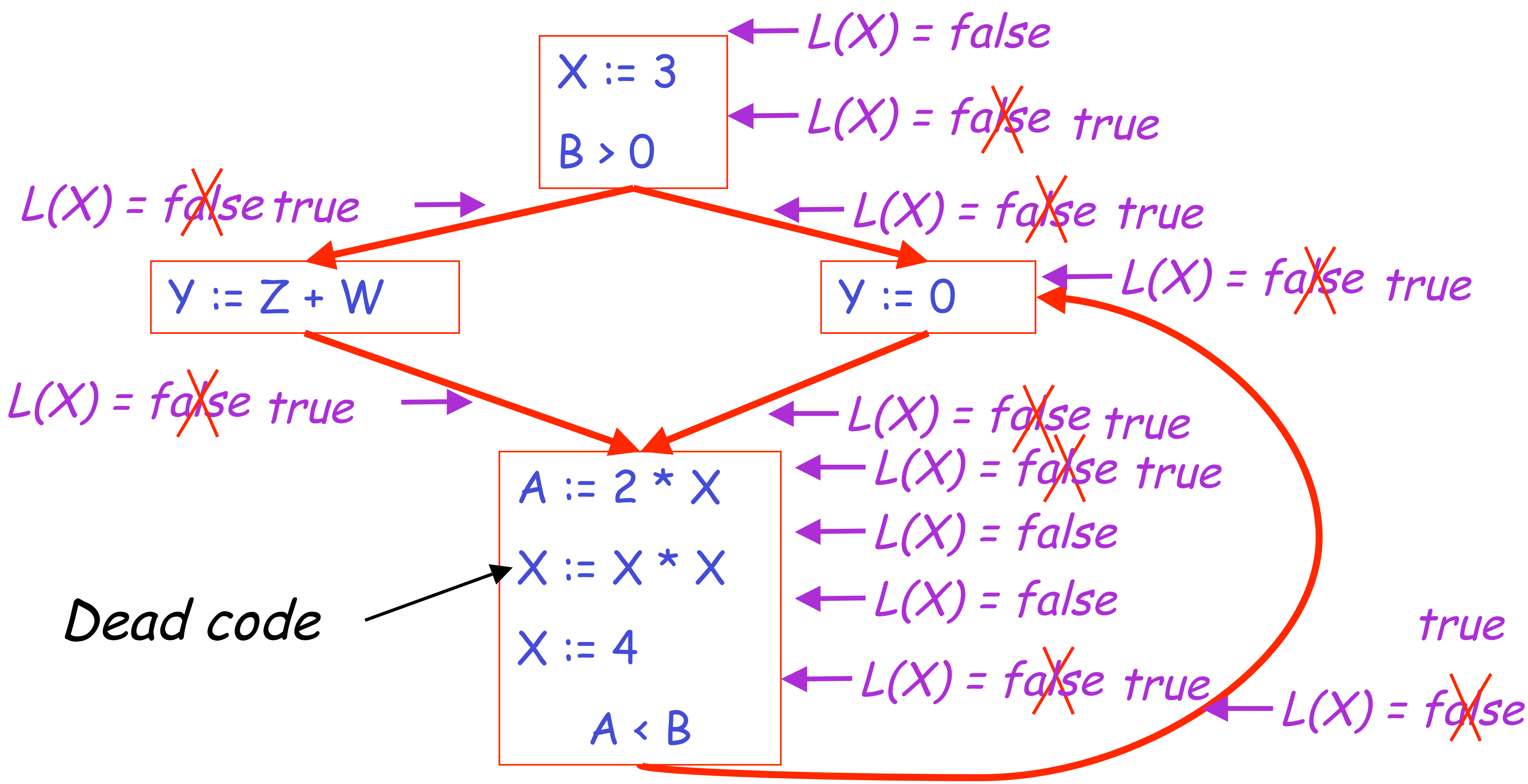
## Algorithm

1. Let all L_(…) = false initially

2. Repeat until all statements s satisfy rules 1-4

   Pick s where one of 1-4 does not hold and update using the appropriate rule

# Another Example



## Termination

- A value can change from false to true, but not the other way around
- Each value can change only once, so termination is guaranteed
- Once the analysis is computed, it is simple to eliminate dead code

# Forward vs. Backward Analysis

We've seen two kinds of analysis:

Constant propagation is a *forwards* analysis: information is pushed from inputs to outputs

Liveness is a *backwards* analysis: information is pushed from outputs back towards inputs

## Analysis

- There are many other global flow analyses
- Most can be classified as either forward or backward
- Most also follow the methodology of local rules relating information between adjacent program points