Bottom-Up Parsing

Lecture 8

(From slides by G. Necula & R. Bodik)

Administrivia

- **Test I** during class on 10 March.
- Notes updated (at last)

Bottom-Up Parsing

- We've been looking at general context-free parsing.
- It comes at a price, measured in overheads, so in practice, we design programming languages to be parsed by less general but faster means, like top-down recursive descent.
- Deterministic bottom-up parsing is more general than top-down parsing, and just as efficient.
- Most common form is *LR parsing*
  - L means that tokens are read left to right
  - R means that it constructs a rightmost derivation

An Introductory Example

- LR parsers don't need left-factored grammars and can also handle left-recursive grammars
- Consider the following grammar:

  E → E + (E) | int

  - Why is this not LL(1)?
- Consider the string: int + (int) + (int)

The Idea

- LR parsing *reduces* a string to the start symbol by inverting productions:

sent ← input string of terminals

while sent ≠ S:

  - Identify first β in sent such that $A \to \beta$ is a production and $S \to^* \alpha\ A\ \gamma \to \alpha\ \beta\ \gamma$ = sent
  - Replace β by A in sent (so $\alpha\ A\ \gamma$ becomes new sent)
- Such $\alpha\ \beta$'s are called *handles*

A Bottom-up Parse in Detail (1)

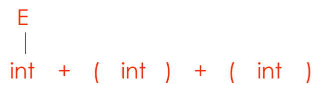
int + (int) + (int)

int + (int) + (int)

A Bottom-up Parse in Detail (2)

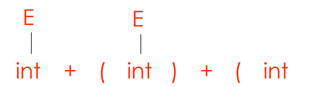
int + (int) + (int)
E + (int) + (int)

(handles in red)



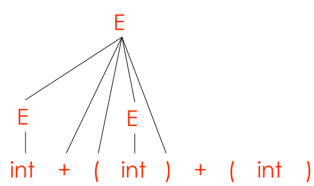
A Bottom-up Parse in Detail (3)

int + (int) + (int)
E + (int) + (int)
E + (E) + (int)



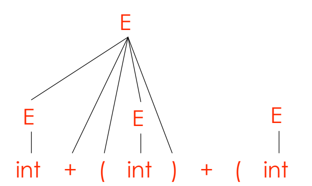
A Bottom-up Parse in Detail (4)

int + (int) + (int)
E + (int) + (int)
E + (E) + (int)
E + (int)



A Bottom-up Parse in Detail (5)

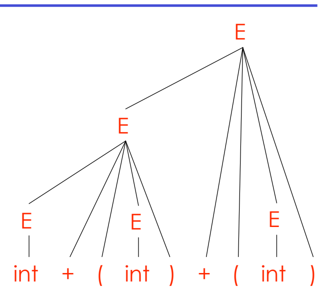
int + (int) + (int)
E + (int) + (int)
E + (E) + (int)
E + (int)
E + (E)



A Bottom-up Parse in Detail (6)

int + (int) + (int)
E + (int) + (int)
E + (E) + (int)
E + (int)
E + (E)
E

A reverse rightmost derivation



Where Do Reductions Happen

Because an LR parser produces a reverse rightmost derivation:
- If $\alpha\beta\gamma$ is step of a bottom-up parse with handle $\alpha\beta$
- And the next reduction is by $A \rightarrow \beta$
- Then γ is a string of terminals !

… Because $\alpha A\gamma \rightarrow \alpha\beta\gamma$ is a step in a right-most derivation

Intuition: We make decisions about what reduction to use *after* seeing all symbols in handle, rather than before (as for LL(1))

Notation

- Idea: Split the string into two substrings
  - Right substring (a string of terminals) is as yet unexamined by parser
  - Left substring has terminals and non-terminals
- The dividing point is marked by a |
  - The | is not part of the string
  - Marks end of next potential handle
- Initially, all input is unexamined: $| x_1 x_2 \ldots x_n$

Shift-Reduce Parsing

- Bottom-up parsing uses only two kinds of actions:

  Shift: Move | one place to the right, shifting a terminal to the left string

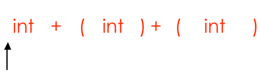
  E + (| int) ⇒ E + (int |)

  Reduce: Apply an inverse production at the handle. If E → E + (E) is a production, then

  E + (E + (E) |) ⇒ E +(E |)

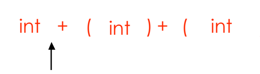
Shift-Reduce Example

| int + (int) + (int)$ shift



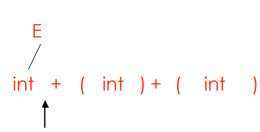
Shift-Reduce Example

| int + (int) + (int)$ shift
int | + (int) + (int)$ red. E → int



Shift-Reduce Example

| int + (int) + (int)$ shift
int | + (int) + (int)$ red. E → int
E | + (int) + (int)$ shift 3 times



Shift-Reduce Example

| int + (int) + (int)$ shift
int | + (int) + (int)$ red. E → int
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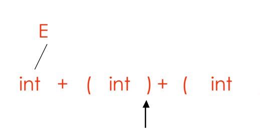
E + (int |) + (int)$ red. E → int



Shift-Reduce Example

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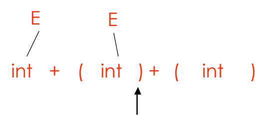
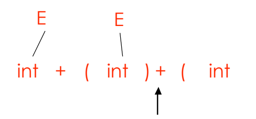
E + (int |) + (int)$ red. E → int
E + (E |) + (int)$ shift



Shift-Reduce Example

| int + (int) + (int)$ shift
int | + (int) + (int)$ red. E → int
E | + (int) + (int)$ shift 3 times
E + (int |) + (int)$ red. E → int
E + (E |) + (int)$ shift
E + (E) | + (int)$ red. E → E + (E)



Shift-Reduce Example

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int | + (int) + (int)$ red. E → int
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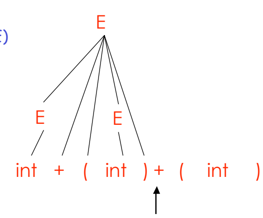
E + (int |) + (int)$ red. E → int
E + (E |) + (int)$ shift
E + (E) | + (int)$ red. E → E + (E)
E | + (int)$ shift 3 times



Shift-Reduce Example

| int + (int) + (int)$ shift
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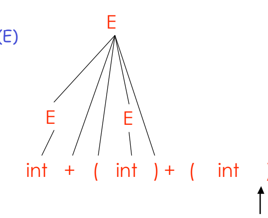
E + (E |) + (int)$ shift
E + (E) | + (int)$ red. E → E + (E)
E | + (int)$ shift 3 times
E + (int |)$ red. E → int



Shift-Reduce Example

| int + (int) + (int)$ shift
int | + (int) + (int)$ red. E → int
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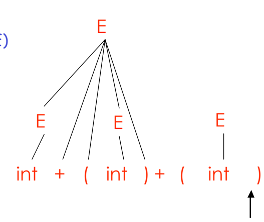
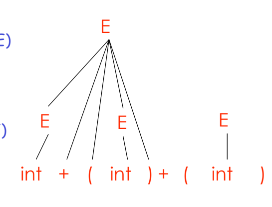
E + (int |) + (int)$ red. E → int
E + (E |) + (int)$ shift
E + (E) | + (int)$ red. E → E + (E)
E | + (int)$ shift 3 times
E + (int |)$ red. E → int
E + (E |)$ shift



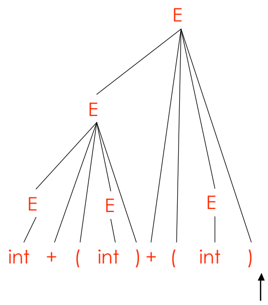
Shift-Reduce Example

| int + (int) + (int)$ shift
int | + (int) + (int)$ red. E → int
E | + (int) + (int)$ shift 3 times
E + (int |) + (int)$ red. E → int
E + (E |) + (int)$ shift
E + (E) | + (int)$ red. E → E + (E)
E | + (int)$ shift 3 times
E + (int |)$ red. E → int
E + (E |)$ shift
E + (E) | $ red. E → E + (E)



Shift-Reduce Example

ı int + (int) + (int)$	shift
int ı + (int) + (int)$	red. E → int
E ı + (int) + (int)$	shift 3 times
E + (int ı) + (int)$	red. E → int
E + (E ı) + (int)$	shift
E + (E) ı + (int)$	red. E → E + (E)
E ı + (int)$	shift 3 times
E + (int ı)$	red. E → int
E + (E ı)$	shift
E + (E) ı $	red. E → E + (E)
E ı $	accept



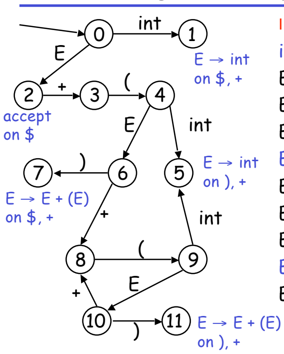
The Stack

- Left string can be implemented as a stack
  - Top of the stack is the ı
- Shift pushes a terminal on the stack
- Reduce pops 0 or more symbols from the stack (production rhs) and pushes a non-terminal on the stack (production lhs)

Key Issue: When to Shift or Reduce?

- Decide based on the left string (the stack)
- Idea: use a finite automaton (DFA) to decide when to shift or reduce
  - The DFA input is the stack up to potential handle
  - DFA alphabet consists of terminals and nonterminals
  - DFA *recognizes complete handles*
- We run the DFA on the stack and we examine the resulting state X and the token tok after ı
  - If X has a transition labeled tok then *shift*
  - If X is labeled with "A → β on tok" then *reduce*

LR(1) Parsing. An Example



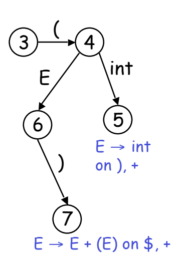
ı int + (int) + (int)$	shift
int ı + (int) + (int)$	E → int
E ı + (int) + (int)$	shift(x3)
E + (int ı) + (int)$	E → int
E + (E ı) + (int)$	shift
E + (E) ı + (int)$	E → E+(E)
E ı + (int)$	shift (x3)
E + (int ı)$	E → int
E + (E ı)$	shift
E + (E) ı $	E → E+(E)
E ı $	accept

Representing the DFA

- Parsers represent the DFA as a 2D table
  - As for table-driven lexical analysis
- Lines correspond to DFA states
- Columns correspond to terminals and non-terminals
- In classical treatments, columns are split into:
  - Those for terminals: action table
  - Those for non-terminals: goto table

Representing the DFA. Example

- The table for a fragment of our DFA:



	int	+	(	)	$	E
…						
3			s4			
4	s5					s6
5		$r_{E \to int}$		$r_{E \to int}$		
6				s7		
7		$r_{E \to E+(E)}$			$r_{E \to E+(E)}$	
…						

The LR Parsing Algorithm

- After a shift or reduce action we rerun the DFA on the entire stack
  - This is wasteful, since most of the work is repeated
- So record, for each stack element, state of the DFA after that state
- LR parser maintains a stack

  $\langle$ sym_1, $state_1$ $\rangle$ . . . $\langle$ sym_n, $state_n$ $\rangle$

  $state_k$ is the final state of the DFA on $sym_1 \ldots sym_k$

The LR Parsing Algorithm

```
Let I = w1w2…wn$ be initial input
Let j = 1
Let DFA state 0 be the start state
Let stack = ⟨ dummy, 0 ⟩
   repeat
        case table[top_state(stack), I[j]] of
                shift k:  push ⟨ I[j], k ⟩; j += 1
                reduce X → α:
                    pop |α| pairs,
                    push ⟨X, table[top_state(stack), X]⟩
                accept: halt normally
                error: halt and report error
```


Parsing Contexts

- Consider the state describing the situation at the I in the stack E + (I int) + (int)
- Context:
  - We are looking for an E → E + (• E)
    - Have have seen E + (from the right-hand side
  - We are also looking for E → • int *or* E → • E + (E)
    - Have seen nothing from the right-hand side
- One DFA state describes a set of such contexts
- (Traditionally, use • to show where the I is.)

LR(1) Items

- An *LR(1) item* is a pair:

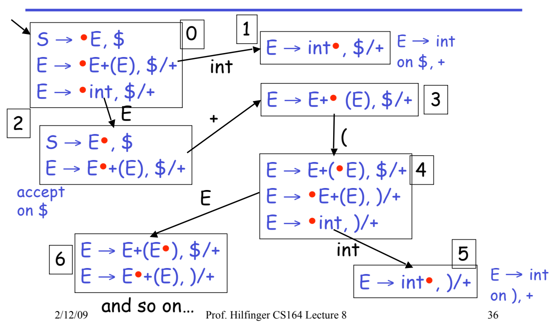
  $X \rightarrow \alpha \bullet \beta$, a
  - $X \rightarrow \alpha\beta$ is a production
  - a is a terminal (the lookahead terminal)
  - LR(1) means 1 lookahead terminal
- [$X \rightarrow \alpha \bullet \beta$, a] describes a context of the parser
  - We are trying to find an X followed by an a, and
  - We have α already on top of the stack
  - Thus we need to see next a prefix derived from βa

Convention

- We add to our grammar a fresh new start symbol S and a production S → E
  - Where E is the old start symbol
  - No need to do this if E had only one production
- The initial parsing context contains:

  S → • E, $
  - Trying to find an S as a string derived from E$
  - The stack is empty

Constructing the Parsing DFA. Example.



LR Parsing Tables. Notes

- Parsing tables (i.e. the DFA) can be constructed automatically for a CFG
- But we still need to understand the construction to work with parser generators
  - E.g., they report errors in terms of sets of items
- What kind of errors can we expect?

Shift/Reduce Conflicts

- If a DFA state contains both
  [X → α•aβ, b] and [Y → γ•, a]
- Then on input "a" we could either
  - Shift into state [X → αa•β, b], or
  - Reduce with Y → γ
- This is called a *shift-reduce conflict*

Shift/Reduce Conflicts

- Typically due to ambiguities in the grammar
- Classic example: the dangling else
  S → if E then S | if E then S else S | OTHER
- Will have DFA state containing
  [S → if E then S•, else]
  [S → if E then S• else S, $]
- If else follows then we can shift or reduce

More Shift/Reduce Conflicts

- Consider the ambiguous grammar
  E → E + E | E * E | int
- We will have the states containing
  [E → E * • E, +] [E → E * E•, +]
  [E → • E + E, +] ⇒^{E} [E → E• + E, +]
  … …
- Again we have a shift/reduce on input +
  - We need to reduce (* binds more tightly than +)
  - Solution: declare the precedence of * and +

More Shift/Reduce Conflicts

- In bison declare precedence and associativity of *terminal symbols*:

```
%left +
%left *
```

- Precedence *of a rule* = that of its last terminal
  - See bison manual for ways to override this default
- Resolve shift/reduce conflict with a *shift* if:
  - input terminal has higher precedence than the rule
  - the precedences are the same and right associative

Using Precedence to Solve S/R Conflicts

- Back to our example:
  [E → E * • E, +] [E →E * E•, +]
  [E → • E + E, +] ⇒^{E} [E →E • + E, +]
  … …
- Will choose reduce because precedence of rule E → E * E is higher than of terminal +

Using Precedence to Solve S/R Conflicts

- Same grammar as before
  E → E + E | E * E | int
- We will also have the states
  [E → E + • E, +]   [E → E + E•, +]
  [E → • E + E, +]   ⇒^{E} [E → E • + E, +]
  …   …
- Now we also have a shift/reduce on input +
  - We choose reduce because E → E + E and + have the same precedence and + is left-associative

Using Precedence to Solve S/R Conflicts

- Back to our dangling else example
  [S → if E then S•,   else]
  [S → if E then S• else S, x]
- Can eliminate conflict by declaring else with higher precedence than then
- However, best to avoid overuse of precedence declarations or you'll end with unexpected parse trees

Reduce/Reduce Conflicts

- If a DFA state contains both
  [X → α•, a] and [Y → β•, a]
  - Then on input "a" we don't know which production to reduce
- This is called a *reduce/reduce conflict*

Reduce/Reduce Conflicts

- Usually due to gross ambiguity in the grammar
- Example: a sequence of identifiers
  S → ε | id | id S
- There are two parse trees for the string id
  S → id
  S → id S → id
- How does this confuse the parser?

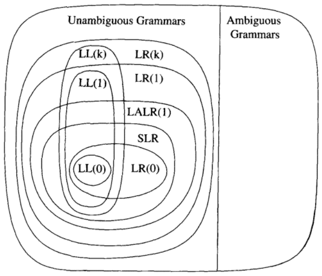
More on Reduce/Reduce Conflicts

- Consider the states   [S → id •, $]
  [S' → • S, $]   [S → id • S, $]
  [S → •, $]   ⇒^{id} [S → •, $]
  [S → • id, $]   [S → • id, $]
  [S → • id S, $]   [S → • id S, $]
- Reduce/reduce conflict on input $
  S' → S → id
  S' → S → id S → id
- Better rewrite the grammar: S → ε | id S

Relation to Bison

- Bison builds this kind of machine.
- However, for efficiency concerns, collapses many of the states together.
- Causes some additional conflicts, but not many.
- The machines discussed here are LR(1) engines. Bison's optimized versions are LALR(1) engines.

A Hierarchy of Grammar Classes



From Andrew Appel, "Modern Compiler Implementation in Java"

Notes on Parsing

- Parsing
  - A simple parser: LL(1), recursive descent
  - A more powerful parser: LR(1)
  - An efficiency hack: LALR(1)
  - We use LALR(1) parser generators
  - Earley's algorithm provides a complete algorithm for parsing context-free languages.