Lecture 3: Finite Automata

Administrivia

- Everyone should now be registered electronically using the link on our webpage. If you haven't, do so today!
- I'd like to have teams formed by next Monday at the latest.
- Please fill out the background survey linked to on the homework page.

An Alternative Style for Describing Languages

- Rather than *giving a single pattern*, we can *give a set of rules*.
- Each rule has the form

$$A: \ \alpha_1\alpha_2\cdots\alpha_n, \ \ n \geq 0,$$

  where

  - A is a symbol that is intended to stand for a language (set of strings)—a *metavariable* or *nonterminal symbol*.
  - Each α_i is either a literal character (like "a") or a nonterminal symbol.

- The interpretation of this rule is

  One way to form a string in $L(A)$ (the language denoted by A) is to concatenate one string each from $L(\alpha_1)$, $L(\alpha_2)$, ...

  (where $L(\texttt{"}c\texttt{"})$ is just the language $\{\texttt{"}c\texttt{"}\}$).

- This is *Backus-Naur Form (BNF)*. A set of rules is a *grammar*.
- **Aside:** You'll see that ':' written many different ways, such as '::=', '$\longrightarrow$', etc. We'll just use the same notation our tools use.

Some Abbreviations

- The basic form from the last slide is good for formal analysis, but not for writing.
- So, we can allow some abbreviations that are obviously exandable into the basic forms:

Abbreviation	Meaning
$A:\ \mathcal{R}_1 \mid \cdots \mid \mathcal{R}_n$	$A:\ \mathcal{R}_1$ $\vdots$ $A:\ \mathcal{R}_n$
$A:\ \cdots(\mathcal{R})\cdots$	$B:\ \mathcal{R}$ $A: \cdots B \cdots$
$A:$ "c_1" | $\cdots$ | "c_n" (likewise other character classes)	$[c_1 \cdots c_n]$

Some Technicalities

- From the definition, each nonterminal in a grammar defines a language. Often, we are interested in just one of them (the *start symbol*), and the others are auxiliary definitions.
- The definition of what a rule means ("One way to form a string in $L(A)$ is...") leaves open the possibility that there are other ways to form items in $L(A)$ than covered in the rule.
- We need that freedom in order to allow multiple rules for A, but we don't really want to include strings that aren't covered by some rule.
- So precise mathematical definitions throw in sentences like:

  A grammar defines the *minimal* languages that contain all strings that satisfy the rules.

A Big Restriction (for now)

- For the time being, we'll also add a restriction. In each rule:

$$A: \ \alpha_1\alpha_2\cdots\alpha_n, \quad n \geq 0,$$

  we'll require that if α_i is a nonterminal symbol, then either
  - All the rules for that symbol have to occured before all the rules for A, or
  - $i = n$ (i.e., is the last item) and α_n is A.
- We call such a restricted grammar a *Type 3* or *regular* grammar. The languages definable by regular grammars are called *regular languages.*

Claim: Regular languages are exactly the ones that can be defined by regular expressions.

Proof of Claim (I)

- Start with a regular expression, $\mathcal{R}$, and make a (possibly not yet valid) rule,

  R: $\mathcal{R}$

- Create a new (preceding) rule for each parenthesized expression.
- This will leave just the constructs '$X*$', '$X+$', and '$X?$'. What do we do with them?

Proof of Claim (II)

Replace construct. . .	. . . with Q, where
$R*$	

Proof of Claim (II)

Replace construct...	... with Q, where
$R*$	`Q :` `Q : R Q`

R+

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Replace construct. . .	**. . . with Q, where**
R*	`Q :` `Q : R Q`
R+	`Q : R` `Q : R Q`

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Proof of Claim (II)

Replace construct. . .	**. . . with Q, where**
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R+	`Q : R` `Q : R Q`
R?	`Q :` `Q : R`

Example

- Consider the regular expression `("+"|"-")?("0"|"1")+`

1. `R: ("+"|"-")?("0"|"1")+` *replace with ...*

2. $\mathtt{Q}_1$: `"+" | "-"`
   $\mathtt{Q}_2$: `"0" | "1"`
   R: $\mathtt{Q}_1$? $\mathtt{Q}_2$+ *replace with ...*

3. $\mathtt{Q}_3$: ϵ | $\mathtt{Q}_1$
   $\mathtt{Q}_4$: $\mathtt{Q}_2$ | $\mathtt{Q}_2$ $\mathtt{Q}_4$
   R: $\mathtt{Q}_3$ $\mathtt{Q}_4$

Classical Pattern-Matching Implementation

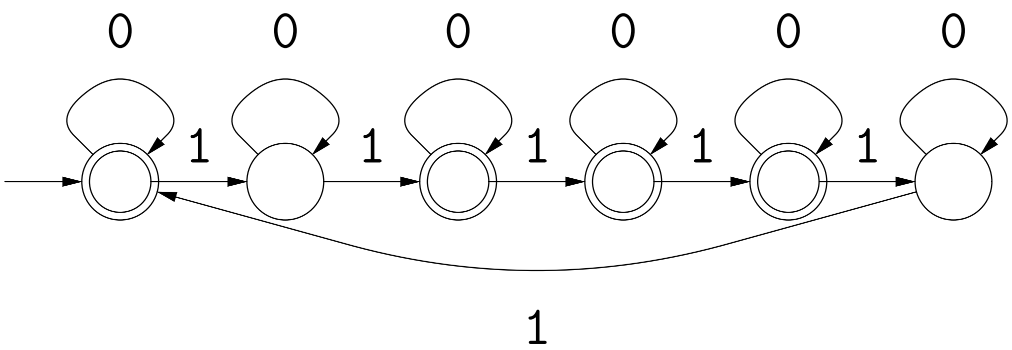
- For compilers, can generally make do with "classical" regular expressions.
- Implementable using *finite(-state) automata* or *FAs*. ("Finite state" = "finite memory").
- Classical construction:

  regular expression $\Rightarrow$ nondeterministic FA (NFA)
  $\Rightarrow$ deterministic FA (DFA) $\Rightarrow$ table-driven program.

Review: FA operation

- A FA is a graph whose nodes are states (of memory) and whose edges are state transitions. There are a finite number of nodes.
- One state is the designated start state.
- Some subset of the nodes are final states.
- Each transition is labeled with a set of symbols (characters, etc.) or ϵ.
- A FA recognizes a string $c_1 c_2 \cdots c_n$ if there is a path (sequence of edges) from the start state to a final state such that the labels of the edges in sequence, aside from ϵ edges, respectively contain $c_1, c_2, \ldots, c_n$.
- If the edges leaving any node have disjoint sets of characters and if there are no ϵ nodes, FA is a DFA, else an NFA.

Example: What does this DFA recognize?



What is the simplest equivalent NFA you can think of?

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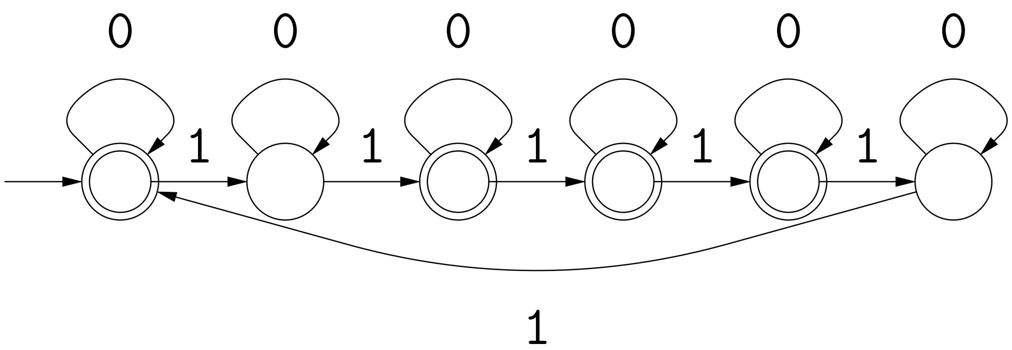

Bit strings with # of 1's divisible by 2 or 3.

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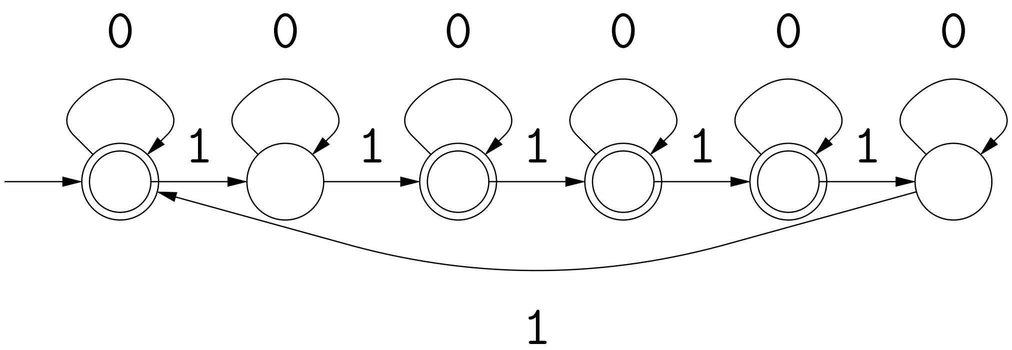

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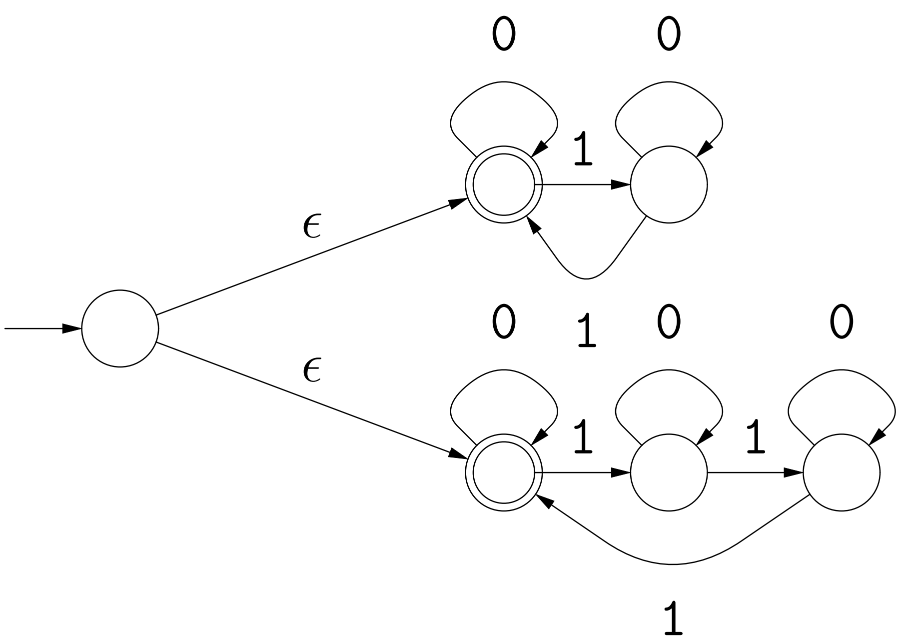

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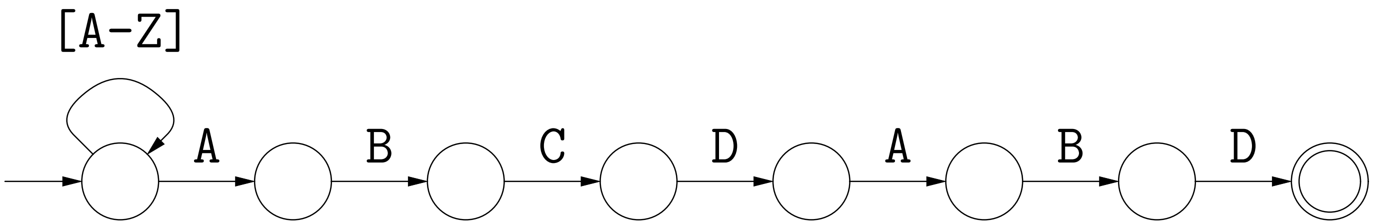

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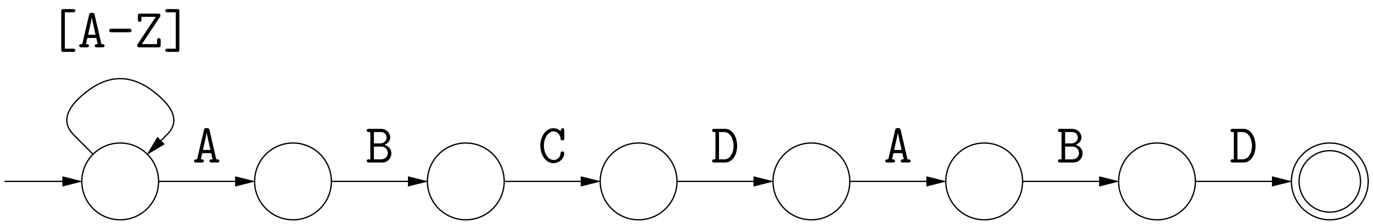

Strings of capitals ending in ABCDABD.

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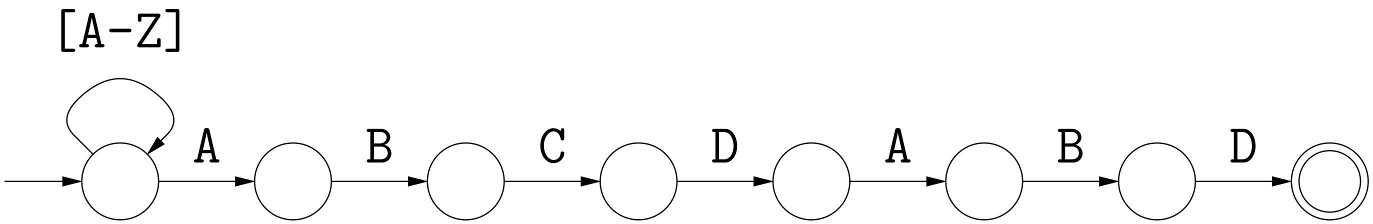

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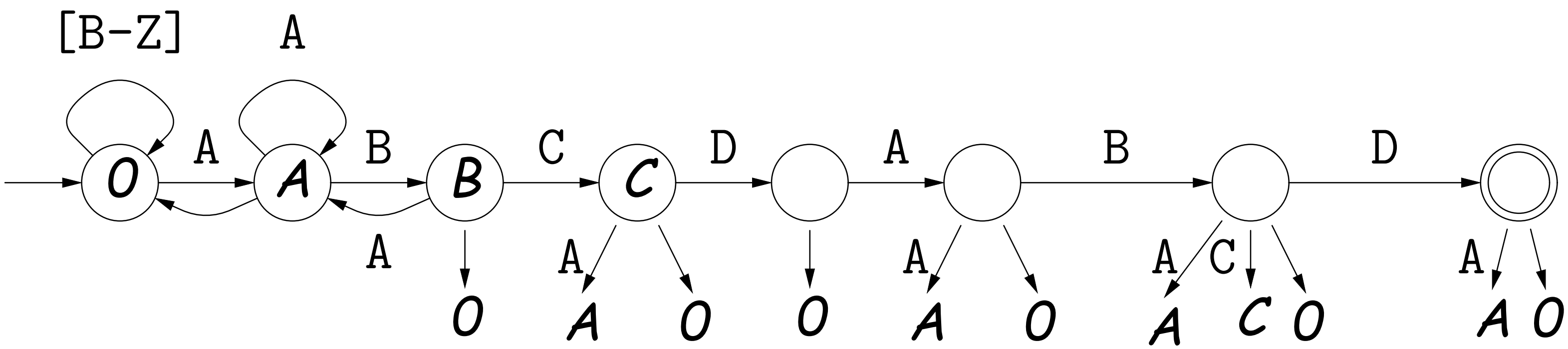

(Edges without labels mean "any character not covered by another edge.")

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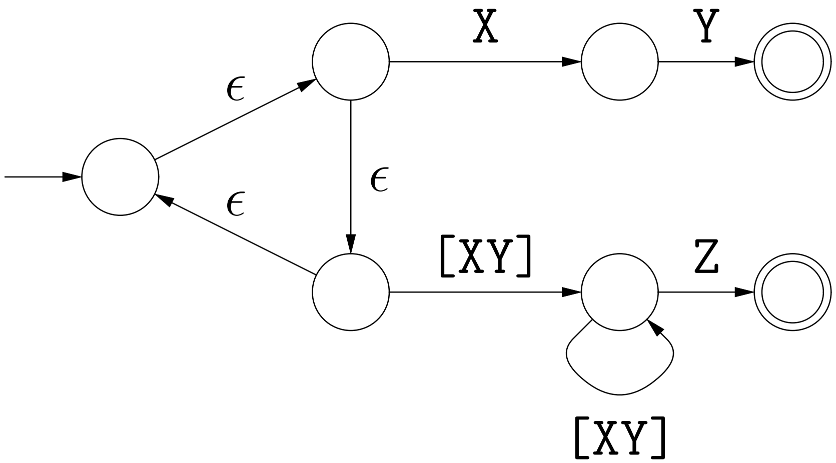

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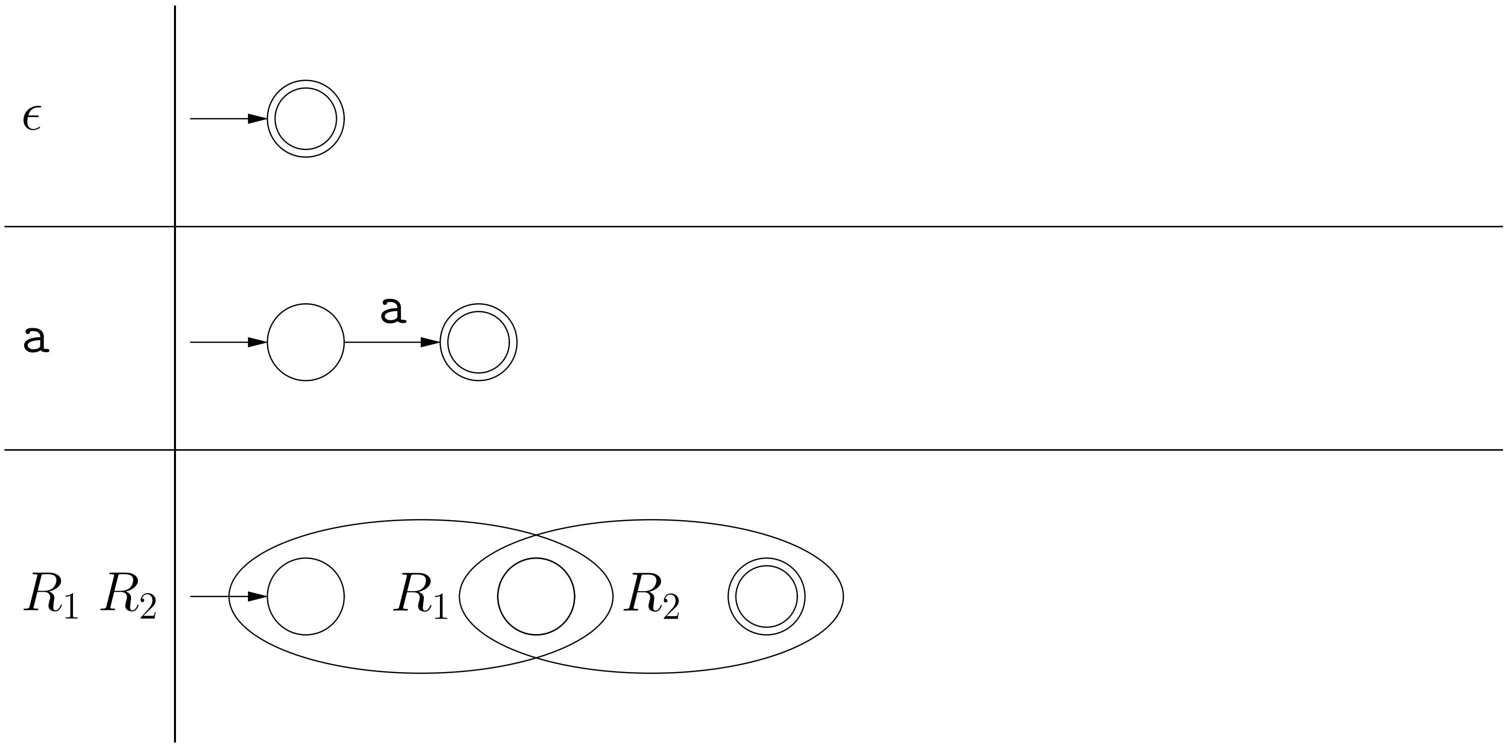
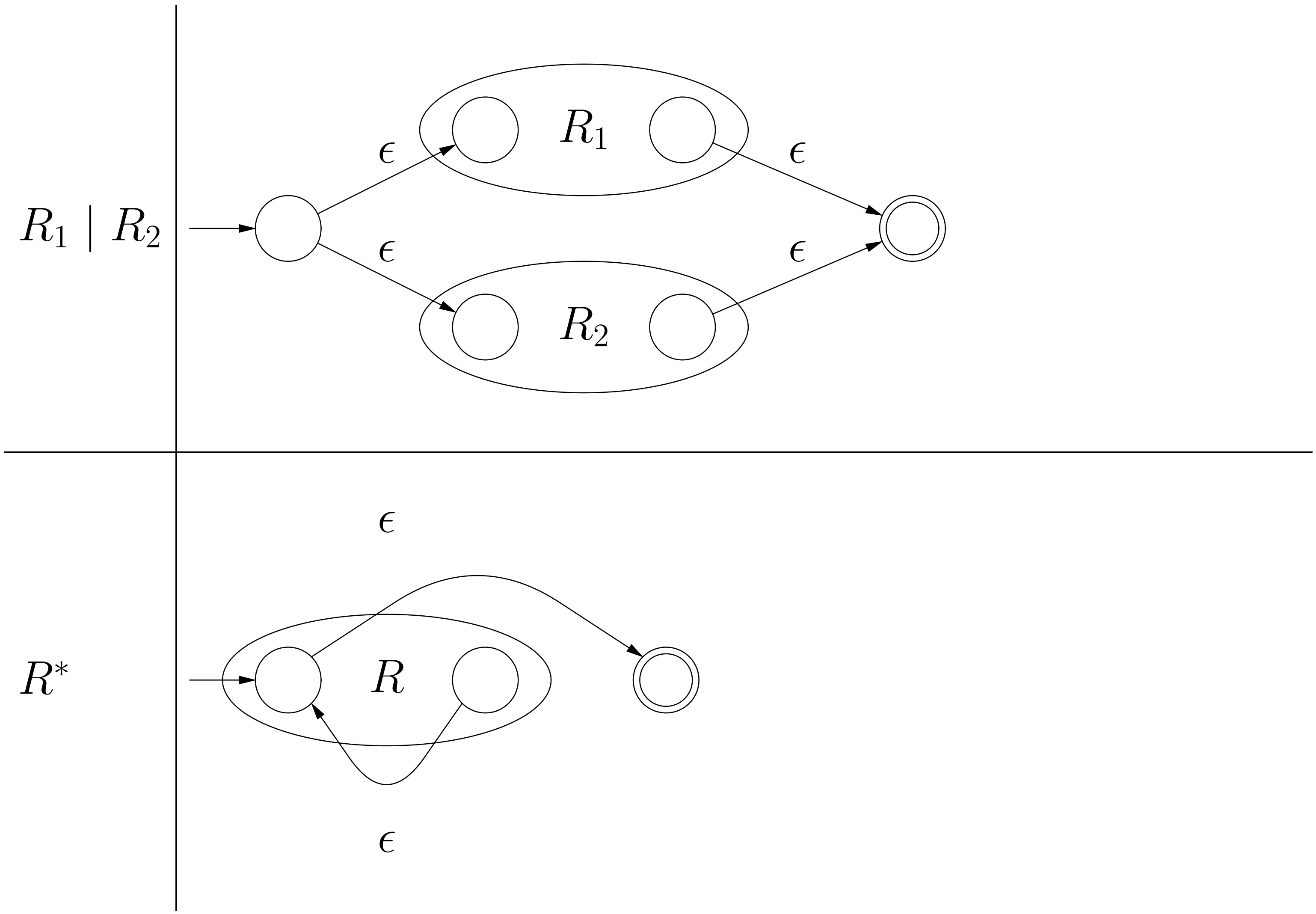
Review: Classical Regular Expressions to NFAs (I)



Review: Classical Regular Expressions to NFAs (II)



Extensions?

- How would you translate ϕ (the empty language, containing no strings) into an FA?
- How could you translate 'R?' into an NFA?
- How could you translate 'R+' into an NFA?
- How could you translate '$R_1|R_2|\cdots|R_n$' into an NFA?

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How would you translate `((ab)*|c)*` into an NFA (using the construction above)?

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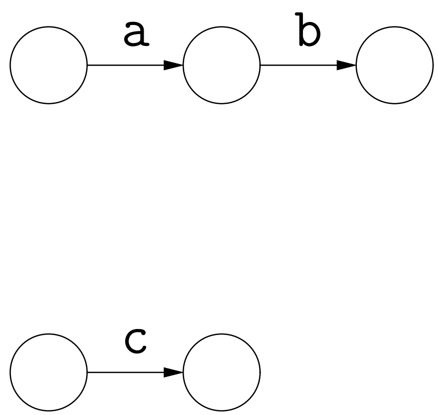

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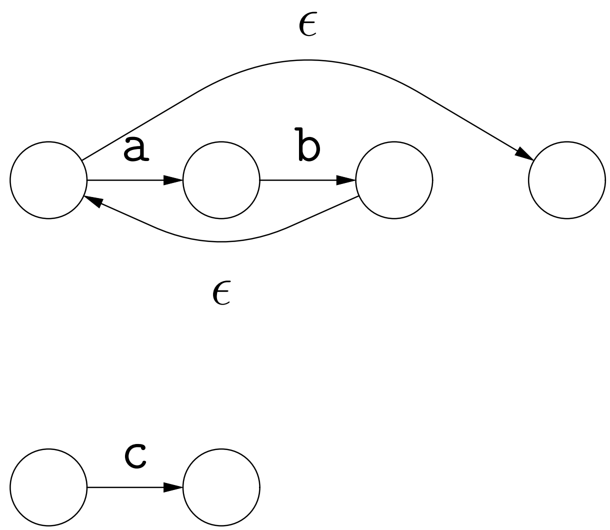

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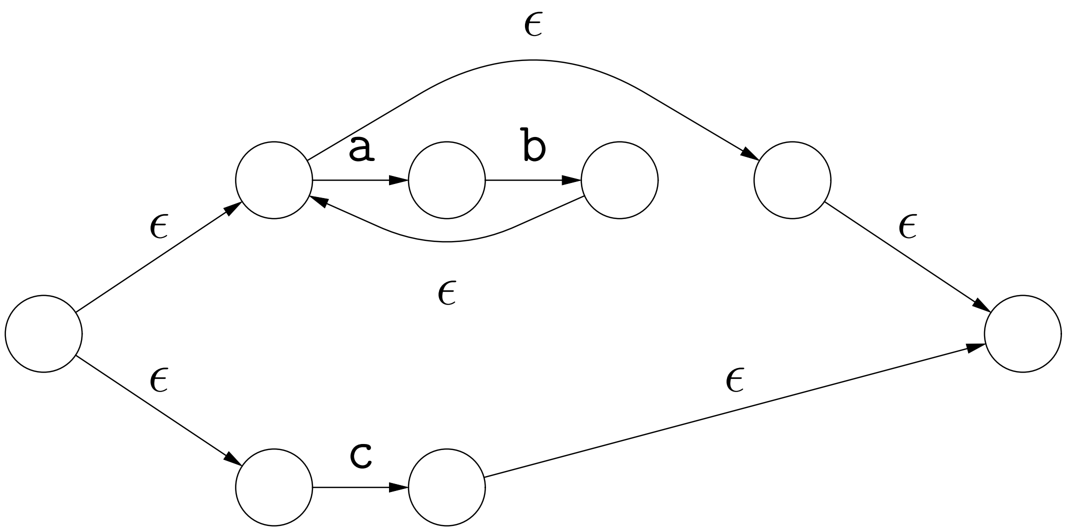

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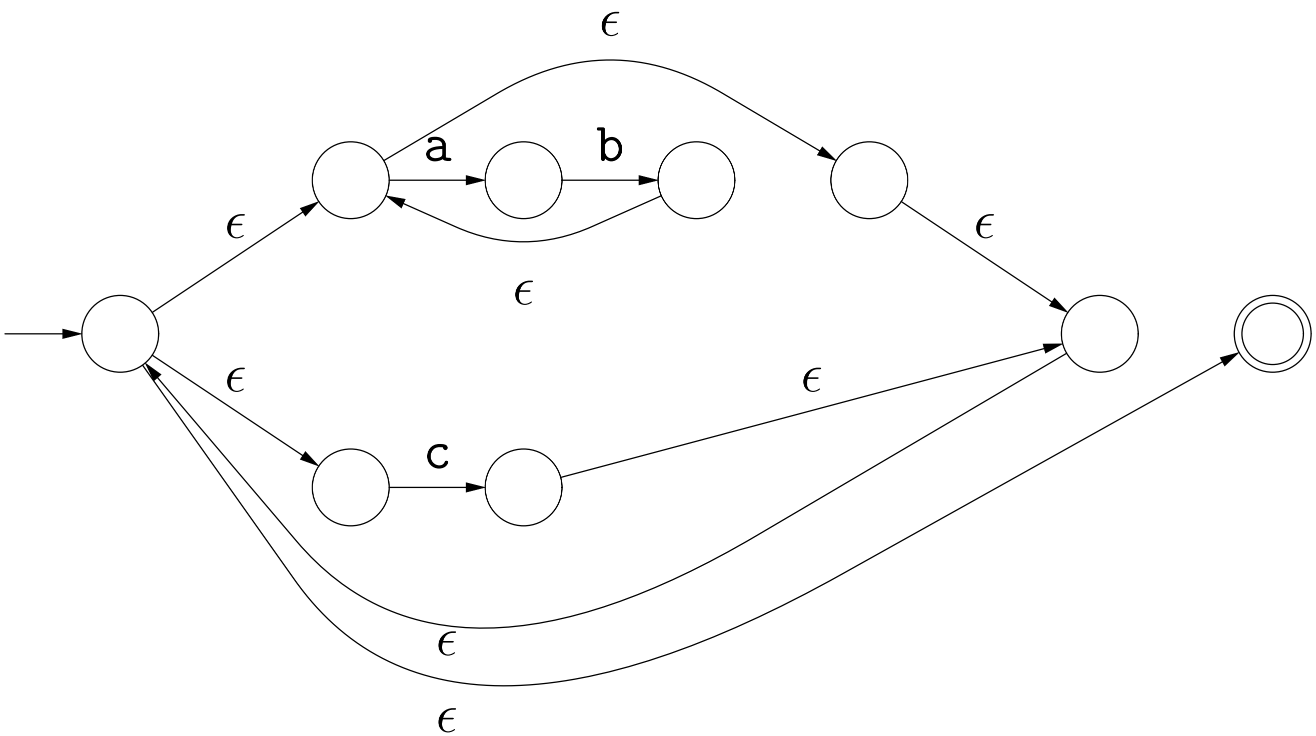
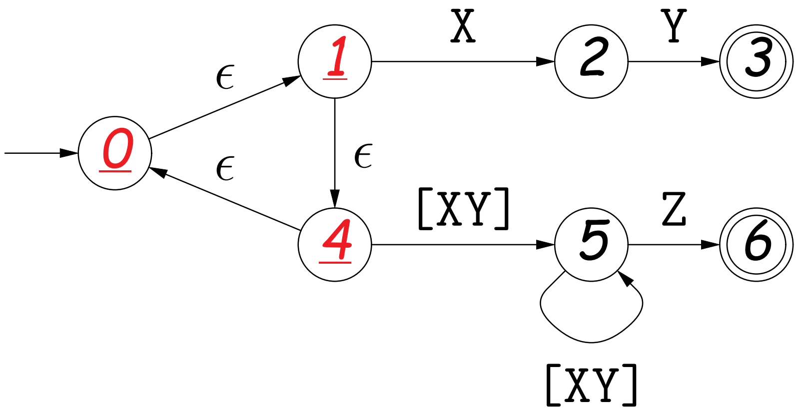
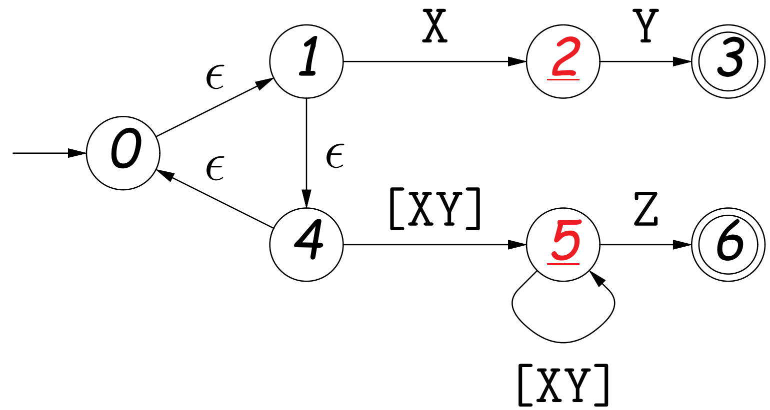
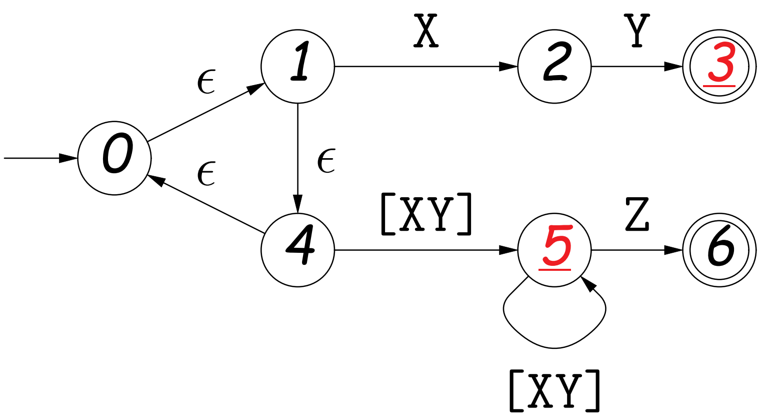
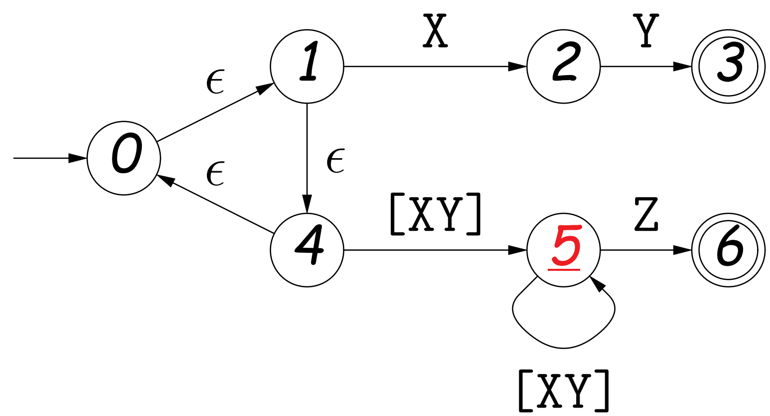


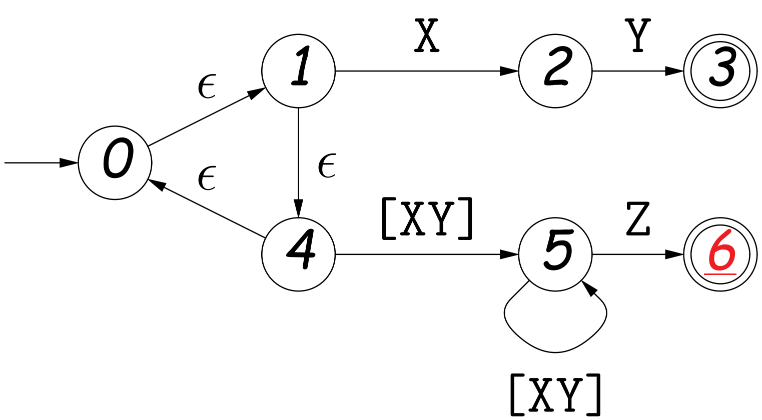
Abstract Implementation of NFAs







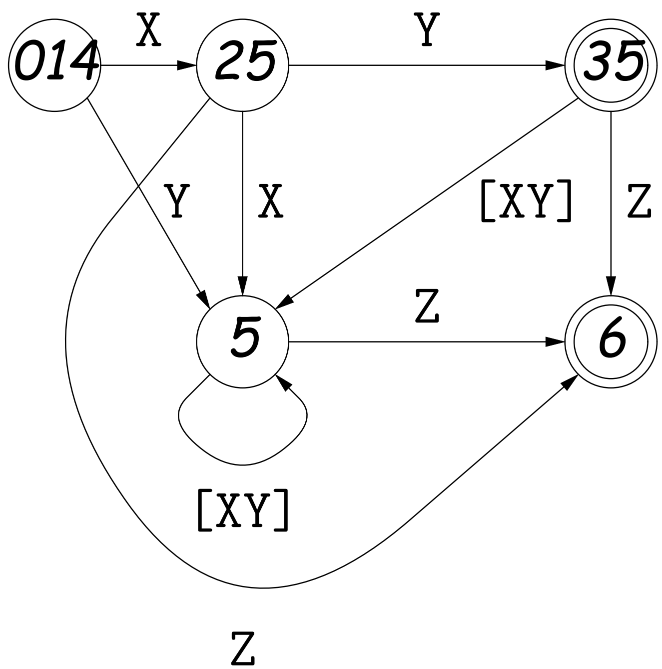




String: XYYZ

Review: Converting to DFAs

- **OBSERVATION:** The set of states that are marked (colored red) changes with each character in a way that depends only on the set and the character.
- In other words, machine on previous slide acted like this DFA:



DFAs as Programs

- Can realize DFA in program with control structure:

```
state = INITIAL;
for (s = input; *s != '\0'; s += 1) {
   switch (state):
   case INITIAL:
      if (*s == 'a') state = A_STATE; break;
   case A_STATE:
      if (*s == 'b') state = B_STATE; else state = INITIAL; break;
   ...
   }
}
return state == FINAL1 || state == FINAL2;
```

- Or with data structure (table driven):

```
state = INITIAL;
for (s = input; *s != '\0'; s += 1)
   state = transition[state][s];
return isfinal[state];
```

What Flex Does

- Flex program specification is giant regular expression of the form $R_1|R_2|\cdots|R_n$, where none of the R_i match ϵ.
- Each final state labeled with some action.
- Converted, by previous methods, into a table-driven DFA.
- But, this particular DFA is used to recognize *prefixes* of the (remaining) input: initial portions that put machine in a final state.
- Which final state(s) we end up in determine action. To deal with multiple actions:
  - Match *longest* prefix ("maximum munch").
  - If there are multiple matches, apply *first* rule in order.

How Do They Do It?

- How can we use a DFA to recognize longest match?
- How can we use DFA to act on first of equal-length matches?
- How can we use a DFA to handle the R_1/R_2 pattern (matches just R_1 but only if followed by R_2, like R_1(?=R_2) in Python)?