Lecture 12: Deterministic Bottom-Up Parsing

- (From slides by G. Necula & R. Bodik)

Avoiding nondeterministic choice: LR

- We've been looking at *general* context-free parsing.
- It comes at a price, measured in overheads, so in practice, we design programming languages to be parsed by less *general* but faster means, like top-down recursive descent.
- Deterministic bottom-up parsing is more *general* than top-down parsing, and just as efficient.
- Most common form is LR parsing
  - L means that tokens are read left to right
  - R means that it constructs a rightmost derivation

An Introductory Example

- LR parsers don't need left-factored grammars and can also handle left-recursive grammars
- Consider the following grammar:

  E : E + (E) | int

  (Why is this not LL(1)?)
- Consider the string: `int + ( int ) + ( int ) .`

The Idea

- LR parsing reduces a string to the start symbol by inverting productions. In the following, sent is a sentential form that starts as the input and is reduced to the start symbol, S:

```
sent = input string of terminals
while sent $\neq$ S:
    Identify first $\beta$ in sent such that $A : \beta$ is a production
        and $S \xRightarrow{*} \alpha A \gamma \Rightarrow \alpha\beta\gamma =$ sent.
    Replace $\beta$ by A in sent (so that $\alpha A \gamma$ becomes new sent).
```

- Such $\alpha\beta$'s are called *handles*.

A Bottom-up Parse in Detail (1)

Grammar:

E : E + (E) | int

`int + (int) + (int)`

int + (int) + (int)

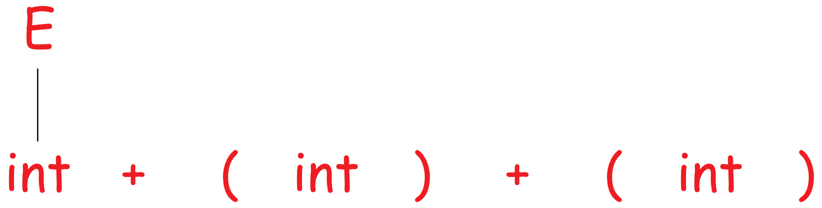
A Bottom-up Parse in Detail (2)

Grammar:

E : E + (E) | int

```
int + (int) + (int)
E + (int) + (int)
```

(handles in red)

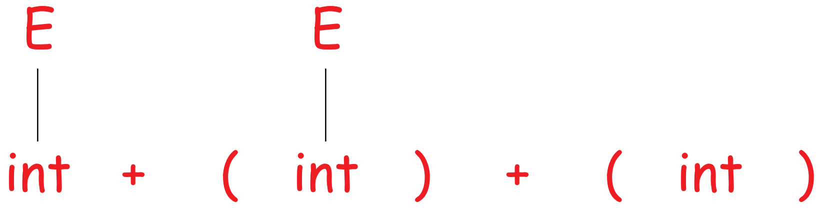


A Bottom-up Parse in Detail (3)

Grammar:

E : E + (E) | int

```
int + (int) + (int)
E + (int) + (int)
E + (E) + (int)
```

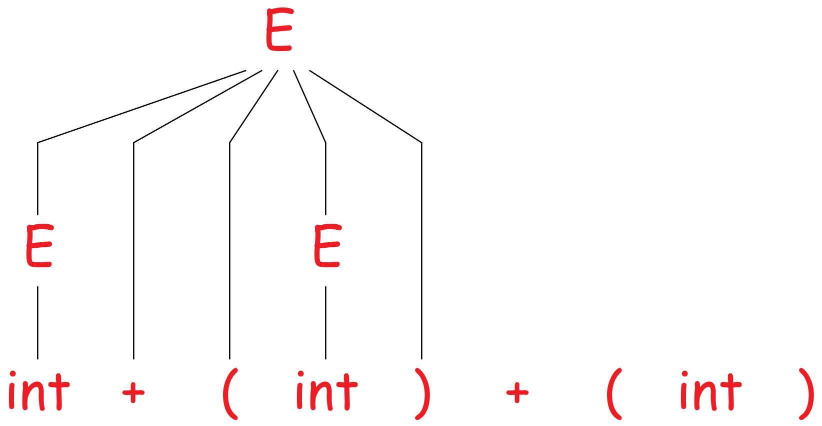


A Bottom-up Parse in Detail (4)

Grammar:

E : E + (E) | int

```
int + (int) + (int)
E + (int) + (int)
E + (E) + (int)
E + (int)
```

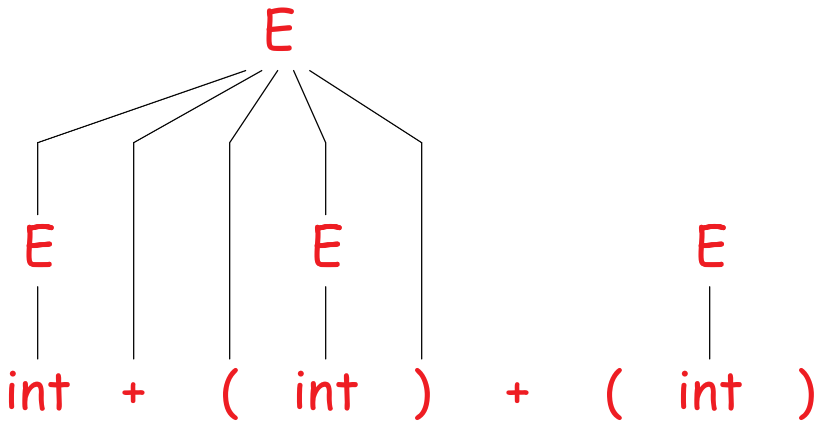


A Bottom-up Parse in Detail (5)

Grammar:

E : E + (E) | int

```
int + (int) + (int)
E + (int) + (int)
E + (E) + (int)
E + (int)
E + (E)
```



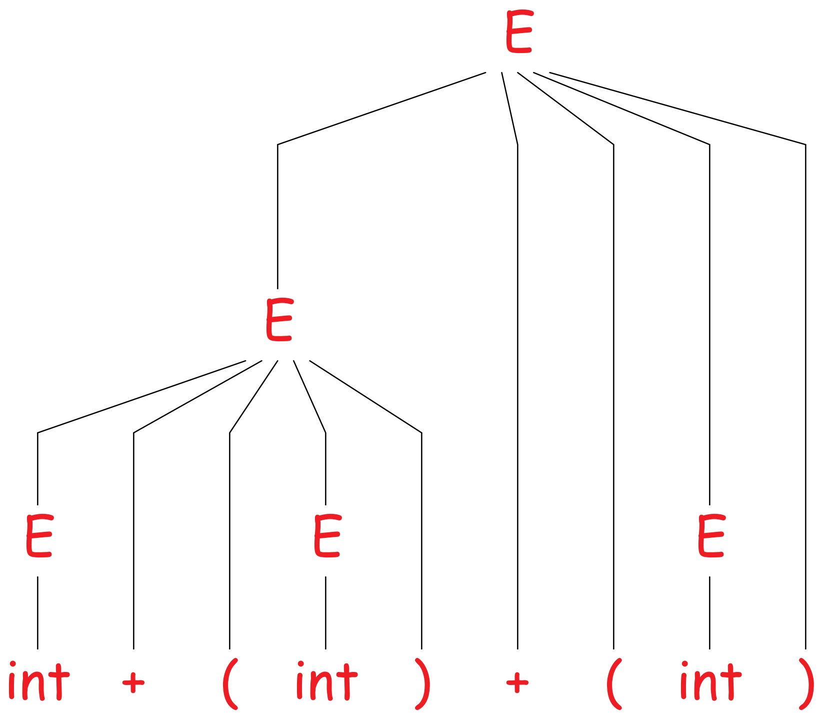
A Bottom-up Parse in Detail (6)

Grammar:

E : E + (E) | int

A reverse rightmost derivation:

```
int + (int) + (int)
E + (int) + (int)
E + (E) + (int)
E + (int)
E + (E)
E
```



Where Do Reductions Happen?

Because an LR parser produces a reverse rightmost derivation:

- If $\alpha\beta\gamma$ is one step of a bottom-up parse with handle $\alpha\beta$
- And the next reduction is by $A : \beta$,
- Then γ must be a string of terminals,
- Because $\alpha A\gamma \Rightarrow \alpha\beta\gamma$ is a step in a rightmost derivation

Intuition: We make decisions about what reduction to use after seeing *all* symbols in the handle, rather after seeing only the first (as for LL(1)).

Notation

- Idea: Split the input string into two substrings
  - Right substring (a string of terminals) is as-yet unprocessed by parser
  - Left substring has terminals and nonterminals
  - (In examples, we'll mark the dividing point with |.)
  - The dividing point marks the end of the next potential handle.
  - Initially, all input is unexamined: $|x_1 x_2 \cdots x_n$

Shift-Reduce Parsing

Bottom-up parsing uses only two kinds of actions:

- *Shift:* Move | one place to the right, shifting a terminal to the left string.
  - For example,

    $\mathrm{E} + (\mid \mathrm{int}) \longrightarrow \mathrm{E} + (\mathrm{int} \mid)$

- *Reduce:* Apply an inverse production at the handle.
  - For example, if E : E + (E) is a production, then we might reduce:

    $\mathrm{E} + (\underline{\mathrm{E} + (\mathrm{E})} \mid) \longrightarrow \mathrm{E} +(\underline{\mathrm{E}} \mid)$

Accepting a String

- The process ends when we reduce all the input to the start symbol.
- For technical convenience, however, we usually add a new start symbol and a hidden production to handle the end-of-file:

  $S' : S \dashv$

- Having done this, we can now stop parsing and accept the string whenever we reduce the entire input to

  $S \mid \dashv$

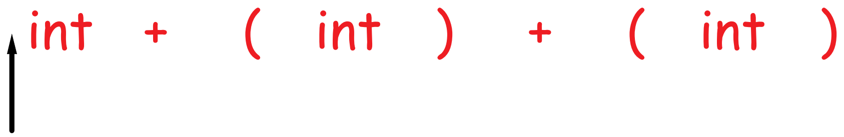
  without bothering to do the final shift and reduce.
- This will be the convention from now on.

Shift-Reduce Example (1)

Sent. Form	Actions
| int + (int) + (int) ⊣	shift

Grammar:

E : E + (E) | int

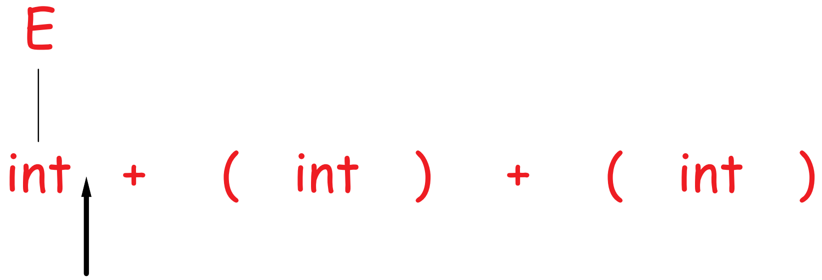


Shift-Reduce Example (2)

Sent. Form	Actions
| int + (int) + (int) ⊣	shift
int | + (int) + (int) ⊣	reduce by E: int

Grammar:

E : E + (E) | int

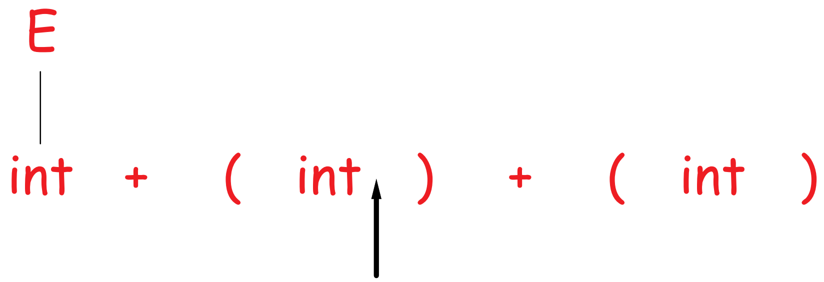


Shift-Reduce Example (3)

<table>
<tr><th>Sent. Form</th><th>Actions</th></tr>
<tr><td>| int + (int) + (int) ⊣</td><td>shift</td></tr>
<tr><td>int | + (int) + (int) ⊣</td><td>reduce by E: int</td></tr>
<tr><td>E | + (int) + (int) ⊣</td><td>shift 3 times</td></tr>
</table>

Grammar:

E : E + (E) | int

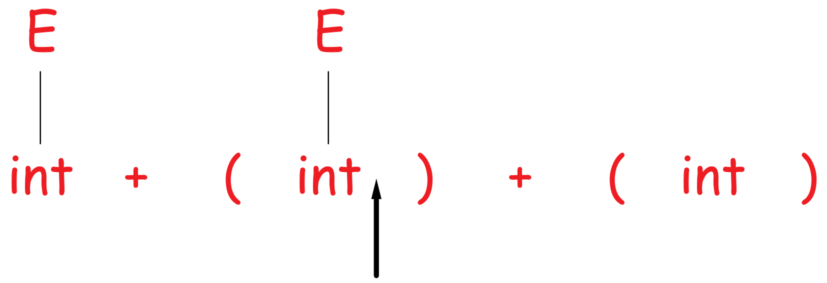


Shift-Reduce Example (4)

<table>
<tr><th>Sent. Form</th><th>Actions</th></tr>
<tr><td>| int + (int) + (int) ⊣</td><td>shift</td></tr>
<tr><td>int | + (int) + (int) ⊣</td><td>reduce by E: int</td></tr>
<tr><td>E | + (int) + (int) ⊣</td><td>shift 3 times</td></tr>
<tr><td>E + (int |) + (int) ⊣</td><td>reduce by E: int</td></tr>
</table>

Grammar:

E : E + (E) | int

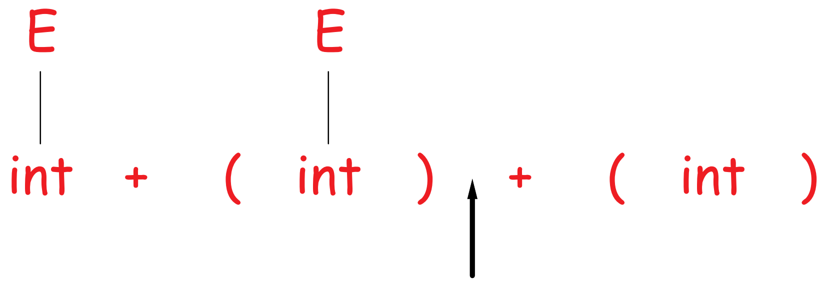


Shift-Reduce Example (5)

Sent. Form	Actions
| int + (int) + (int) ⊣	shift
int | + (int) + (int) ⊣	reduce by E: int
E | + (int) + (int) ⊣	shift 3 times
E + (int |) + (int) ⊣	reduce by E: int
E + (E |) + (int) ⊣	shift

Grammar:

E : E + (E) | int

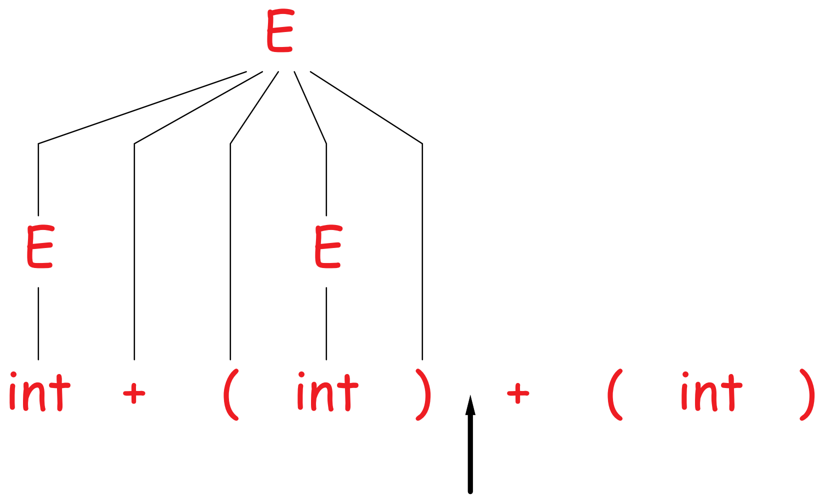


Shift-Reduce Example (6)

Sent. Form	Actions
| int + (int) + (int) ⊣	shift
int | + (int) + (int) ⊣	reduce by E: int
E | + (int) + (int) ⊣	shift 3 times
E + (int |) + (int) ⊣	reduce by E: int
E + (E |) + (int) ⊣	shift
E + (E) | + (int) ⊣	reduce by E: E+(E)

Grammar:

E : E + (E) | int

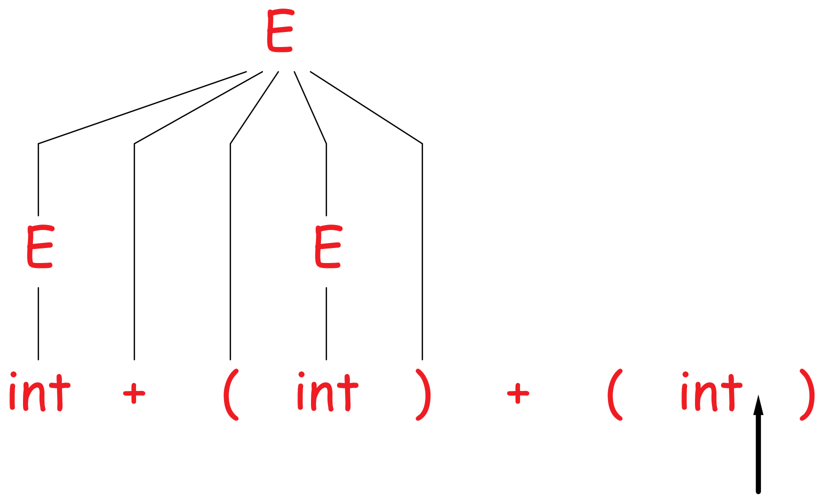


Shift-Reduce Example (7)

Sent. Form	Actions
| int + (int) + (int) ⊣	shift
int | + (int) + (int) ⊣	reduce by E: int
E | + (int) + (int) ⊣	shift 3 times
E + (int |) + (int) ⊣	reduce by E: int
E + (E |) + (int) ⊣	shift
E + (E) | + (int) ⊣	reduce by E: E+(E)
E | + (int) ⊣	shift 3 times

Grammar:

E : E + (E) | int

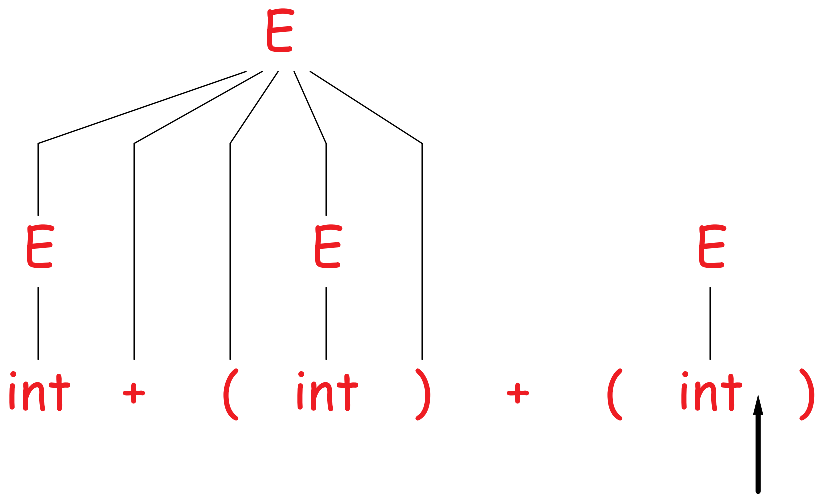


Shift-Reduce Example (8)

Sent. Form	Actions
| int + (int) + (int) ⊣	shift
int | + (int) + (int) ⊣	reduce by E: int
E | + (int) + (int) ⊣	shift 3 times
E + (int |) + (int) ⊣	reduce by E: int
E + (E |) + (int) ⊣	shift
E + (E) | + (int) ⊣	reduce by E: E+(E)
E | + (int) ⊣	shift 3 times
E + (int |) ⊣	reduce by E: int

Grammar:

E : E + (E) | int

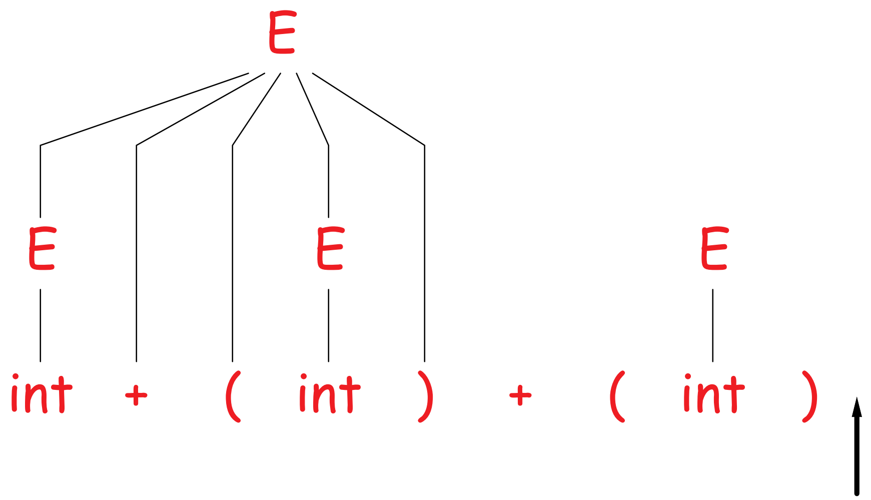


Shift-Reduce Example (9)

Sent. Form	Actions
∣ int + (int) + (int) ⊣	shift
int ∣ + (int) + (int) ⊣	reduce by E: int
E ∣ + (int) + (int) ⊣	shift 3 times
E + (int ∣) + (int) ⊣	reduce by E: int
E + (E ∣) + (int) ⊣	shift
E + (E) ∣ + (int) ⊣	reduce by E: E+(E)
E ∣ + (int) ⊣	shift 3 times
E + (int ∣) ⊣	reduce by E: int
E + (E ∣) ⊣	shift

Grammar:

E : E + (E) | int

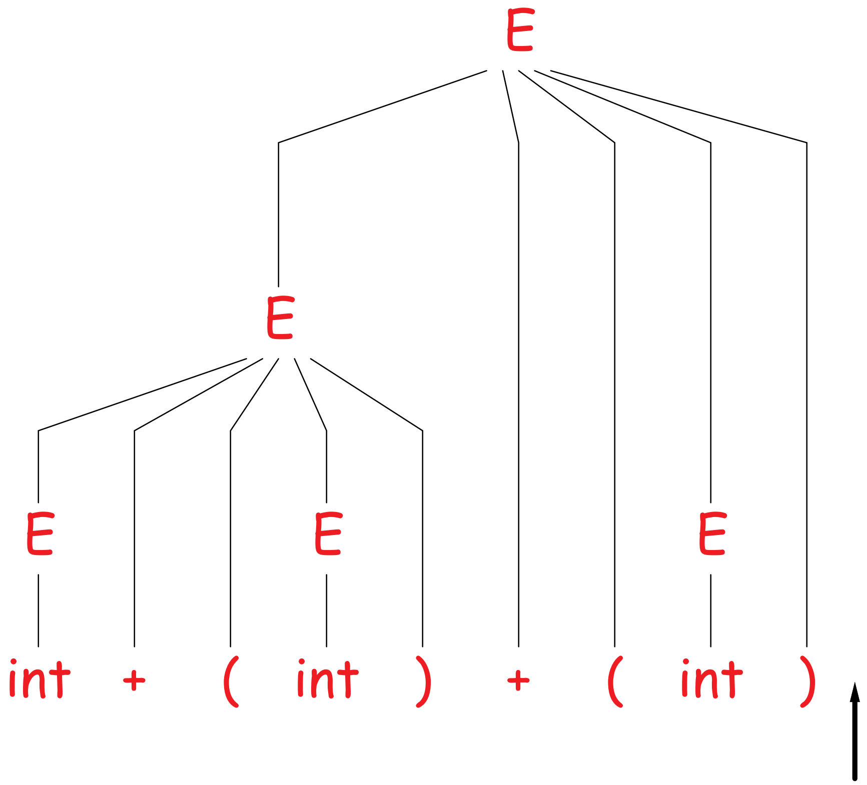


Shift-Reduce Example (10)

Sent. Form	Actions
| int + (int) + (int) ⊣	shift
int | + (int) + (int) ⊣	reduce by E: int
E | + (int) + (int) ⊣	shift 3 times
E + (int |) + (int) ⊣	reduce by E: int
E + (E |) + (int) ⊣	shift
E + (E) | + (int) ⊣	reduce by E: E+(E)
E | + (int) ⊣	shift 3 times
E + (int |) ⊣	reduce by E: int
E + (E |) ⊣	shift
E + (E) |⊣	reduce by E: E+(E)

Grammar:

E : E + (E) | int

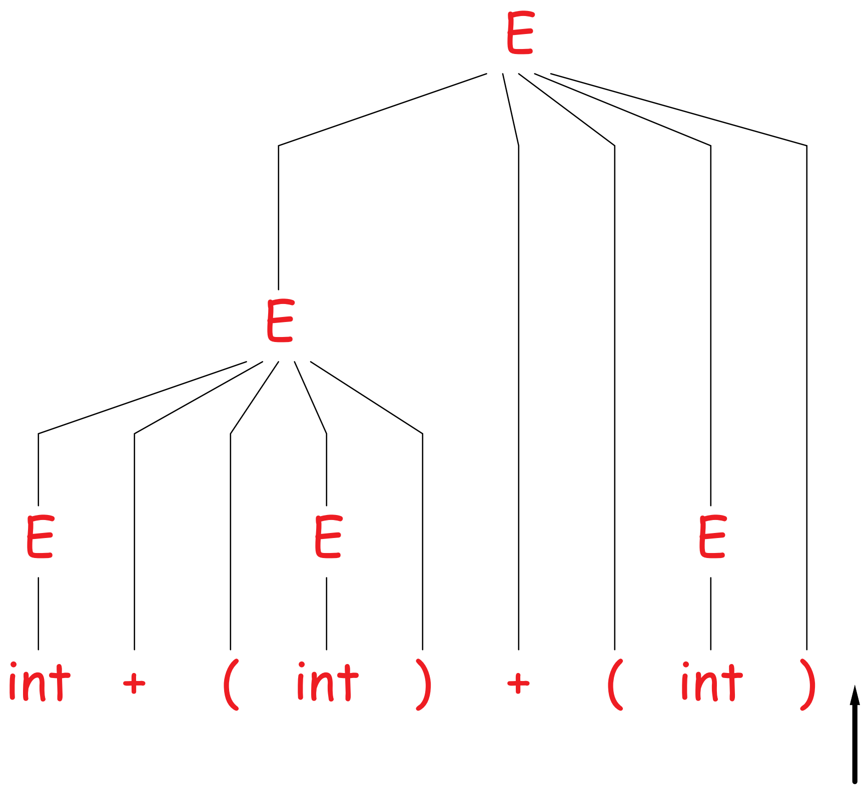


Shift-Reduce Example (11)

Sent. Form	Actions
| int + (int) + (int) ⊣	shift
int | + (int) + (int) ⊣	reduce by E: int
E | + (int) + (int) ⊣	shift 3 times
E + (int |) + (int) ⊣	reduce by E: int
E + (E |) + (int) ⊣	shift
E + (E) | + (int) ⊣	reduce by E: E+(E)
E | + (int) ⊣	shift 3 times
E + (int |) ⊣	reduce by E: int
E + (E |) ⊣	shift
E + (E) |⊣	reduce by E: E+(E)
E |⊣	accept

Grammar:

E : E + (E) | int



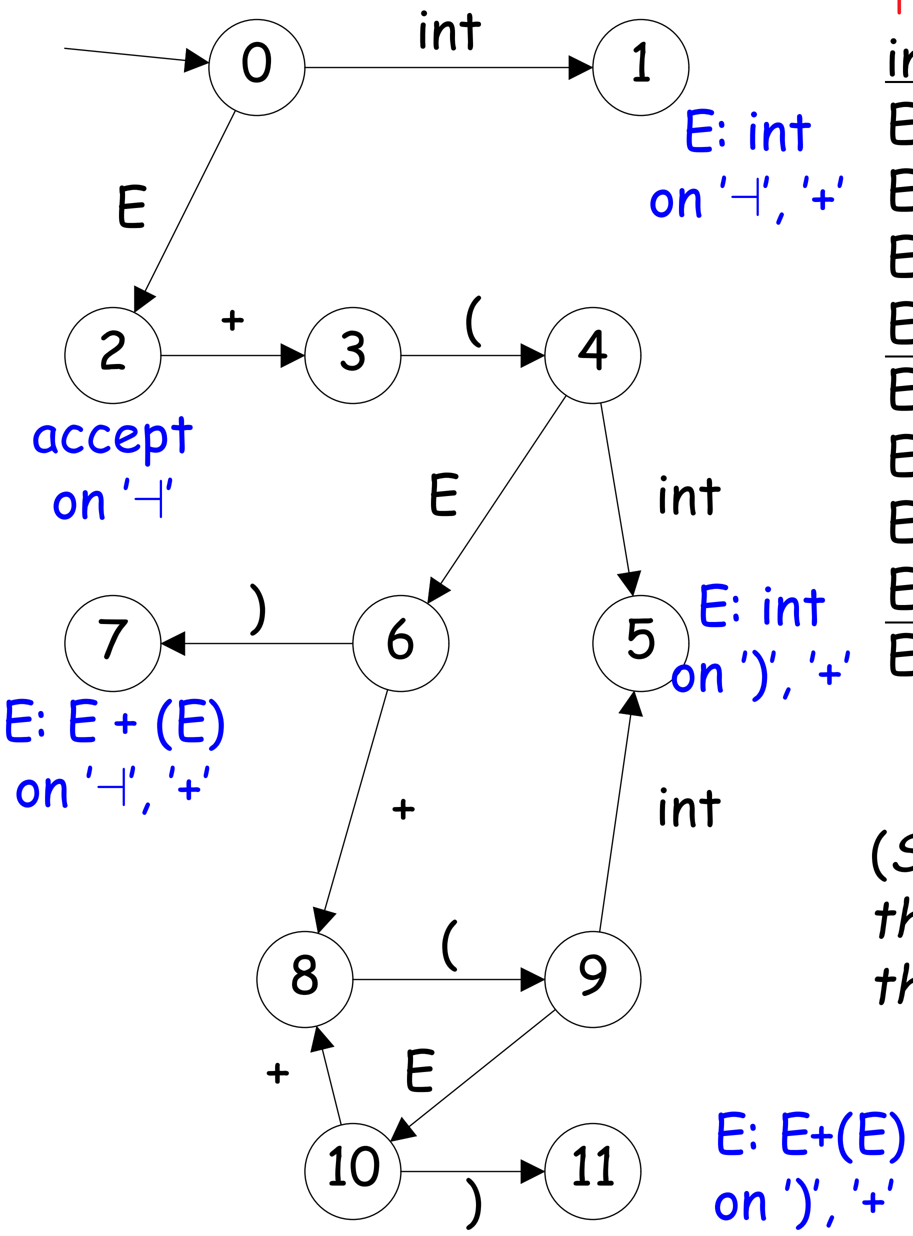
The Parsing Stack

- The left string (left of the |) can be implemented as a stack:
  - Top of the stack is just left of the |.
  - Shift pushes a terminal on the stack.
  - Reduce pops 0 or more symbols from the stack (corresponding to the production's right-hand side) and pushes a nonterminal on the stack (the production's left-hand side).

Key Issue: When to Shift or Reduce?

- Decide based on the left string ("the stack") and some of the remaining input (*lookahead tokens*)—typically one token at most.
- Idea: use a DFA to decide when to shift or reduce:
  - DFA alphabet consists of terminals and nonterminals.
  - The DFA input is the stack up to potential handle.
  - DFA recognizes complete handles.
  - In addition, the final states are labeled with particular productions that might apply, given the possible lookahead symbols.
- We run the DFA on the stack and we examine the resulting state, X and the lookahead token τ after |.
  - If X has a transition labeled τ then shift.
  - If X is labeled with "$A : \beta$ on τ," then reduce.
- So we scan the input from Left to right, producing a (reverse) Rightmost derivation, using 1 symbol of lookahead: giving LR(1) parsing.

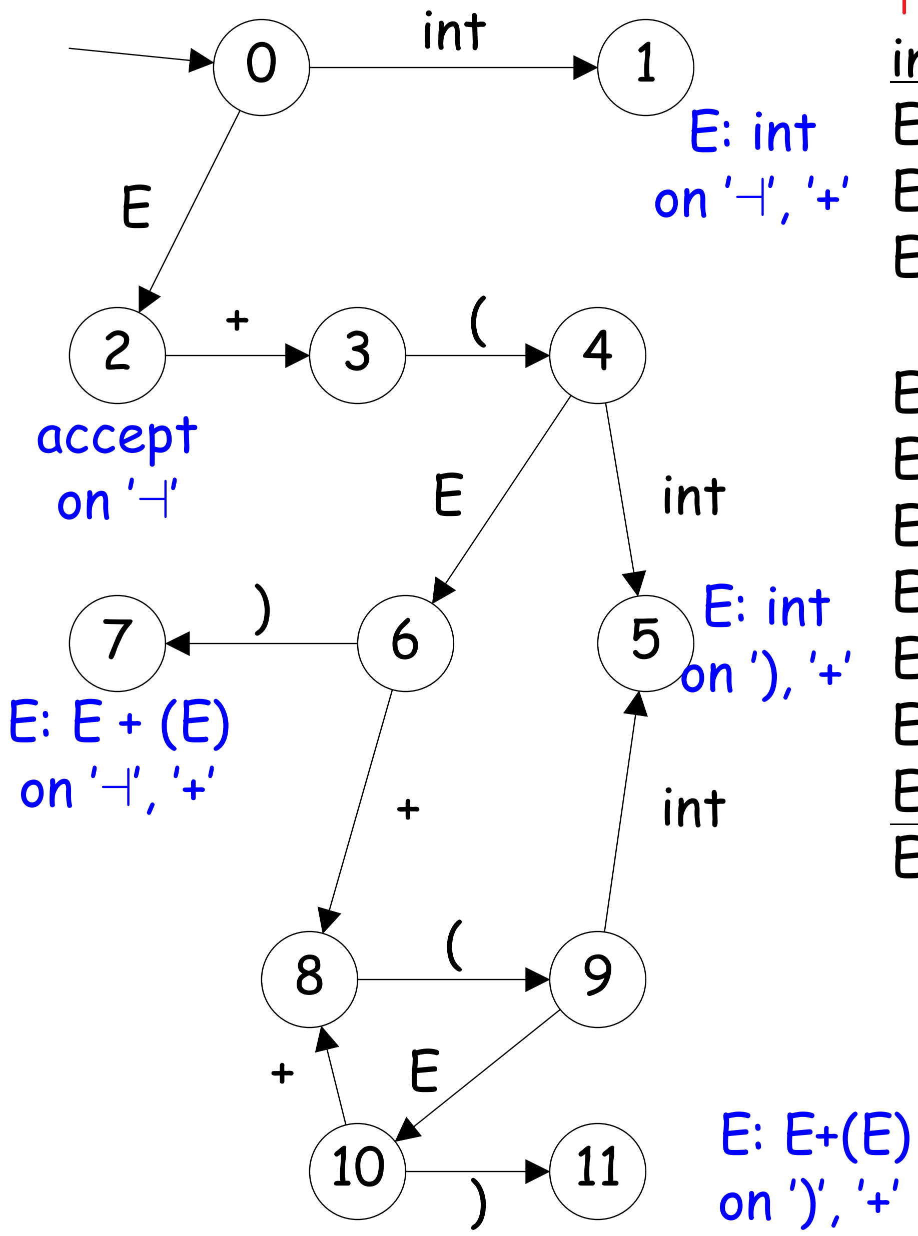
LR(1) Parsing. An Example



$\vert_0$ $\underline{\text{int}}$ + (int) + (int) ⊣	shift
$\underline{\text{int}}$ $\vert_1$ + (int) + (int) ⊣	red. by E: int
E $\vert_2$ $\underline{\text{+ (int}}$) + (int) ⊣	shift 3 times
E + ($\underline{\text{int}}$ $\vert_5$) + (int) ⊣	red. by E: int
E + (E $\vert_6$ $\underline{)}$ + (int) ⊣	shift
$\underline{\text{E + (E)}}$ $\vert_7$ + (int) ⊣	red. by E: E+(E)
E $\vert_2$ $\underline{\text{+ (int}}$) ⊣	shift 3 times
E + ($\underline{\text{int}}$ $\vert_5$) ⊣	red. by E: int
E + (E $\vert_6$ $\underline{)}$ ⊣	shift
$\underline{\text{E + (E)}}$ $\vert_7$ ⊣	red. by E: E+(E)
E $\vert_2$ ⊣	accept

(Subscripts on | show the states that the DFA reaches by scanning the left string.)

LR(1) Parsing. Another Example



$\mid_0$ $\underline{\text{int}}$ + (int + (int + (int))) ⊣	shift
$\underline{\text{int}}$ $\mid_1$ + (int + (int + (int)))⊣	red. by E: int
E $\mid_2$ $\underline{\text{+ (int)}}$ + (int + (int))) ⊣	shift 3 times
E + ($\underline{\text{int}}$ $\mid_5$) + (int + (int))) ⊣	red. by E: int
E + (E $\mid_6$ $\underline{\text{)}}$ + (int + (int))) ⊣	shift
⋮	⋮
E + (E + (E + ($\underline{\text{int}}$$\mid_5$))) ⊣	red. by E: int
E + (E + (E + (E$\mid_{10}$$\underline{\text{)}}$)) ⊣	shift
E + (E + ($\underline{\text{E + (E)}}$ $\mid_{11}$)) ⊣	red. by E: E + (E)
E + (E + (E $\mid_{10}$ $\underline{\text{)}}$)) ⊣	shift
E + ($\underline{\text{E + (E)}}$$\mid_{11}$) ⊣	red. by E: E + (E)
E + (E$\mid_6$$\underline{\text{)}}$ ⊣	shift
$\underline{\text{E + (E)}}$$\mid_7$ ⊣	red. by E: E + (E)
E $\mid_2$ ⊣	accept

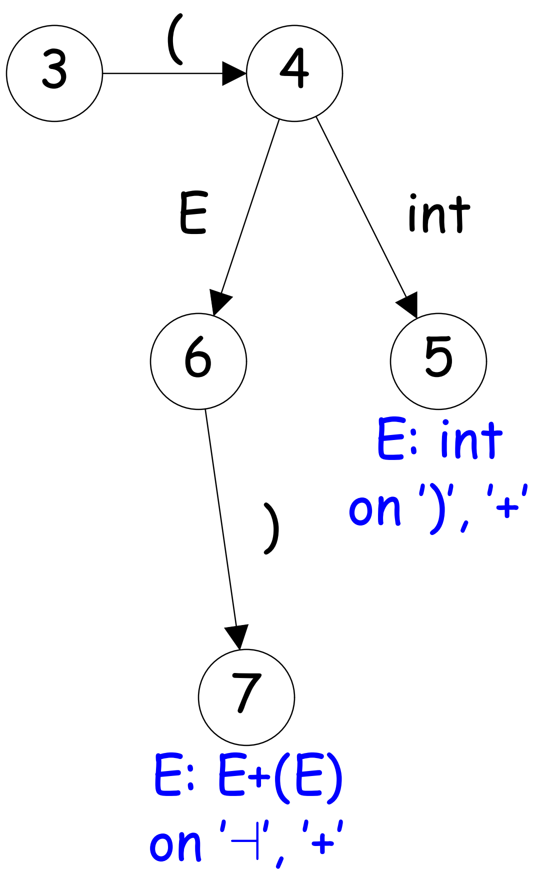
Representing the DFA

- Parsers represent the DFA as a 2D table, as for table-driven lexical analysis
- Lines correspond to DFA states
- Columns correspond to terminals and nonterminals
- Classical treatments (like Aho, *et al*) split the columns into:
  - Those for terminals: the *action table*.
  - Those for nonterminals: the *goto table*.

  The goto table contains only shifts, but conceptually, the tables are very much alike as far as the DFA is concerned.
- The classical division has some advantages when it comes to table compression.

Representing the DFA. Example

Here's the table for a fragment of our DFA:



	int	+	(	)	⊣	E
...						
3			s4			
4	s5					s6
5		$r_{\text{E: int}}$		$r_{\text{E: int}}$		
6				s7		
7		$r_{\text{E: E+(E)}}$			$r_{\text{E: E+(E)}}$	
...						

Legend: 'sN' means "shift (or go to) state N."
'r$_P$' means "reduce using production P."
blank entries indicate errors.

A Little Optimization

- After a shift or reduce action we rerun the DFA on the entire stack.
- This is wasteful, since most of the work is repeated, so
- Memoize: instead of putting terminal and nonterminal symbols on the stack, put the DFA states you get to after reading those symbols.
- For example, when we've reached this point:

  $$E + (E + (E + (\underline{\text{int}} \mid_{5})\ )) \dashv$$

  store the part to the left of | as

  0 2 3 4 6 8 9 10 8 9 5

- And don't throw any of these away until you reduce them.

The Actual LR Parsing Algorithm

```
Let I = $w_1 w_2 \ldots w_n$ be initial input
Let j = 1
Let stack = < 0 >

repeat
   case table[top_state(stack), I[j]] of
      s$k$:
           push $k$ on the stack; j += 1
      r$_{\text{X: } \alpha}$:
           pop len($\alpha$) symbols from stack
           push $j$ on stack, where table[top_state(stack), X] is s$j$.
      accept:
           return normally
      error:
           return parsing error indication
```

Parsing Contexts

- Consider the state describing the situation at the | in the stack E + (| int)+(int), which tells us
  - We are looking to reduce E: E + (E), having already seen E + (from the right-hand side.
  - Therefore, we expect that the rest of the input starts with something that will eventually reduce to E:

    E: int or E: E+(E)

    after which we expect to find a ')',
  - but we have as yet seen nothing from the right-hand sides of either of these two possible productions.
- One DFA state captures a set of such contexts in the form of a set of *LR(1) items*, like this:

```
[ E: E + ( • E ), ... ]       [ E: • int, '+' ] (why?)
[ E: • int, ')' ]             [ E: • E+(E), '+' ] (why?)
[ E: • E+(E), ')' ]
```

- (Traditionally, use • in items to show where the | is.)

LR(1) Items

- An LR(1) item is a pair:

  X: $\alpha\bullet\beta$, a

  - X: $\alpha\beta$ is a production.
  - a is a terminal symbol (an expected lookahead).

- It says we are trying to find an X followed by a.
- and that we have already accumulated α on top of the parsing stack.
- Therefore, we need to see next a prefix of something derived from βa.
- (As an abbreviation, we'll usually write

  X: $\alpha\bullet\beta$, a/b

  to mean the *two* LR(1) items

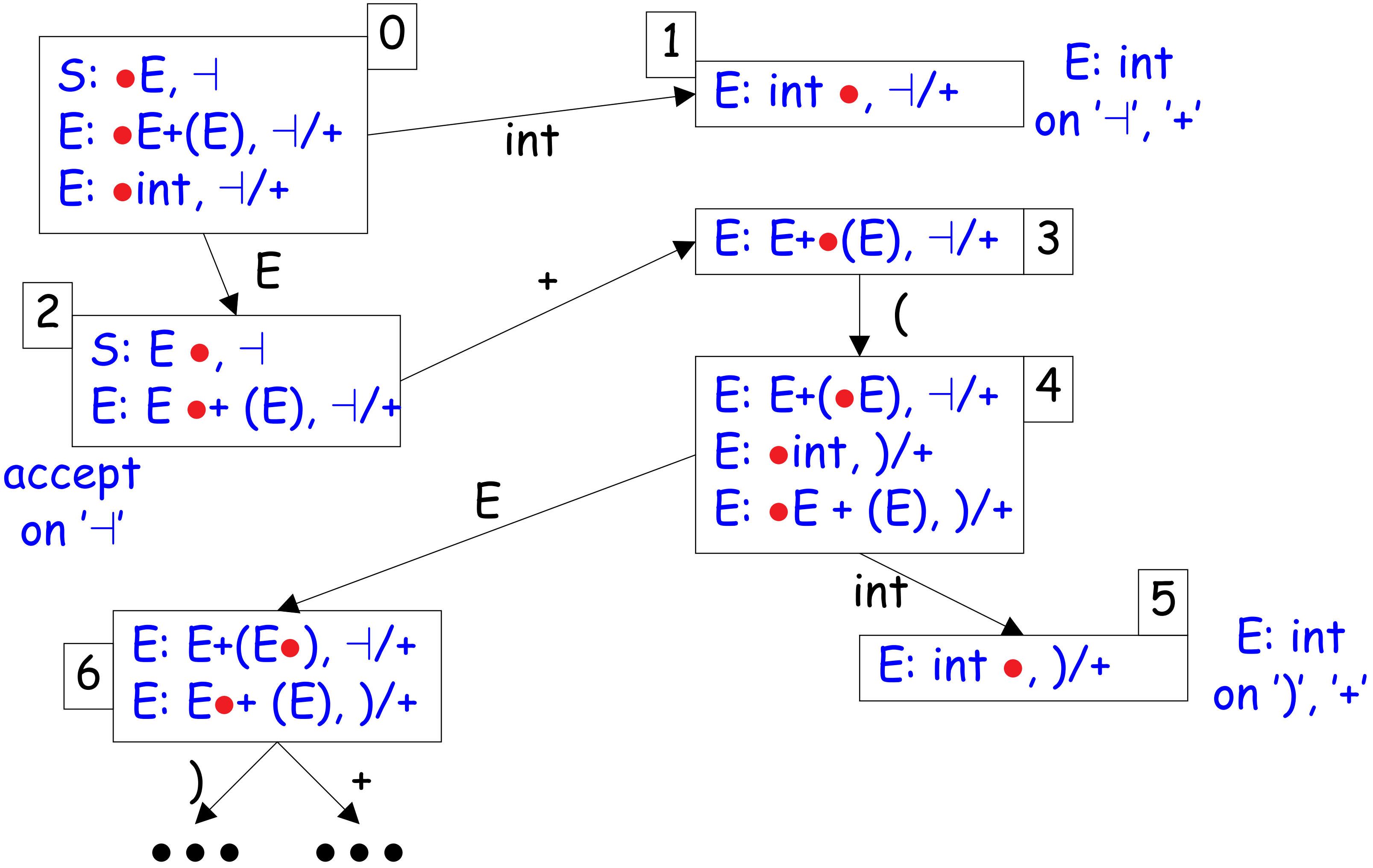
  X: $\alpha\bullet\beta$, a
  X: $\alpha\bullet\beta$, b

)

Constructing the Parsing DFA

- The idea is to borrow from Earley's algorithm (where we've already seen this notation!).
- We throw away a lot of the information that Earley's algorithm keeps around (notably where in the input each current item got introduced), because when we have a handle, there will only be one possible reduction to take based on what we've seen so far.
- This allows the set of possible item sets to be finite.
- Each state in the DFA has an item set that is derived from what Earley's algorithm would do, but collapsed because of the information we throw away.

Constructing the Parsing DFA: Partial Example



LR Parsing Tables. Notes

- We really want to construct parsing tables (i.e. the DFA) from CFGs automatically, since this construction is tedious.
- But still good to understand the construction to work with parser generators, which report errors in terms of sets of items.
- What kind of errors can we expect?

Shift/Reduce Conflicts

- If a DFA state contains both [X: $\alpha\bullet a\beta$, b] and [Y: γ, a], then we have two choices when the parser gets into that state at the | and the next input symbol is a:
  - Shift into the state containing [X: $\alpha a\bullet\beta$, b], or
  - Reduce with Y: γ.
- This is called a *shift-reduce conflict*.
- Often due to ambiguities in the grammar. Classic example: the dangling else

  S: "if" E "then" S | "if" E "then" S "else" S | . . .

- This grammar gives rise to a DFA state containing

  [S: "if" E "then" S•, "else"] and [S: "if" E "then" S•"else" S, . . .]

- So if "else" is next, we can shift or reduce.

More Shift/Reduce Conflicts

- Consider the ambiguous grammar

  E : E + E | E * E | int

- We will have states containing

$$\begin{array}{lcl} [E\!: E + \bullet E,\ */+] & & [E\!: E + E \bullet,\ */+] \\ [E\!: \bullet E + E,\ */+] & \xRightarrow{E} & [E\!: E \bullet + E,\ */+] \\ [E\!: \bullet E * E,\ */+] & & [E\!: E \bullet * E,\ */+] \\ \ldots & & \ldots \end{array}$$

- Again we have a shift/reduce conflict on input '*' or '+' (in the item set on the right).
- We probably want to shift on '*' (which is usually supposed to bind more tightly than '+')
- We probably want to reduce on '+' (left-associativity).
- Solution: provide extra information (the precedence of '*' and '+') that allows the parser generator to decide what to do.

Using Precedence in Bison/Horn

- In Bison or Horn, you can declare precedence and associativity of both terminal symbols and rules,
- For terminal symbols (tokens), there are precedence declarations, listed from lowest to highest precedence:

  ```
  %left '+'
  %left '*'
  %right "**"
  ```

  Symbols on each such line have the same precedence.
- For a rule, precedence = that of its last terminal (Can override with `%prec` if needed, cf. the Bison manual).
- Now, we resolve shift/reduce conflict with a shift if:
  - The next input token has higher precedence than the rule, or
  - The next input token has the same precedence as the rule and the relevent precedence declaration was `%right`.

  and otherwise, we choose to reduce the rule.

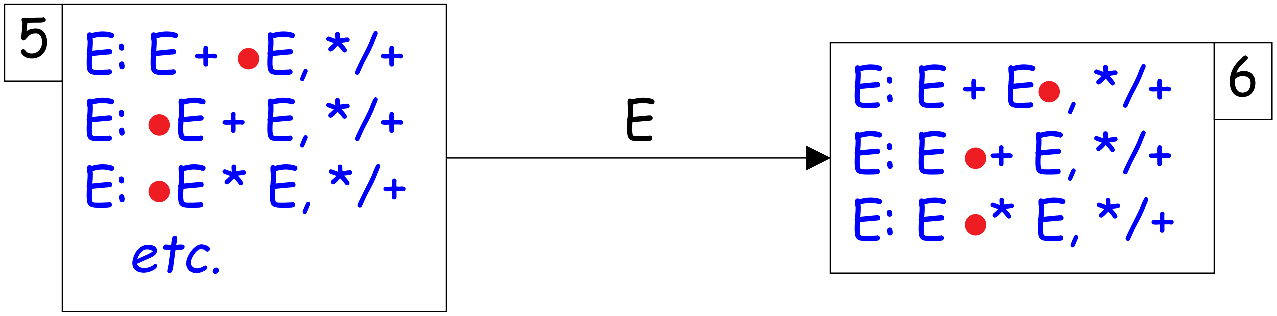
Example of Using Precedence to Solve S/R Conflict (1)

- *Assuming we've declared*

```
%left '+'
%left '*'
```

the rule E: E + E will have precedence 1 (left-associative) and the rule E: E * E will have precedence 2.

- So, when the parser confronts the choice in state 6 w/next token '*',



5

E: E + ●E, */+
E: ●E + E, */+
E: ●E * E, */+
etc.

E

E: E + E●, */+
E: E ●+ E, */+
E: E ●* E, */+

6

it will choose to shift because the '*' has higher precedence than the rule E + E.

- On the other hand, with input symbol '+', it will choose to reduce, because the input token then has the same precedence as the rule to be reduced, and is left-associative.

Example of Using Precedence to Solve S/R Conflict (2)

- Back to our dangling else example. We'll have the state

```
10  S: "if" E "then" S •, "else"
    S: "if" E "then" S•"else" S, "else"
      etc.
```

- Can eliminate conflict by declaring the token "else" to have higher precedence than "then" (and thus, than the first rule above).

- HOWEVER: best to limit use of precedence to these standard examples (expressions, dangling elses). If you simply throw them in because you have a conflict you don't understand, you're like to end up with unexpected parse trees or syntax errors.

Reduce/Reduce Conflicts

- The lookahead symbols in LR(1) items are only considered for reductions in items that end in '•'.
- If a DFA state contains both

  [X: α•, a] and [Y: β•, a]

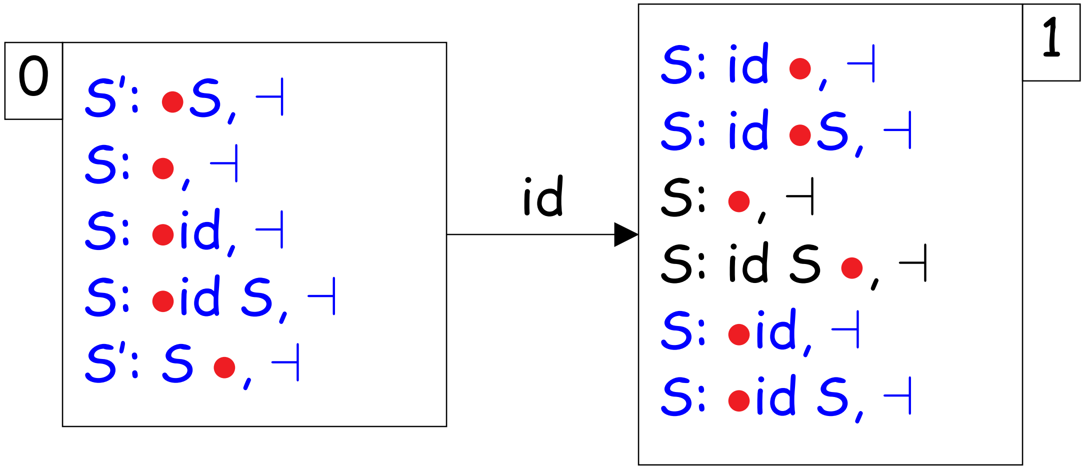
  then on input 'a' we don't know which production to reduce.
- Such *reduce/reduce conflicts* are often due to a gross ambiguity in the grammar.
- Example: defining a sequence of identifiers with

  S: ϵ | id | id S
- There are two parse trees for the string id:

  S $\Rightarrow$id or S $\Rightarrow$id S $\Rightarrow$S.

Reduce/Reduce Conflicts in DFA

- For this example, you'll get states:



0

S′: •S, ⊣

S: •, ⊣

S: •id, ⊣

S: •id S, ⊣

S′: S •, ⊣

id

1

S: id •, ⊣

S: id •S, ⊣

S: •, ⊣

S: id S •, ⊣

S: •id, ⊣

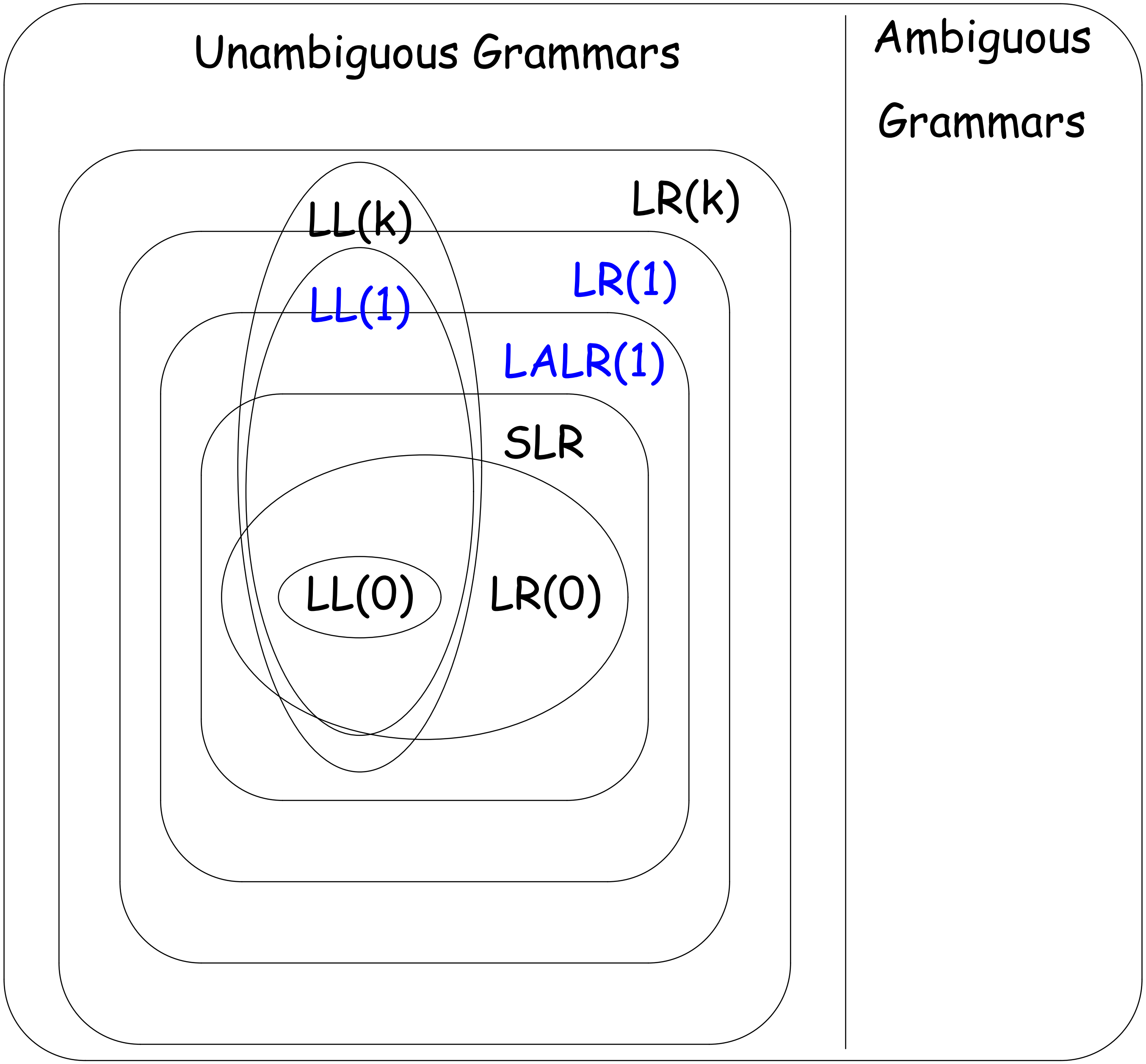
S: •id S, ⊣

- Reduce/reduce conflict on input '⊣'.
- Better rewrite the grammar: S: ϵ | id S.

Relation to Bison

- Bison builds this kind of machine.
- However, for efficiency concerns, collapses many of the states together, namely those that differ only in lookahead sets, but otherwise have identical sets of items. Result is called an *LALR(1) parser* (as opposed to LR(1)).
- Causes some additional conflicts, but not these are rare.

A Hierarchy of Grammar Classes



From Andrew Appel, "Modern Compiler Implementation in Java"

Summary

- Parsing provides a means of tying translation actions to syntax clearly.
- A simple parser: LL(1), recursive descent
- A more powerful parser: LR(1)
- An efficiency hack: LALR(1), as in Bison.
- Earley's algorithm provides a complete algorithm for parsing all context-free languages.
- We can get the same effect in Bison by other means (the `%glr-parser` option, for Generalized LR), as seen in one of the examples from lecture #5.