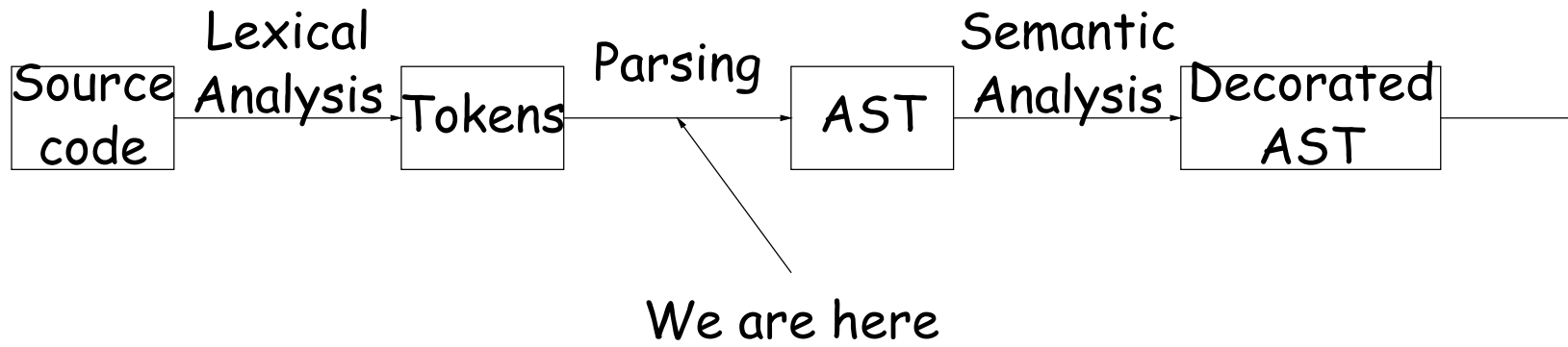


Lecture 6: Parsing

Administrivia

- If you do not have a group, please post a request on the Piazza board with tag #group, giving your email. (Also try contacting anyone you find there about forming a group). Please take your name down if you find a group.
- We will assign orphans to groups randomly in a few days.

A Glance at the Map



Review: BNF

- BNF is another pattern-matching language;
- Alphabet typically set of *tokens*, such as from lexical analysis, referred to as *terminal symbols* or *terminals*.
- Matching rules have form:

$$X : \alpha_1\alpha_2\cdots\alpha_n,$$

where X is from a set of *nonterminal symbols* (or *nonterminals* or *meta-variables*), $n \geq 0$, and each α_i is a terminal or nonterminal symbol.

- For emphasis, may write $X : \epsilon$ when $n = 0$.
- Read $X : \alpha_1\alpha_2\cdots\alpha_n$, as
"An X may be formed from the concatenation of an $\alpha_1, \alpha_2, \dots, \alpha_n$."
- Designate one nonterminal as the *start symbol*.
- Set of all matching rules is a *context-free grammar*.

Review: Derivations

- String (of terminals) T is in the language described by grammar G , ($T \in L(G)$) if there is a *derivation of T* from the start symbol of G .
- Derivation of $T = \tau_1 \cdots \tau_k$ from nonterminal A is sequence of *sentential forms*:

$$A \Rightarrow \alpha_{11}\alpha_{12} \dots \Rightarrow \alpha_{21}\alpha_{22} \dots \Rightarrow \dots \Rightarrow \tau_1 \dots \tau_k$$

where each α_{ij} is a terminal or nonterminal symbol.

- We say that

$$\alpha_1 \cdots \alpha_{m-1} B \alpha_{m+1} \cdots \alpha_n \Rightarrow \alpha_1 \cdots \alpha_{m-1} \beta_1 \cdots \beta_p \alpha_{m+1} \cdots \alpha_n$$

if $B : \beta_1 \cdots \beta_p$ is a production. ($1 \leq m \leq n$).

- If Φ and Φ' are sentential forms, then $\Phi_1 \xRightarrow{*} \Phi_2$ means that 0 or more \Rightarrow steps turns Φ_1 into Φ_2 . $\Phi_1 \xRightarrow{+} \Phi_2$ means 1 or more \Rightarrow steps does it.
- So if S is start symbol of G , then $T \in L(G)$ iff $S \xRightarrow{+} T$.

Example of Derivation

1. $e : s \text{ ID}$
2. $e : s \text{ ' (' e ') '}$
3. $e : e \text{ '/ ' e}$
4. $s :$
5. $s : \text{'+'}$
6. $s : \text{'-'}$

Alternative Notation

$e : s \text{ ID}$
 $| s \text{ ' (' e ') '}$
 $| e \text{ '/ ' e}$
 $s : \epsilon \mid \text{'+'} \mid \text{'-'}$

Problem: Derive $- \text{ ID } / (\text{ ID } / \text{ ID })$

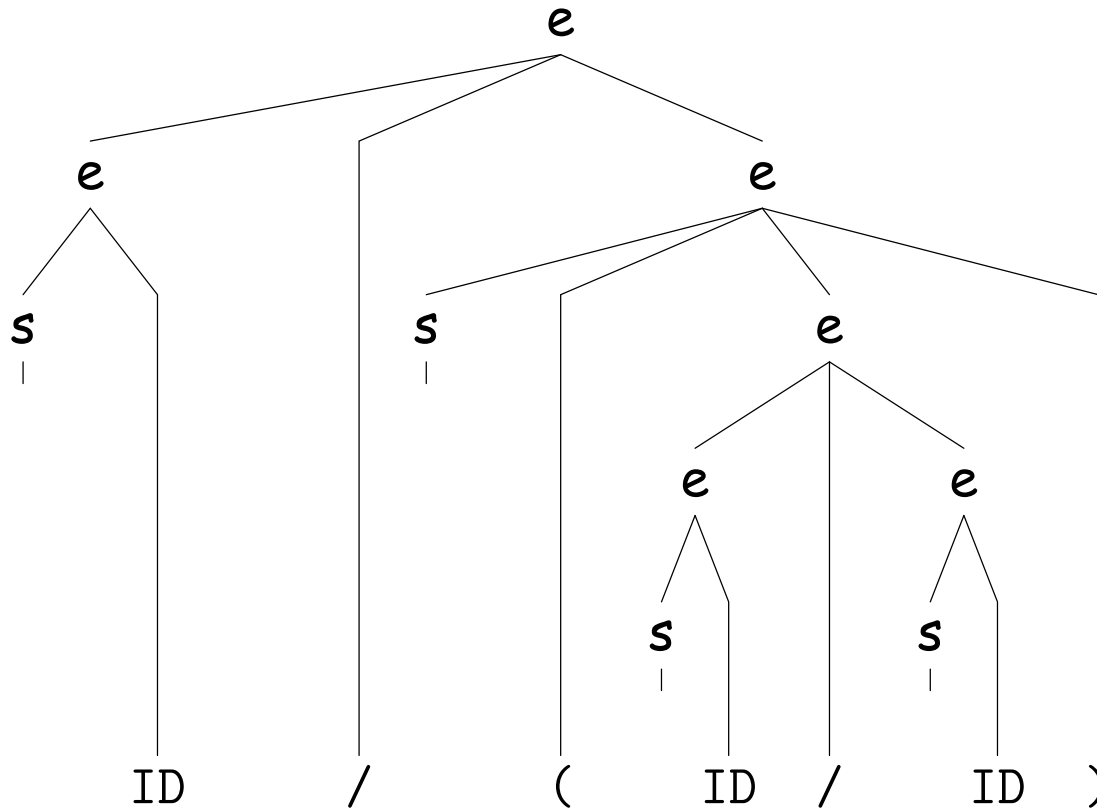
$$\begin{aligned}
 e &\xrightarrow{3} e / e \xrightarrow{1} s \text{ ID} / e \xrightarrow{6} - \text{ ID} / e \xrightarrow{2} - \text{ ID} / s (e) \\
 &\xrightarrow{4} - \text{ ID} / (e) \xrightarrow{3} - \text{ ID} / (e / e) \xrightarrow{1} - \text{ ID} / (s \text{ ID} / e) \\
 &\xrightarrow{4} - \text{ ID} / (\text{ ID} / e) \xrightarrow{1} - \text{ ID} / (\text{ ID} / s \text{ ID}) \\
 &\xrightarrow{4} - \text{ ID} / (\text{ ID} / \text{ ID})
 \end{aligned}$$

Types of Derivation

- *Context free* means can replace nonterminals in any order (i.e., regardless of context) to get same result (as long as you use same productions).
- So, if we use a particular rule for selecting nonterminal to “produce” from, can characterize derivation by just listing productions.
- Previous example was *leftmost derivation*: always choose leftmost nonterminals. Completely characterized by list of productions: 3, 1, 6, 2, 4, 3, 1, 4, 1, 4.

Derivations and Parse Trees

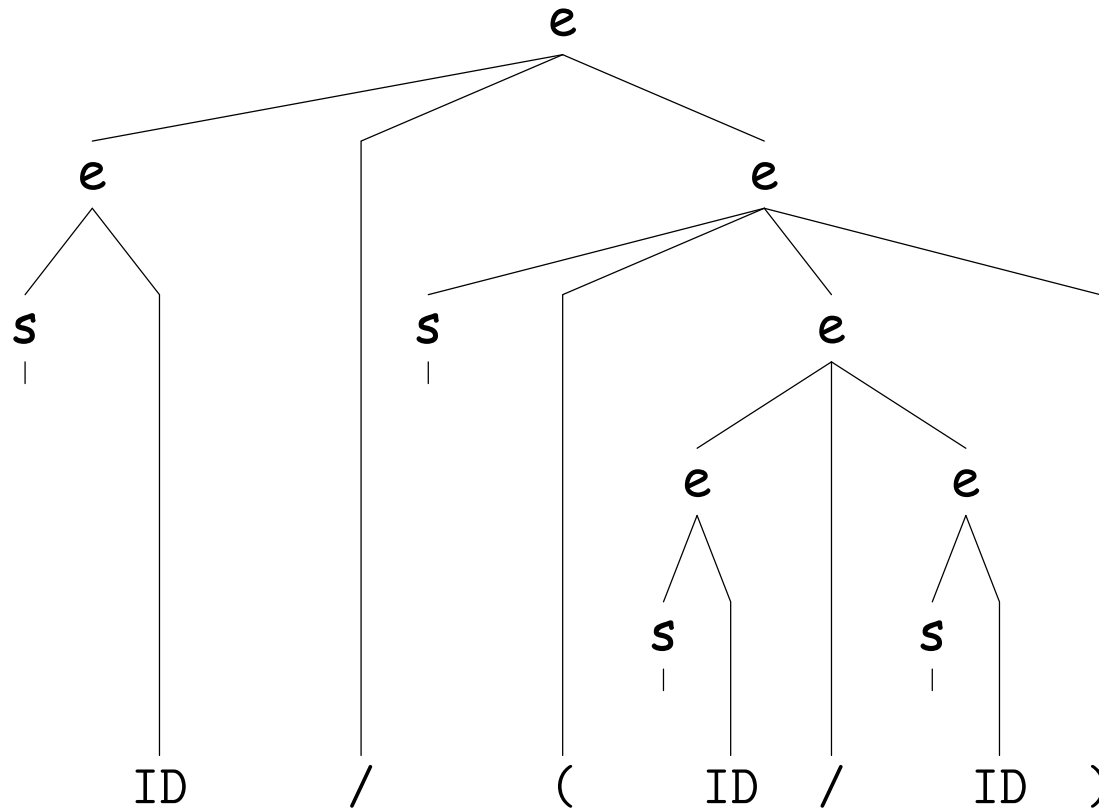
- A leftmost derivation also completely characterized by parse tree:



- What is the rightmost derivation for this?

Derivations and Parse Trees

- A leftmost derivation also completely characterized by parse tree:

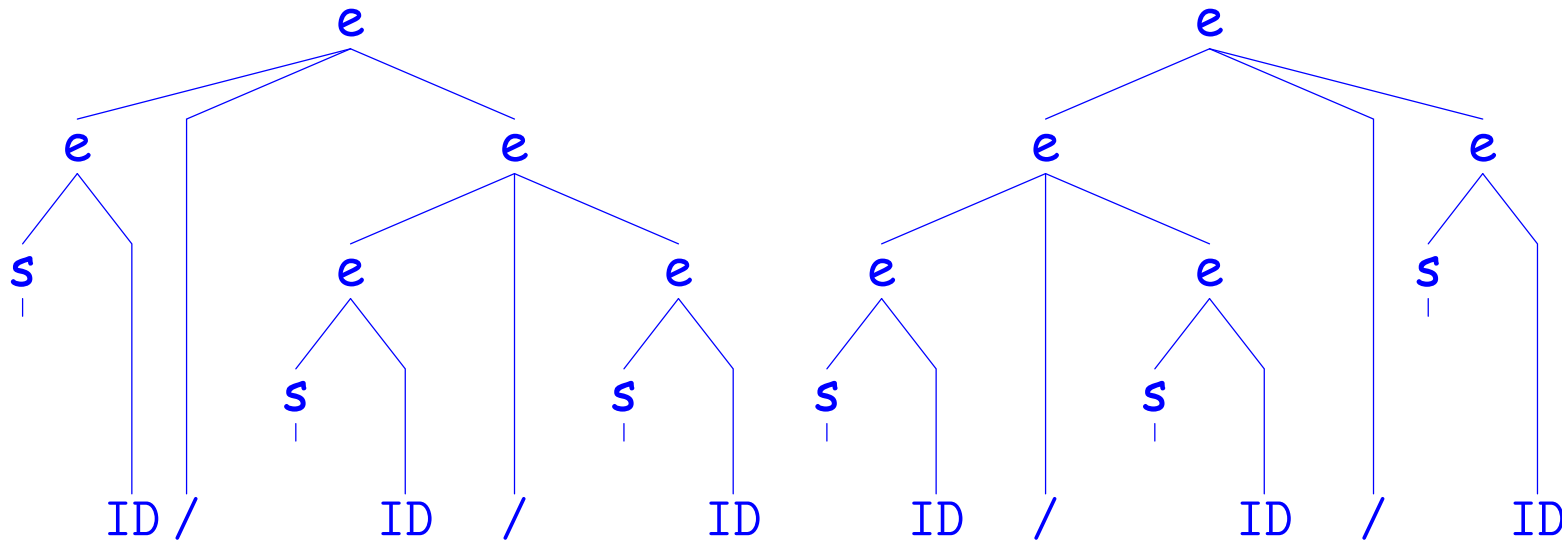


- What is the rightmost derivation for this?

$$\begin{aligned}
 e &\xrightarrow{3} e / e \xrightarrow{2} e / s (e) \xrightarrow{3} e / s (e / e) \\
 &\xrightarrow{1} e / s (e / s ID) \xrightarrow{4} e / s (e / ID) \\
 &\xrightarrow{1} e / s (s ID / ID) \xrightarrow{4} e / s (ID / ID) \\
 &\xrightarrow{4} e / (ID / ID) \xrightarrow{1} s ID / (ID / ID) \xrightarrow{6} - ID / (ID / ID)
 \end{aligned}$$

Ambiguity

- Only one derivation for previous example.
- What about 'ID / ID / ID'?
- Claim there are two parse trees, corresponding to two leftmost derivations. What are they?



- If there exists even one string like ID / ID / ID in $L(G)$, we say G is ambiguous (even if other strings only have one parse tree).

Review: Syntax-Directed Translation

- Want the structure of sentences, not just whether they are in the language, because this drives translation.
- Associate translation rules to each production, just as Flex associated actions with matching patterns.
- Bison notation:

```
e : e '/' e          { $$ = doDivide($1, $3); }
```

provides way to refer to and set *semantic values* on each node of a parse tree.

- Compute these semantic values from leaves up the parse tree.
- Same as the order of a *rightmost derivation in reverse* (a.k.a a *canonical derivation*).
- Alternatively, just perform arbitrary actions in the same order.

Example: Conditional statement

Problem: `if-else` or `if-elif-else` statements in Python (`else` optional). Assume that only (indented) suites may be used for then and else clauses, that nonterminal `stmt` defines an individual statement (one per line), and that nonterminal `expr` defines an expression. Lexer supplies `INDENTs` and `DEDENTs`. A `cond` is a kind of `stmt`.

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```
expr : ...
stmt : ... | cond | ...
cond : "if" expr ':' suite elifs else
suite: INDENT stmts DEDENT
stmts: stmt | stmts stmt
elifs:  $\epsilon$  | "elif" expr ':' suite elifs
else :  $\epsilon$  | "else" ':' suite
```

Example: Conditional statement in Java

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```
expr : ...  
stmt : ... | cond | ...  
cond : "if" '(' expr ')' stmt else  
else :  $\epsilon$  | "else" stmt
```

But this doesn't quite work: recognizes correct statements and rejects incorrect ones, but is ambiguous. E.g.,

```
if (foo) if (bar) walk(); else chewGum();
```

Do we chew gum if `foo` is false? That is, is this equivalent to

```
if (foo) { if (bar) walk(); } else chewGum();  
/*or*/ if (foo) { if (bar) walk(); else chewGum(); } ?
```

Example resolved: Conditional statement in Java

The rule is supposed to be "each 'else' attaches to the nearest open 'if' on the left," which is captured by:

```
expr : ...
stmt : ... | cond | ...
stmt_closed : ... | cond_closed | ...
cond_closed : "if" '(' expr ')' stmt_closed "else" stmt_closed
cond : "if" '(' expr ')' stmt
      | "if" '(' expr ')' stmt_closed "else" stmt
```

This does not allow us to interpret

```
if (foo) if (bar) walk(); else chewGum();
```

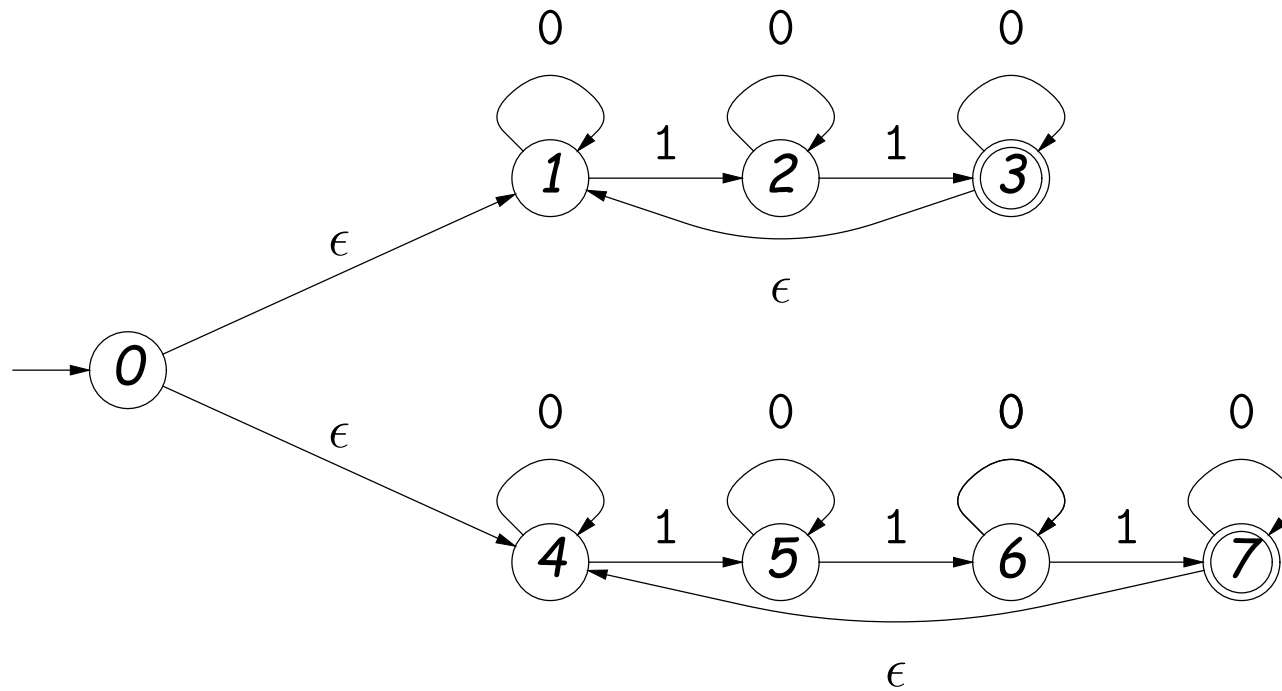
as

```
if (foo) { if (bar) walk(); } else chewGum();
```

But it's not exactly clear, is it?

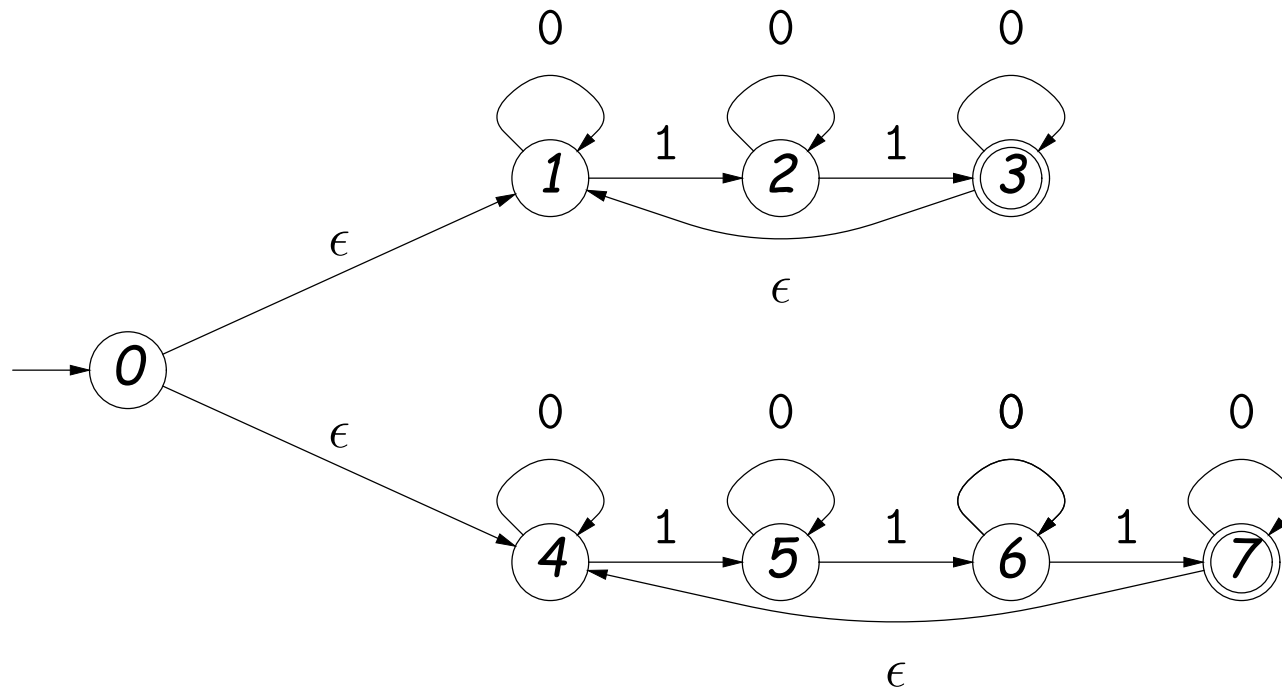
Puzzle: NFA to BNF

Problem: What BNF grammar accepts the same string as this NFA?



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A conventional answer (from class):

S: S2s Z | S3s Z

S2: Z '1' Z '1'

Z: '0' Z | ϵ

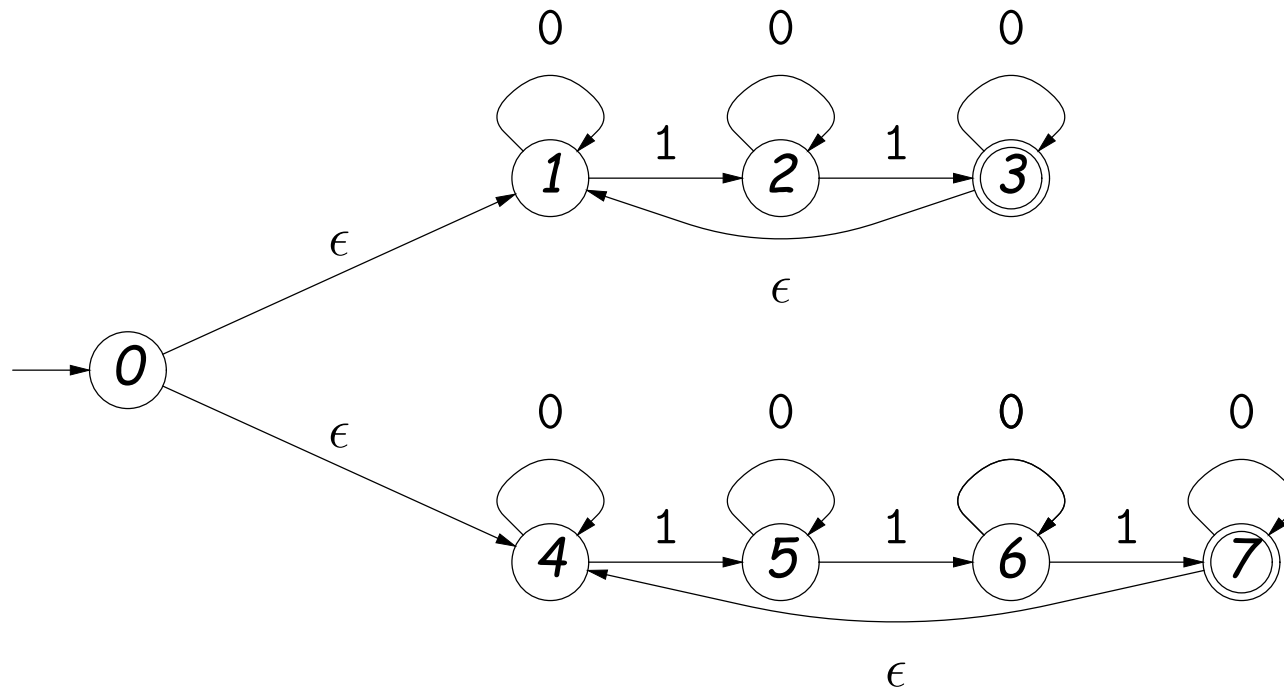
S3: Z '1' Z '1' Z '1'

S2s: S2 | S2 S2s

S3s: S3 | S3 S3s

Puzzle: NFA to BNF

Problem: What BNF grammar accepts the same string as this NFA?



General answer (adaptable to any NFA), with one nonterminal per state:

S0: S1 | S4

S1: '1' S2 | '0' S1

S2: '1' S3 | '0' S2

S3: S1 | '0' S3 | ϵ

S4: '1' S5 | '0' S4

S5: '1' S6 | '0' S5

S6: '1' S7 | '0' S6

S7: S4 | '0' S7 | ϵ

Nonterminal S_k is "the set of strings that will get me from S_k in the NFA to a final state in the NFA."