Global Optimization

Lecture 27

(From notes by R. Bodik & G. Necula)

Administrative

- HW #6 was posted online this evening.
- Version #2 of project 3 files: representation now complete, but still a bit of work for me to do.

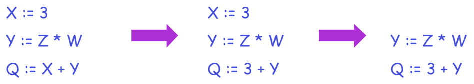
Lecture Outline

- Global flow analysis
- Global constant propagation
- Liveness analysis

Local Optimization

Simple basic-block optimizations…

- Constant propagation
- Dead code elimination



```
X := 3
Y := Z * W
Q := X + Y
```

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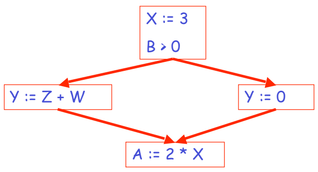
Global Optimization

… extend to entire control-flow graphs



Global Optimization

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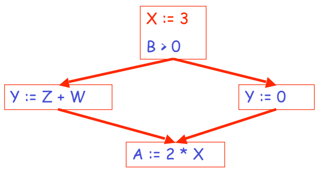
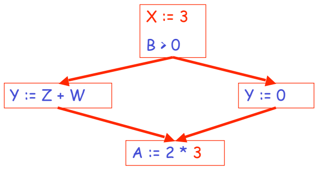


Global Optimization

… extend to entire control-flow graphs



X := 3
B > 0

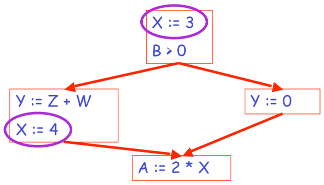
Y := Z + W

Y := 0

A := 2 * 3

Correctness

- How do we know it is OK to globally propagate constants?
- There are situations where it is incorrect:



X := 3
B > 0

Y := Z + W
X := 4

Y := 0

A := 2 * X

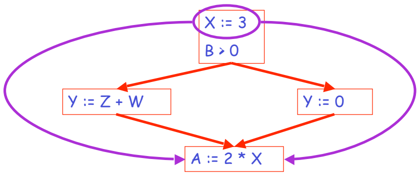
Correctness (Cont.)

To replace a use of x by a constant k we must know that:

Constant Replacement Condition (CR):

On every path to the use of x, the last assignment to x is x := k

Example 1 Revisited



X := 3
B > 0

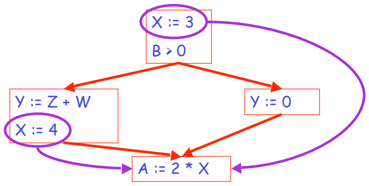
Y := Z + W

Y := 0

A := 2 * X

so replacing X by 3 is OK

Example 2 Revisited



X := 3
B > 0

Y := Z + W
X := 4

Y := 0

A := 2 * X

so replacing X by 3 is not OK

Discussion

- The correctness condition is not trivial to check
- "All paths" includes paths around loops and through branches of conditionals
- Checking the condition requires global analysis
  - An analysis of the entire control-flow graph for one method body

Global Analysis

Global optimization tasks share several traits:

- The optimization depends on knowing a property P at a particular point in program execution
- Proving P at any point requires knowledge of the entire method body
- Property P is typically undecidable !

Undecidability of Program Properties

- Rice's theorem: Most interesting dynamic properties of a program are undecidable:
  - Does the program halt on all (some) inputs?
    - This is called the halting problem
  - Is the result of a function F always positive?
    - Assume we can answer this question precisely
    - Take function H and find out if it halts by testing function F(x) { H(x); return 1; } whether it has positive result
- Syntactic properties are decidable !
  - E.g., How many occurrences of "x" are there?
- Theorem does not apply in absence of loops

Conservative Program Analyses

- So, we cannot tell for sure that "x" is always 3
  - Then, how can we apply constant propagation?
- It is OK to be *conservative*. If the optimization requires P to be true, then want to know either
  - P is definitely true
  - Don't know if P is true or false
- It is always correct to say "don't know"
  - We try to say don't know as rarely as possible
- All program analyses are conservative

Global Analysis (Cont.)

- *Global dataflow analysis* is a standard technique for solving problems with these characteristics

- Global constant propagation is one example of an optimization that requires global dataflow analysis

Global Constant Propagation

- Global constant propagation can be performed at any point where CR condition holds

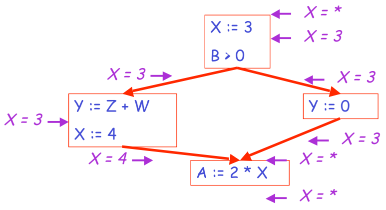
- Consider the case of computing CR condition for a single variable X at all program points

Global Constant Propagation (Cont.)

- To make the problem precise, we associate one of the following values with X at every program point

value	*interpretation*
#	No value has reached here (yet)
c	X = constant c
*	Don't know if X is a constant

Example



Using the Information

- Given global constant information, it is easy to perform the optimization
  - Simply inspect the x = _ associated with a statement using x
  - If x is constant at that point replace that use of x by the constant
- But how do we compute the properties x = _

The Idea

The analysis of a complicated program can be expressed as a combination of simple rules relating the change in information between adjacent statements

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Explanation

- The idea is to "push" or "transfer" information from one statement to the next
- For each statement s, we compute information about the value of x immediately before and after s

$C_{in}(x,s)$ = value of x before s

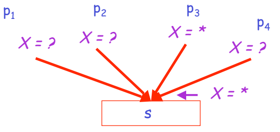
$C_{out}(x,s)$ = value of x after s

(we care about values #, *, k)

Transfer Functions

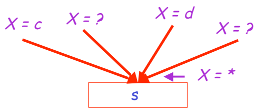
- Define a *transfer function* that transfers information from one statement to another
- In the following rules, let statement s have immediate predecessor statements $p_1,\ldots,p_n$

Rule 1



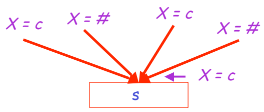
if $C_{out}(x, p_i) = *$ for some i, then $C_{in}(x, s) = *$

Rule 2



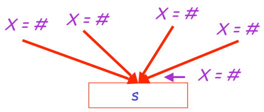
If $C_{out}(x, p_i) = c$ and $C_{out}(x, p_j) = d$ and $d \neq c$
then $C_{in}(x, s) = *$

Rule 3



if $C_{out}(x, p_i) = c$ for at least one i and is c or $\#$ for all i, then $C_{in}(x, s) = c$

Rule 4

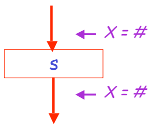


if $C_{out}(x, p_i) = \#$ for all i,
then $C_{in}(x, s) = \#$

The Other Half

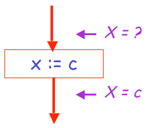
- Rules 1-4 relate the *out* of one statement to the *in* of the successor statement
  - they propagate information *forward* across CFG edges
- Now we need rules relating the *in* of a statement to the *out* of the same statement
  - to propagate information across statements

Rule 5



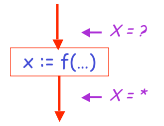
$C_{out}(x, s) = \#$ if $C_{in}(x, s) = \#$

Rule 6



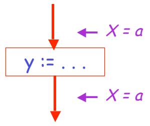
$C_{out}(x, x := c) = c$ if c is a constant

Rule 7



$C_{out}(x, x := f(\ldots)) = *$

Rule 8



$C_{out}(x, y := \ldots) = C_{in}(x, y := \ldots)$ if $x \neq y$

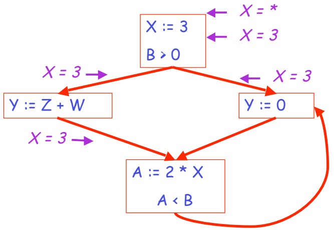
An Algorithm

1. For every entry s to the program, set $C_{in}(x, s) = *$
2. Set $C_{in}(x, s) = C_{out}(x, s) = \#$ everywhere else
3. Repeat until all points satisfy 1-8:

   Pick s not satisfying 1-8 and update using the appropriate rule

The Value

- To understand why we need #, look at a loop



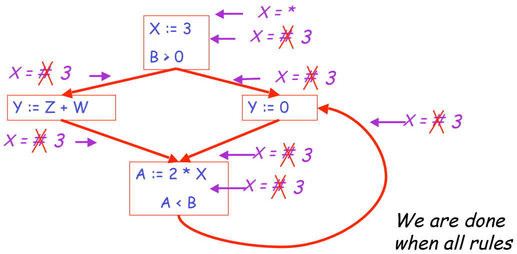
The Value # (Cont.)

- Consider the statement Y := 0
- To compute whether X is constant at this point, we need to know whether X is constant at the two predecessors
  - X := 3
  - A := 2 * X
- But info for A := 2 * X depends on its predecessors, including Y := 0!

The Value # (Cont.)

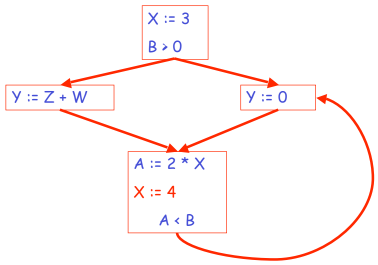
- Because of cycles, all points must have values at all times
- Intuitively, assigning some initial value allows the analysis to break cycles
- The initial value # means "So far as we know, control never reaches this point"

Example

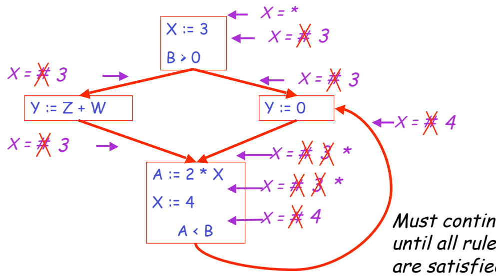


We are done when all rules are satisfied !

Another Example



Another Example



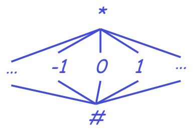
Must continue until all rules are satisfied !

Orderings

- We can simplify the presentation of the analysis by ordering the values

  # < c < *

- Drawing a picture with "smaller" values drawn lower, we get



a *lattice*

Orderings (Cont.)

- * is the largest value, # is the least
  - All constants are in between and incomparable
- Let *lub* be the least-upper bound in this ordering
- Rules 1-4 can be written using lub:

  $C_{in}(x, s) = \text{lub} \{ C_{out}(x, p) \mid p \text{ is a predecessor of } s \}$

Termination

- Simply saying "repeat until nothing changes" doesn't guarantee that eventually nothing changes
- The use of lub explains why the algorithm terminates
  - Values start as # and only *increase*
  - # can change to a constant, and a constant to *
  - Thus, C_(x, s) can change at most twice

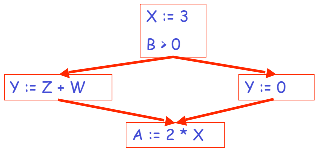
Termination (Cont.)

Thus the algorithm is linear in program size

Number of steps
= Number of C_(….) values computed * 2
= Number of program statements * 4

Liveness Analysis

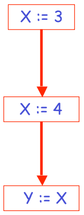
Once constants have been globally propagated, we would like to eliminate dead code



After constant propagation, X := 3 is dead (assuming this is the entire CFG)

Live and Dead

- The first value of x is *dead* (never used)
- The second value of x is *live* (may be used)



Liveness

A variable x is *live at statement s* if
  - There exists a statement s' that uses x
  - There is a path from s to s'
  - That path has no intervening assignment to x

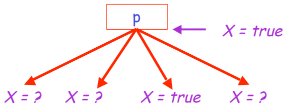
Global Dead Code Elimination

- A statement x := … is dead code if x is dead after the assignment
- Dead statements can be deleted from the program
- But we need liveness information first . . .

Computing Liveness

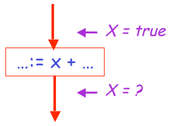
- We can express liveness as a function of information transferred between adjacent statements, just as in copy propagation
- Liveness is simpler than constant propagation, since it is a boolean property (true or false)

Liveness Rule 1



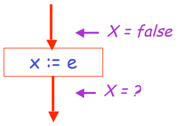
$L_{out}(x, p) = \vee \{ L_{in}(x, s) \mid s \text{ a successor of } p \}$

Liveness Rule 2



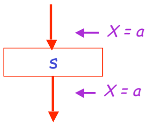
$L_{in}(x, s) = true$ if s refers to x on the rhs

Liveness Rule 3



$L_{in}(x, x := e) = false$ if e does not refer to x

Liveness Rule 4

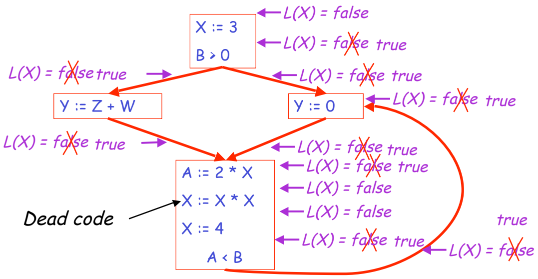


$L_{in}(x, s) = L_{out}(x, s)$ if s does not refer to x

Algorithm

1. Let all L_(…) = false initially
2. Repeat until all statements s satisfy rules 1-4
   Pick s where one of 1-4 does not hold and update using the appropriate rule

Another Example



Termination

- A value can change from false to true, but not the other way around
- Each value can change only once, so termination is guaranteed
- Once the analysis is computed, it is simple to eliminate dead code

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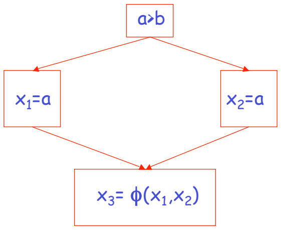
SSA and Global Analysis

- For local optimizations, the single static assignment (SSA) form was useful.
- But how can it work with a full CFG?
  - E.g., how do we avoid two assignments to the temporary holding x after this conditional?

```
if a>b:
    x = a
else:
    x = b
# where is x at this point?
```

A Small Kludge: ϕ "functions"

- For the preceding example, we get a CFG like this:



a>b

x_1=a

x_2=a

x_3= $\phi(x_1,x_2)$

ϕ "functions"

- An artificial device to allow SSA notation in CFGs.
- In a basic block, each variable is associated with *one* definition,
- ϕ-functions in effect associate each variable with a *set* of possible definitions.
- In general, one tries to introduce them in strategic places so as to minimize total number.
- Although this device increases number of assignments in IL, register allocation can remove many by assigning related IL registers to the same real register.

Common Subexpression Elimination (CSE)

- Easy to tell (conservatively) if two IL assignments compute the same value: just see if they have the same right-hand side.
- Thanks to SSA, same variables indicate same values.

Forward vs. Backward Analysis

We've seen two kinds of analysis:

Constant propagation is a *forwards* analysis: information is pushed from inputs to outputs

Liveness is a *backwards* analysis: information is pushed from outputs back towards inputs

Analysis

- There are many other global flow analyses
- Most can be classified as either forward or backward
- Most also follow the methodology of local rules relating information between adjacent program points