Linear Programming

1939 Kantorovich – planning
Koopmans – classic economy

large organizations
airlines
1975 Nobel prize Economics

George Danzig: simplex

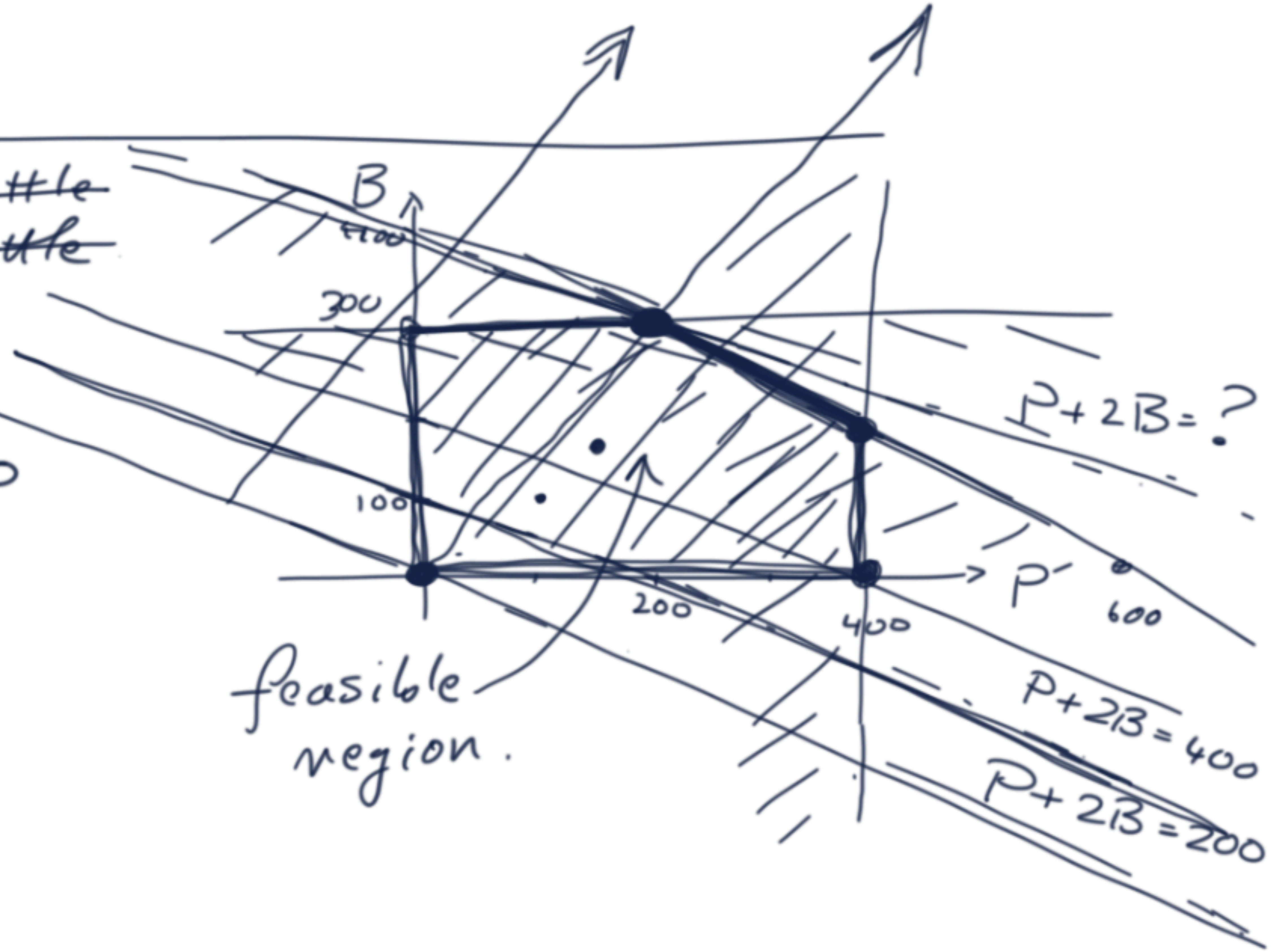
Example:

P 1/case ~~$1 / bottle~~
B 2/case ~~$2 / bottle~~

Demand: $P \le 400$
$B \le 300$

$2P + 3B \le 1200$

$\max\,(P + 2B)$



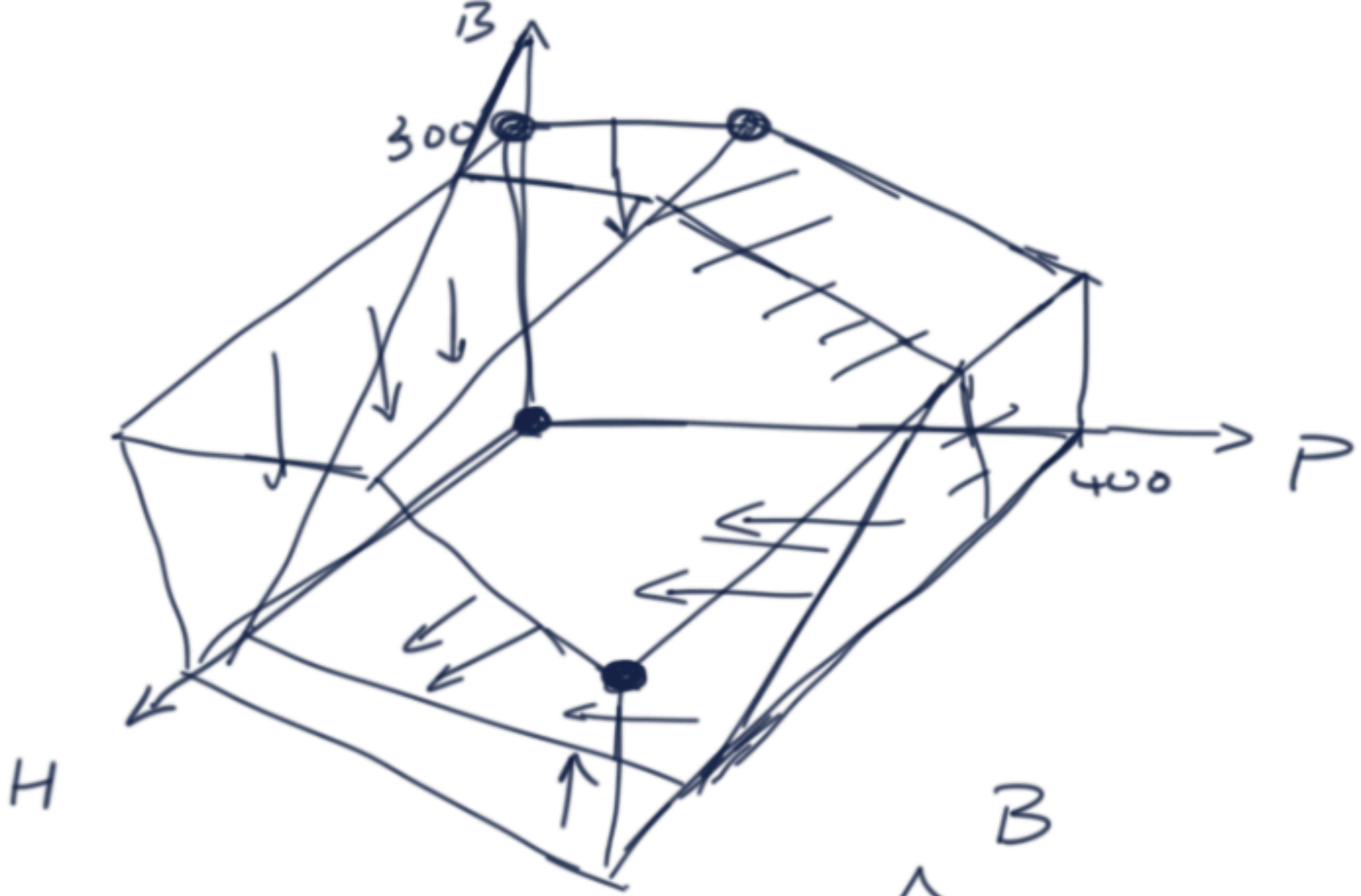
max $P + 2B + 3H$

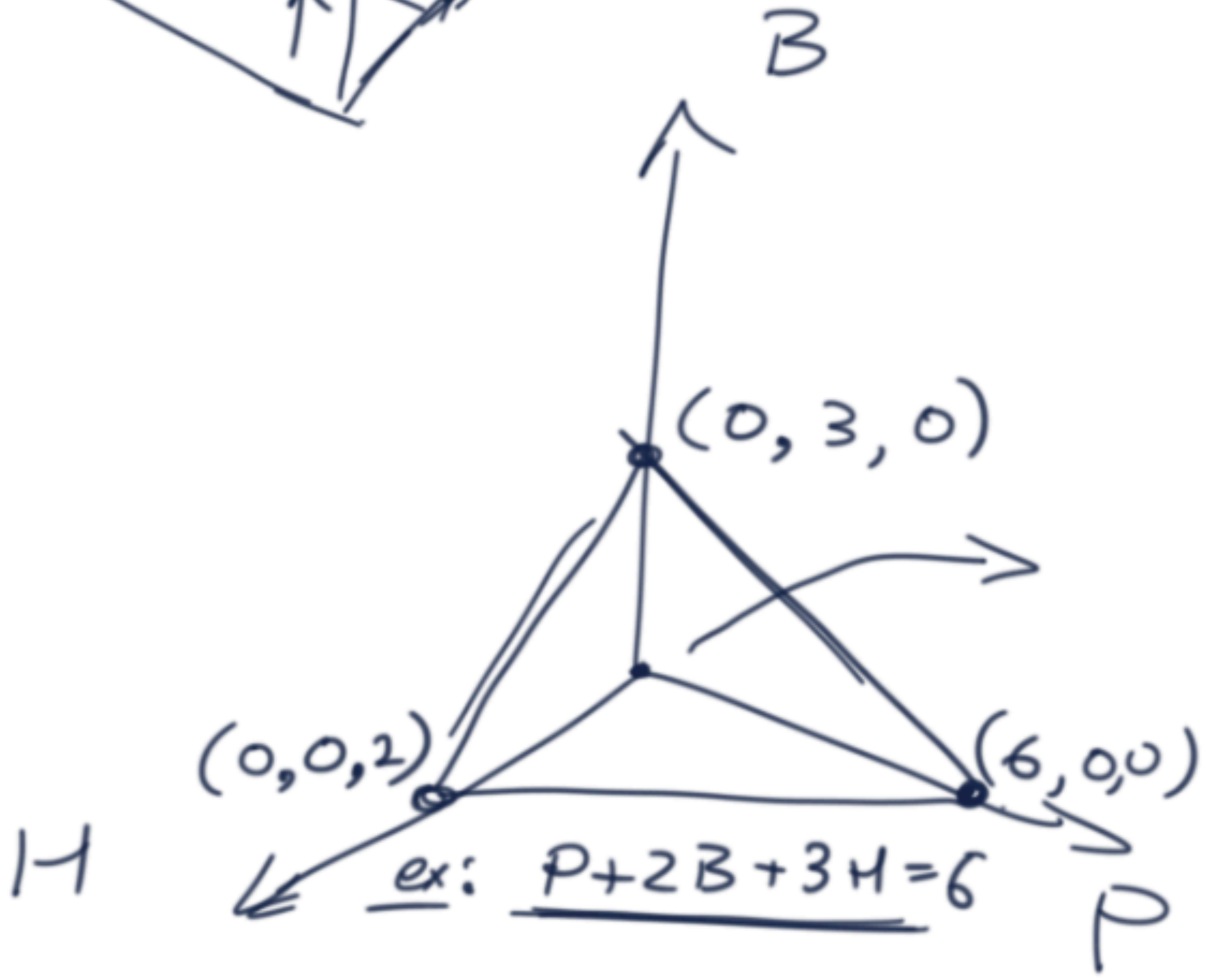
$P \leq 400$

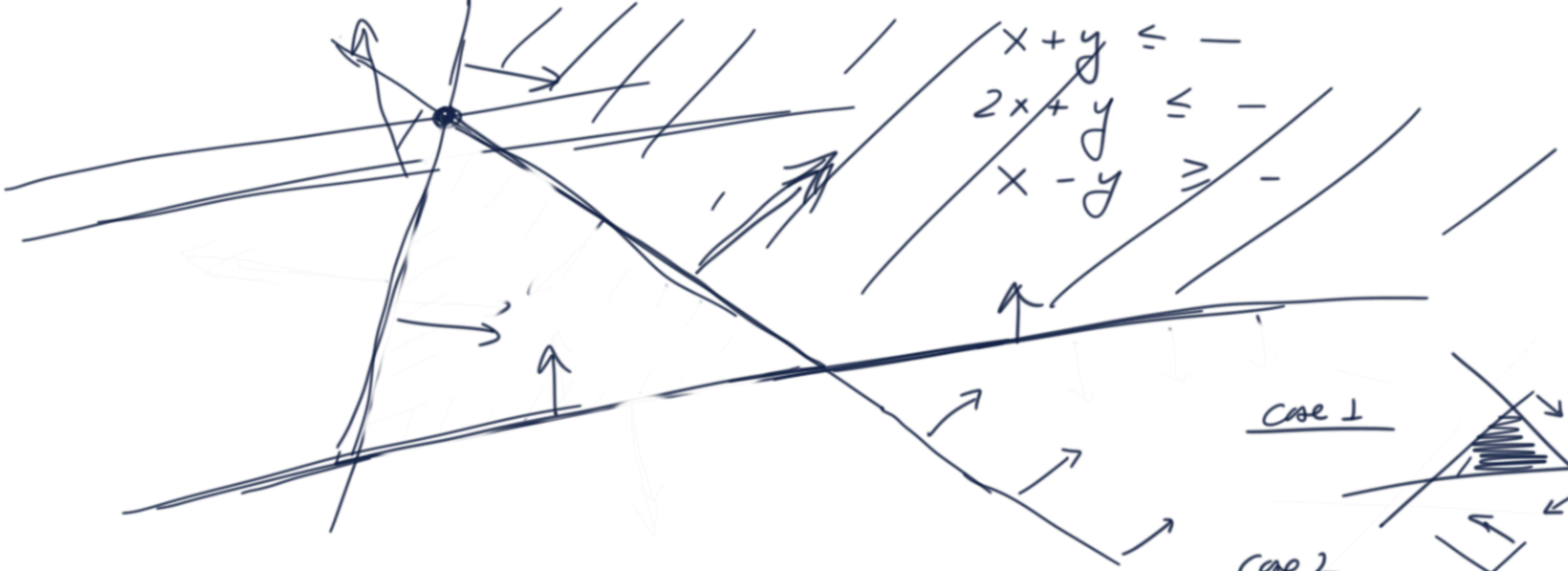
$B \leq 300$

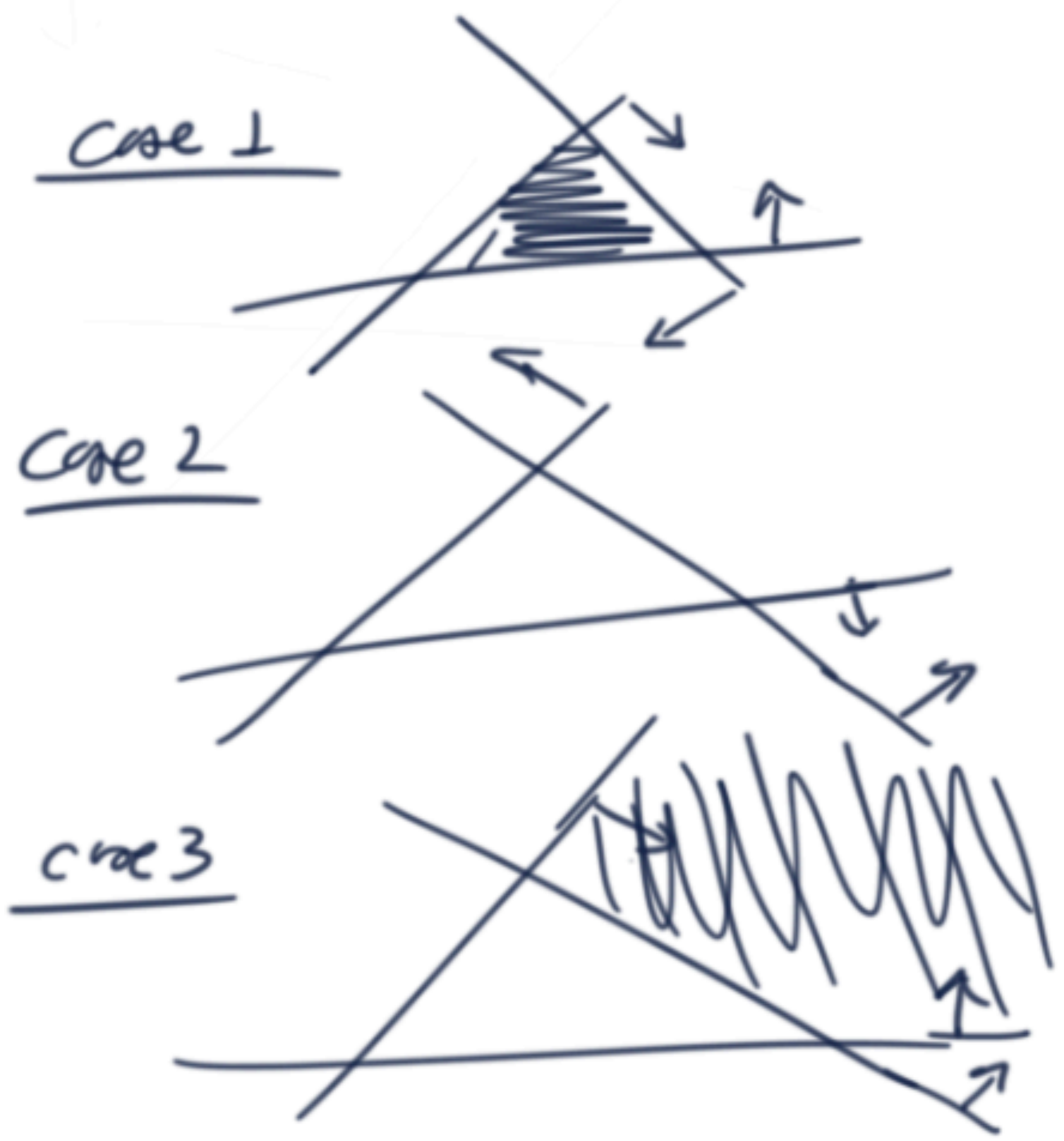
$2P + 3B \leq 1200$

$B + 2H \leq 400$





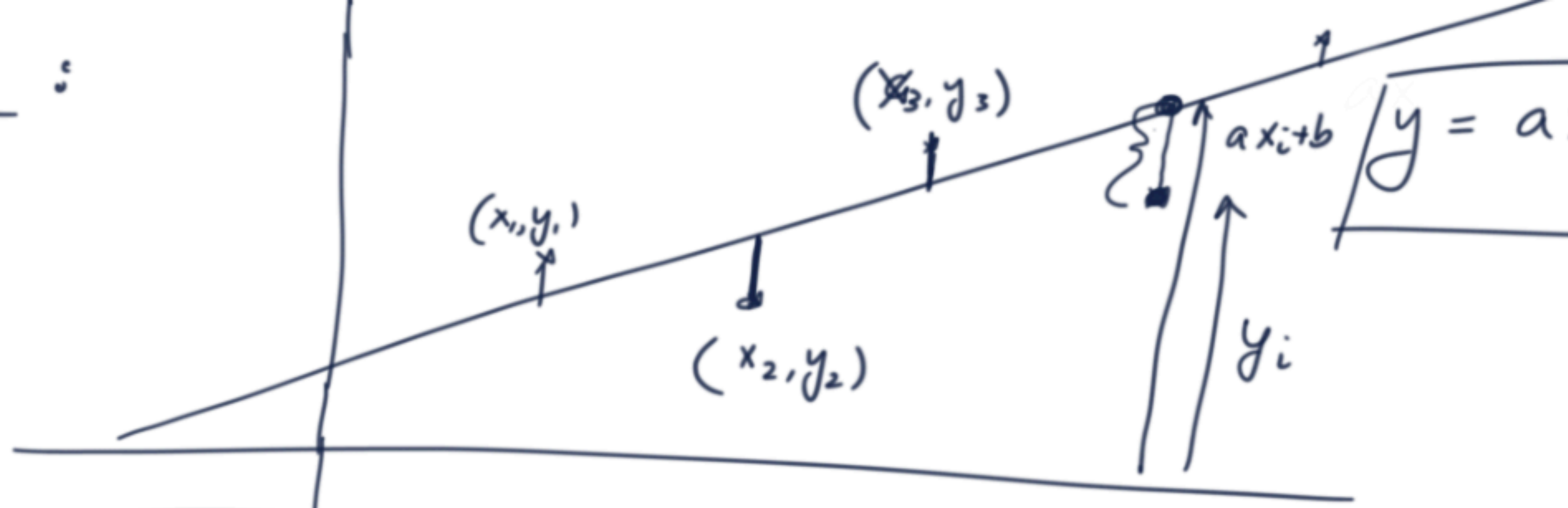




3 possibilities

1. optimal solution at a vertex
2. No feasible region
3. solution at ∞ (unbounded)

Line fitting :



example :

$(x_1,y_1) = (1,1)$

$(x_2,y_2) = (3,2)$

$(x_3,y_3) = (5,6)$

Pick a, b :

$\max\limits_i |y_i - (a x_i + b)|$ as small as possible. $= e$

$$\min_{\underline{a,b}} \max_i |y_i - (a x_i + b)|$$

3 variables : a, b, e

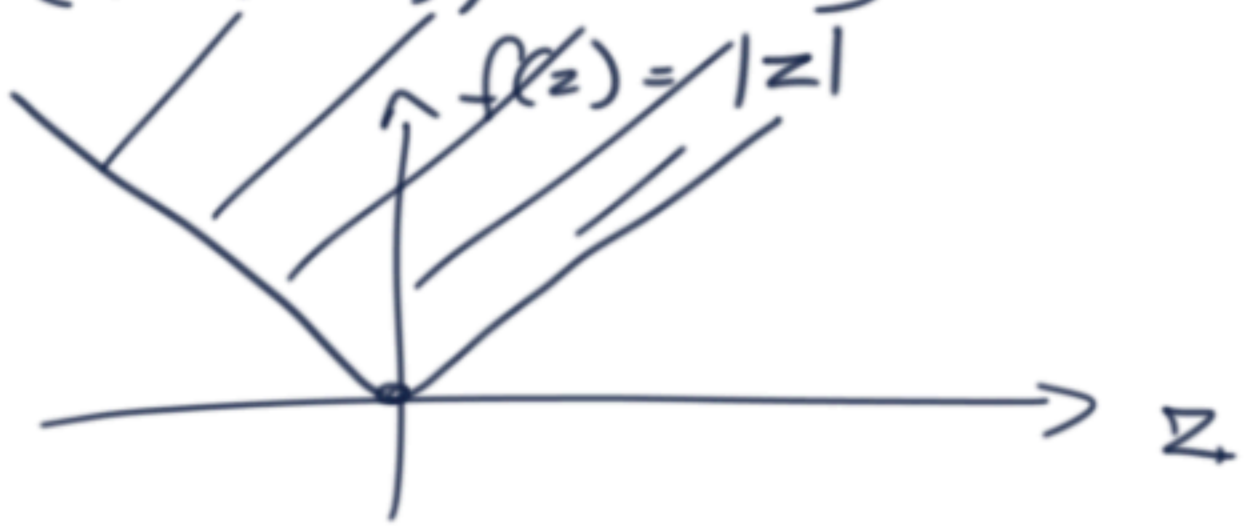
(x_i, y_i) are constant specified as input to problem.

2 constraints for each (x_i, y_i)

$\therefore$ LP has 3 vars, $2n$ constraints

$$\min \ e$$

$$\forall i \quad \left.\begin{array}{l} y_i - (a x_i + b) \le e \\ -(y_i - (a x_i + b)) \le e \end{array}\right\} \sim \quad e \ge |y_i - (a x_i + b)|$$



$e \ge |z| \leftarrow$ enforce

$\underline{e \ge z}$ & $\underline{e \ge -z}$

$z = 5$ $e \ge 5$ & $e \ge -5$

$z = -5$ $e \ge -5$ & $e \ge -(-5) = 5$