

Lecture 11: Types¹

Administrivia

- Reminder: Test #1 in class on Wednesday, 10 Oct.
- The autograder will run a couple of times between now and the deadline, and continually thereafter.

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Type Checking Phase

- Determines the type of each expression in the program, (each node in the AST that corresponds to an expression)
- Finds type errors.
 - Examples?
- The *type rules* of a language define each expression's type and the types required of all expressions and subexpressions.

Types and Type Systems

- A type is a set of *values* together with a set of *operations* on those values.
- E.g., fields and methods of a Java class are meant to correspond to values and operations.
- A language's *type system* specifies which operations are valid for which types.
- Goal of type checking is to ensure that operations are used with the correct types, enforcing intended interpretation of values.
- Notion of "correctness" often depends on what programmer has in mind, rather than what the representation would allow.
- Most operations are legal only for values of some types
 - Doesn't make sense to add a function pointer and an integer in C
 - It does make sense to add two integers
 - But both have the same assembly language implementation:

```
movl y, %eax; addl x, %eax
```

Uses of Types

- Detect errors:
 - Memory errors, such as attempting to use an integer as a pointer.
 - Violations of abstraction boundaries, such as using a private field from outside a class.
- Help compilation:
 - When Python sees $x+y$, its type systems tells it almost nothing about types of x and y , so code must be general.
 - In C, C++, Java, code sequences for $x+y$ are smaller and faster, because representations are known.

Review: Dynamic vs. Static Types

- A *dynamic type* attaches to an object reference or other value. It's a run-time notion, applicable to any language.
- The *static type* of an expression or variable is a constraint on the possible dynamic types of its value, enforced at compile time.
- Language is *statically typed* if it enforces a "significant" set of static type constraints.
 - A matter of degree: assembly language might enforce constraint that "all registers contain 32-bit words," but since this allows just about any operation, not considered static typing.
 - C sort of has static typing, but rather easy to evade in practice.
 - Java's enforcement is pretty strict.
- In early type systems, $\text{dynamic_type}(\mathcal{E}) = \text{static_type}(\mathcal{E})$ for all expressions \mathcal{E} , so that in all executions, \mathcal{E} evaluates to exactly type of value deduced by the compiler.
- Gets more complex in advanced type systems.

Subtyping

- Define a relation $X \preceq Y$ on classes to say that:
 - An object (value) of type X could be used when one of type Y is acceptable
 - or equivalently
 - X conforms to Y
- In Java this means that X extends Y .
- Properties:
 - $X \preceq X$
 - $X \preceq Y$ if X inherits from Y .
 - $X \preceq Z$ if $X \preceq Y$ and $Y \preceq Z$.

Example

```
class A { ... }
class B extends A { ... }
class Main {
  void f () {
    A x;           // x has static type A.
    x = new A();  // x's value has dynamic type A.
    ...
    x = new B();  // x's value has dynamic type B.
    ...
  }
}
```

Variables, with static type A can hold values with dynamic type $\preceq A$, or in general...

Type Soundness

Soundness Theorem on Expressions.

$$\forall E. \text{dynamic_type}(E) \preceq \text{static_type}(E)$$

- Compiler uses $\text{static_type}(E)$ (call this type C).
- All operations that are valid on C are also valid on values with types $\preceq C$ (e.g., attribute (field) accesses, method calls).
- Subclasses only add attributes.
- Methods may be overridden, but only with same (or compatible) signature.

Typing Options

- **Statically typed:** almost all type checking occurs at compilation time (C, Java). Static type system is typically rich.
- **Dynamically typed:** almost all type checking occurs at program execution (Scheme, Python, Javascript, Ruby). Static type system can be trivial.
- **Untyped:** no type checking. What we might think of as type errors show up either as weird results or as various runtime exceptions.

"Type Wars"

- Dynamic typing proponents say:
 - Static type systems are restrictive; can require more work to do reasonable things.
 - Rapid prototyping easier in a dynamic type system.
 - Use **duck typing**: define types of things by what operations they respond to ("if it walks like a duck and quacks like a duck, it's a duck").
- Static typing proponents say:
 - Static checking catches many programming errors at compile time.
 - Avoids overhead of runtime type checks.
 - Use various devices to recover the flexibility lost by "going static:" **subtyping**, **coercions**, and **type parameterization**.
 - Of course, each such wrinkle introduces its own complications.

Using Subtypes

- In languages such as Java, can define types (classes) either to
 - Implement a type, or
 - Define the operations on a family of types without (completely) implementing them.
- Hence, relaxes static typing a bit: we may know that something **is a Y** without knowing precisely which subtype it has.

Implicit Coercions

- In Java, can write

```
int x = 'c';
float y = x;
```
- But relationship between **char** and **int**, or **int** and **float** not usually called subtyping, but rather **conversion** (or **coercion**).
- Such implicit coercions avoid cumbersome casting operations.
- Might cause a change of value or representation,
- But usually, such coercions allowed implicitly only if type coerced to contains all the values of the that coerced from (a **widening coercion**).
- Inverses of widening coercions, which typically lose information (e.g., **int**→**char**), are known as **narrowing coercions**. and typically required to be explicit.
- **int**→**float** a traditional exception (implicit, but can lose information and is neither a strict widening nor a strict narrowing.)

Coercion Examples

```
Object x = ...; String y = ...;
int a = ...; short b = 42;
x = y; a = b; // OK
y = x; b = a; // ERRORS{ x = (Object) y; // {OK
a = (int) b; // OK
y = (String) x; // OK but may cause exception
b = (short) a; // OK but may lose information
```

Possibility of implicit coercion complicates type-matching rules (see C++).

Type Inference

- Types of expressions and parameters need not be explicit to have static typing. With the right rules, might *infer* their types.
- The appropriate formalism for type checking is logical rules of inference having the form

If Hypothesis is true, then Conclusion is true

- For type checking, this might become rules like

If E_1 and E_2 have types T_1 and T_2 , then E_3 has type T_3 .

- The standard notation used in scholarly work looks like this:

$$\frac{\Gamma \vdash E_1 : T_1, \quad \Gamma \vdash E_2 : T_2}{\Gamma \vdash E_3 : T_3}$$

Here, Γ stands for some set of assumptions about the types of free names, generically known as a *type environment* and $A \vdash B$ means "from A we may infer that B " or " A entails B ."

- Given proper notation, easy to read (with practice), so easy to check that the rules are accurate.
- Can even be mechanically translated into programs.

Prolog: A Declarative Programming Language

- Prolog is the most well-known *logic programming language*.
- Its statements "declare" facts about the desired solution to a problem. The system then figures out the solution from these facts.
- You saw this in CS61A.
- General form:

Conclusion :- Hypothesis₁, ..., Hypothesis_k.

for $k \geq 0$ means Means "may infer Conclusion by first establishing each Hypothesis." (when $k = 0$, we generally leave off the ':-').

Prolog: Terms

- Each conclusion and hypothesis is a kind of *term*, represent both programs and data. A term is:
 - A constant, such as `a`, `foo`, `bar12`, `=`, `+`, `'(`, `12`, `'Foo'`.
 - A variable, denoted by an unquoted symbol that starts with a capital letter or underscore: `E`, `Type`, `_foo`.
 - The nameless variable (`_`) stands for a different variable each time it occurs.
 - A structure, denoted in prefix form: `symbol(term1, ..., termk)`.
Very general: can represent ASTs, expressions, lists, facts.
- Constants and structures can also represent conclusions and hypotheses, just as some list structures in Scheme can represent programs.

Prolog Sugaring

- For convenience, allows structures written in infix notation, such as `a + X` rather than `+(a,X)`.
- List structures also have special notation:
 - Can write as `.(a,(b,(c,[])))` or `.(a,(b,(c,X)))`
 - But more commonly use `[a, b, c]` or `[a, b, c | X]`.

Inference Databases

- Can now express *ground* facts, such as `likes(brian, potstickers)`.
- *Universally quantified* facts, such as `eats(brian, X)`.
(for all `X`, brian eats `X`).
- Rules of inference, such as `eats(brian, X) :- isfood(X), likes(brian, X)`.
(you may infer that brian eats `X` if you can establish that `X` is a food and brian likes it.)
- A collection (database) of these constitutes a Prolog program.

Examples: From English to an Inference Rule

- "If `e1` has type `int` and `e2` has type `int`, then `e1+e2` has type `int`:"
`typeof(E1 + E2, int) :- typeof(E1, int), typeof(E2, int)`.
- "All integer literals have type `int`:"
`typeof(X, int) :- integer(X)`.
(`integer` is a built-in predicate on terms).
- In general, our `typeof` predicate will take an AST and a type as arguments.

Soundness

- We'll say that our definition of `typeof` is *sound* if
 - Whenever rules show that `typeof(e,t)`, `e` always evaluates to a value of type `t`
- We only want sound rules,
- But some sound rules are better than others; here's one that's not very useful:
`typeof(X, any) :- integer(X)`.
Instead, would be better to be more general, as in
`typeof(X, any)`.
(that is, any expression `X` is an `any`.)

Example: A Few Rules for Java (Classic Notation)

$$\frac{\vdash X : \text{boolean}}{\vdash !X : \text{boolean}}$$

$$\frac{\vdash X : T}{\vdash X : \text{void}}$$

$$\frac{\vdash E : \text{boolean} \quad \vdash S : \text{void}}{\vdash \text{while}(E, S) : \text{void}}$$

$$\frac{\vdash E_1 : \text{int} \quad \vdash E_2 : \text{int}}{\vdash E_1 + E_2 : \text{int}}$$

Example: A Few Rules for Java (Prolog)

- `typeof(! X, boolean) :- typeof(X, boolean).`
- `typeof(while(E, S), void) :- typeof(E, boolean), typeof(S, void).`
- `typeof(X,void) :- typeof(X,Y)`

The Environment

- What is the type of a variable instance? E.g., how do you show that `typeof(x, int)`?
- Ans: You can't, in general, without more information.
- We need a hypothesis of the form "we are in the scope of a declaration of x with type T."
- A *type environment* gives types for free names:
- a mapping from identifiers to types.
- (A variable is *free* in an expression if the expression contains an occurrence of the identifier that refers to a declaration outside the expression.
 - In the expression `x`, the variable `x` is free
 - In `lambda x: x + y` only `y` is free (Python).
 - In `map(lambda x: g(x,y), x)`, `x`, `y`, `map`, and `g` are free.

Defining the Environment in Prolog

- Can define a predicate, say, `defn(I,T,E)`, to mean "I is defined to have type T in environment E."
- We can implement such a defn in Prolog like this:

```
defn(I, T, [def(I,T) | _]).
defn(I, T, [def(I1,_)|R]) :- dif(I,I1), defn(I,T,R).
```

(`dif` is built-in, and means that its arguments differ).
- Now we revise `typeof` to have a 3-argument predicate: `typeof(E, T, Env)` means "E is of type T in environment Env," allowing us to say

```
typeof(I, T, Env) :- defn(I, T, Env).
```

Examples Revisited (Classic)

$$\frac{\Gamma \vdash X : \text{boolean}}{\Gamma \vdash !X : \text{boolean}}$$
$$\frac{\Gamma \vdash X : T}{\Gamma \vdash X : \text{void}}$$
$$\frac{\Gamma \vdash E : \text{boolean} \quad \Gamma \vdash S : \text{void}}{\Gamma \vdash \text{while}(E, S) : \text{void}}$$
$$\frac{\Gamma \vdash E_1 : \text{int} \quad \Gamma \vdash E_2 : \text{int}}{\Gamma \vdash E_1 + E_2 : \text{int}}$$
$$\overline{\Gamma \vdash I : \text{int}}$$

(where I is an integer literal and Γ is a type environment)

Examples Revisited (Prolog)

```
typeof(E1 + E2, int, Env)
  :- typeof(E1, int, Env), typeof(E2, int, Env).
typeof(X, int, _) :- integer(X).
typeof(!X, boolean, Env) :- typeof(X, boolean, Env).
typeof(while(E,S), void, Env) :-
  typeof(E, boolean, Env), typeof(S, boolean, Env).
```

Example: lambda (Python)

```
typeof(lambda(X,E1), any->T, Env) :-
  typeof(E1, T, [def(X, any) | Env]).
```

In effect, `[def(X, any) | Env]` means “`Env` modified to map `x` to `any` and behaving like `Env` on all other arguments.”

Example: Same Idea for 'let' in the Cool Language

- Cool is an object-oriented language sometimes used for the project in this course.
- The statement `let x : T0 in e1` creates a variable `x` with given type `T0` that is then defined throughout `e1`. Value is that of `e1`.
- Rule (assuming that “`let(X, T0, E1)`” is the AST for `let`):

```
typeof(let(X, T0, E1), T1, Env) :-
  typeof(E1, T1, [def(X, T0) | Env]).
```

“type of `let X: T0 in E1` is `T1`, assuming that the type of `E1` would be `T1` if free instances of `X` were defined to have type `T0`”.

Example of a Rule That's Too Conservative

- Let with initialization (also from Cool):

```
let x : T0 ← e0 in e1
```

- What's wrong with this rule?

```
typeof(let(X, T0, E0, E1), T1, Env) :-  
  typeof(E0, T0, Env),  
  typeof(E1, T1, [def(X, T0) | Env]).
```

(Hint: I said Cool was an object-oriented language).

Loosening the Rule

- Problem is that we haven't allowed type of initializer to be subtype of T0.

- Here's how to do that:

```
typeof(let(X, T0, E0, E1), T1, Env) :-  
  typeof(E0, T2, Env), T2 <= T0,  
  typeof(E1, T1, [def(X, T0) | Env]).
```

- Still have to define subtyping (written here as <=), but that depends on other details of the language.

As Usual, Can Always Screw It Up

```
typeof(let(X, T0, E0, E1), T1, Env) :-  
  typeof(E0, T2, Env), T2 <= T0,  
  typeof(E1, T1, Env).
```

This allows incorrect programs and disallows legal ones. Examples?

Function Application

- Consider only the one-argument case (Java).
- AST uses 'call', with function and list of argument types.

```
typeof(call(E1, [E2]), T, Env) :-  
  typeof(E1, T1->T, Env), typeof(E2, T1a, Env),  
  T1a <= T1.
```


Conditional Expressions

- Consider:

$e1$ if $e0$ else $e2$

or (from C) $e0 ? e1 : e2$.

- The result can be value of either $e1$ or $e2$.
- The dynamic type is either $e1$'s or $e2$'s.
- Either constrain these to be equal (as in ML):

```
typeof(if(E0,E1,E2), T, Env) :-  
  typeof(E0,bool,Env), typeof(E1,T,Env), typeof(E2,T,Env) .
```

- Or use the *smallest supertype* at least as large as both of these types—the *least upper bound (lub)* (as in Cool):

```
typeof(if(E0,E1,E2), T, Env) :-  
  typeof(E0,bool,Env), typeof(E1,T1,Env), typeof(E2,T2,Env),  
  lub(T,T1,T2) .
```