Due: Wednesday, 27 November 2013

1. I produced the following program using gcc -S foo.c (with an older version of gcc):
```
.globl f
    .type f, @function
f:
    pushl %ebp
    movl %esp, %ebp
    subl $16,%esp
    movl $0, -4(%ebp)
    movl $0, -8(%ebp)
    jmp .L2
.L3:
    movl -8(%ebp), %eax
    sall $2,%eax
    addl 8(%ebp), %eax
    movl (%eax), %eax
    addl %eax, -4(%ebp)
    incl -8(%ebp)
.L2:
    movl -8(%ebp), %eax
    cmpl 12(%ebp), %eax
    jl .L3
    movl -4(%ebp), %eax
    leave
    ret
```

Produce a plausible definition (in C) of function $f$, one that might have produced this output. The function does return a value.
2. In lecture, we talked about array descriptors, which are data structures containing all the information one needs to access (get the address of) an array element $A[i, j]$ in an implementation that allocates all elements of a new array contiguously. In C, multidimensional arrays are composed of rows of rows, so that $\mathrm{A}[\mathrm{i}, \mathrm{j}]$ (or A[i] [j] in C) is located at address $\left(A_{0,0}\right)+N \cdot S \cdot i+S \cdot j$, where the array in A is $M \times N$ and each element has size $S$. Thus, the three constants data address $\left(A_{0,0}\right)$ (the virtual origin), $N \cdot S$ (the row stride), and $S$ (the column stride) can be precomputed into an array descriptor, which the program can use to generate array accesses and can pass as a parameter to functions that expect to receive the array as a by-reference parameter. Show the IL code that you'd use to access array element $\mathrm{A}[\mathrm{i}][\mathrm{j}]$, assuming that the $d, t_{i}$, and $t_{j}$ are IL registers containing the address of the array descriptor for $A$, the value of $i$, and the value of $j$.
3. These exercises involve operations on array descriptors to give different view of an array. Just describe the calculations; we don't need actual IL code.
a. Suppose that a certain array descriptor contains the information ( $V O, S_{1}, S_{2}$ ) for accessing two-dimensional array B. Show how to create a new array descriptor that accesses column number $j$ of B. This will be a one-dimensional array descriptor (having only one stride).
b. Show how to create a new array descriptor that accesses the transpose of B.
c. Show how to create a new array descriptor (for array view B') that accesses the rows and columns of $B$ in reverse, so that $B^{\prime}[0,0]$ is the same as the last column of the last row of $B$.
4. A definition (that is, an assignment) of a simple variable is said to reach a point in the program if it might be the last assignment to that variable executed before execution reaches that point in the program. So for example, definition $A$ below reaches points $B$ and $C$, but not $D$ :

```
x = 3 # A
if a < 2:
    x = 2
    pass # D
else:
    y = 5
    pass # B
pass # C
```

Suppose we want to compute $R(p)$, the set of all definitions that reach point $p$ in a program. Give forward rules (in the style of the lecture) for computing the reaching definitions, $R_{\text {out }}(s)$ for a statement $s$ (the set of definitions that reach the point immediately after the statement) as a function of $R_{\text {in }}(s)$ (the definitions that reach the beginning) for each assignment statement $s$ and give the rules for computing $R_{\mathrm{in}}(s)$ as a function of the $R_{\text {out }}$ values of its predecessors.
5. Suppose that $L$ is a set of basic blocks, a subset of some large control-flow graph, $G$. Suppose also that $P$ is a basic block outside of $L$ with a single successor, that this successor is in $L$, and that $P$ dominates the blocks in $L$, meaning that all paths from the entrance block of $G$ to a block in $L$ go through $P$ first (typically $L$ is a loop, and we call $P$ a preheader). Finally, suppose that you have computed all reaching definitions (see last exercise) at all points in the program. How do you use this information to determine whether the calculation of a certain expression in one of the blocks of $L$, such as the right-hand side of the assignment statement

$$
\mathrm{x}:=\mathrm{a} * \mathrm{~b}
$$

may be moved out of $L$ and to the end of $P$ ?
6. Consider the loop

```
for i := 0 to n-1 do
    for j := 0 to n-1 do
        for k := 0 to n-1 do
            c[i,j] := c[i,j] + a[i,k] * b[k,j]
```

In this nested loop, $\mathrm{a}, \mathrm{b}$, and c are two-dimensional arrays of 4-byte integers. Here is a translation into intermediate code (assume that $a, b$, and $c$ are addresses of static memory, and that all other variables are in registers):

| Entry: |  | t11 := 4 * n | \#17 |
| :---: | :---: | :---: | :---: |
| i : $=0$ | \#1 | t 12 := t11 * k | \#18 |
| goto L6 | \#2 | t13 := 4 * j | \#19 |
| L1: |  | t 14 : $=\mathrm{t} 12+\mathrm{t} 13$ | \#20 |
| j := 0 | \#3 | t15 := *(t14 + b) | \#21 |
| goto L5 | \#4 | t 16 := t10 * t15 | \#22 |
| L2: |  | t 17 := t5 + t16 | \#23 |
| k := 0 | \#5 | t18 := 4 * n | \#24 |
| goto L4 | \#6 | t 19 := t18 * i | \#25 |
| L3: |  | t20 := 4 * j | \#26 |
| t1 := 4 * n | \#7 | t 21 := t19 + t20 | \#27 |
| t2 := t1 * i | \#8 | * $\mathrm{t} 21+\mathrm{c})$ : $=\mathrm{t} 17$ | \#28 |
| t3 := 4 * j | \#9 | $\mathrm{k}:=\mathrm{k}+1$ | \#29 |
| t4 := t2 + t3 | \#10 | L4: |  |
| t5 : $=$ * (t4 + c) | \#11 | if $\mathrm{k}<\mathrm{n}$ : goto L3 | \#30 |
| $\mathrm{t6}:=4 * \mathrm{n}$ | \#12 | $j \quad:=$ j + 1 | \#31 |
| t7 := t6 * i | \#13 | L5: |  |
| t8 : $=4$ * k | \#14 | if j < n : goto L2 | \#32 |
| $\mathrm{t9}$ : $=\mathrm{t} 7$ + t8 | \#15 | i := i + 1 | \#33 |
| t10 := * t 9 + a$)$ | \#16 | L6: <br> if i < n: goto L1 | \#34 |
|  |  | Exit: |  |

To notate accesses to memory, we've used C-like notation:

```
r1 := *(r2+K)
*(r1+K) := r2
*K := r3
r3 := *K
```

K is an integer literal, and L is a static-storage label (a constant address in memory). Unlike C , the additions here are just straight addition: no automatic scaling by word size.
a. According to this code, how are the elements of the three two-dimensional arrays laid out in memory (in what order do the elements of the arrays appear)?
b. Divide the instructions into basic blocks (feel free to refer to them by number) and show the flow graph.
c. The program is almost in SSA form, except for variables i, j, and k. Introduce new variables and $\phi$ functions as needed to put the program into SSA form (try to minimize $\phi$ functions).
d. Now optimize this code as best you can, moving assignments of invariant expressions out of loops, eliminating common subexpressions, removing dead statements, performing copy propogation, etc.

