

To Draw a curve:

For  $u = u_{\text{beg}}; u < u_{\text{end}}; u += \Delta u$   
 drawline( $x(u), x(\text{Min}(u_{\text{end}}, u + \Delta u))$ );

Simple but not so great:

$$\text{let } \Delta x = x(u + \Delta u) - x(u)$$

if  $|\Delta x|$  too small  $\rightarrow$  wasted effort  
 if  $|\Delta x|$  too large  $\rightarrow$  curve looks bad

Desired value for  $|\Delta x|$  depends on application

eg rendering - measure error in screen space

recall inv under Proj. x F. ?

Manufacturing - given error tolerance

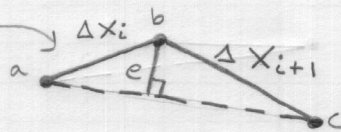
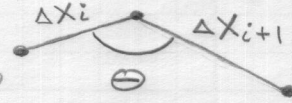
Adaptive: Pick  $\Delta u$  so that  $\Delta x$  "is good"

eg  $|\Delta x| \sim 1 \text{ pixel}$

or  $\theta \sim 180 \text{ deg}$

or  $|e| \leq 1 \text{ pixel}$

or  $e = b - (a+c)/2$



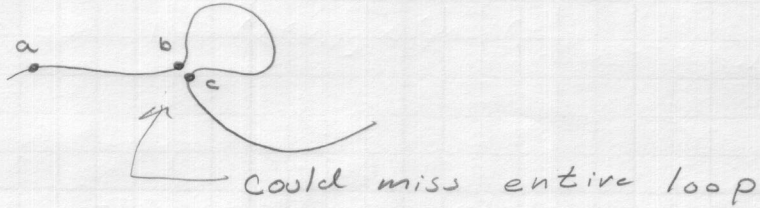
Good metric depends on application

& Cost of Metric

⊗  $\Delta u$  varies over curve based on metric

$\rightarrow$  What do these approximate?

Could still have problems :



Bounding boxes -- small or flat → use a line

To draw a surface

For  $u = u_{\text{beg}} ; u < u_{\text{end}} ; u += \Delta u$

For  $v = v_{\text{beg}} ; v < v_{\text{end}} ; v += \Delta v$

draw Quad (  $x(u, v), x(u,$

$x(\min(u_{\text{end}}, u + \Delta u), v),$

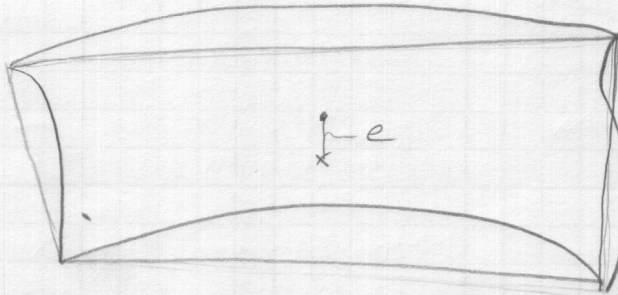
$x(u, \min(v_{\text{end}}, v + \Delta v)),$

$x(\min(u_{\text{end}}, u + \Delta u), \min(v_{\text{end}}, v + \Delta v)) ) ;$

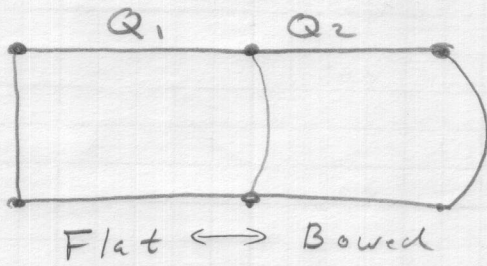
\* Quads may not be flat

Again, simple but not so good...

→ Need to adjust  $\Delta u$  &  $\Delta v$  to get good approx to surface.



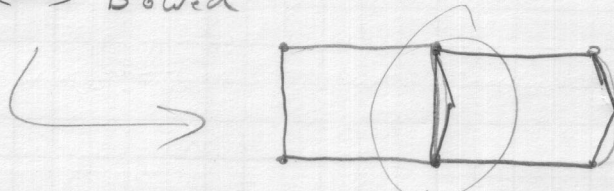
# Problem: "Cracking"



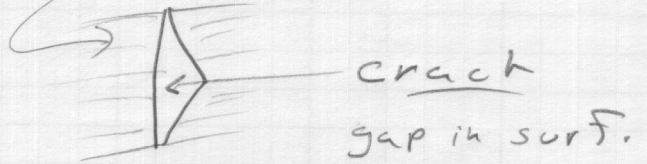
Consider:

Q<sub>1</sub> Passes

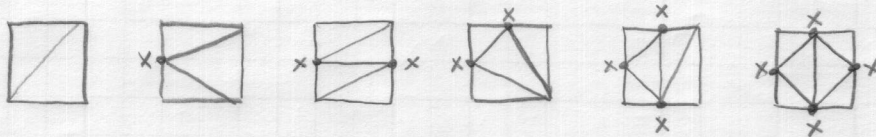
Q<sub>2</sub> Fails



③ What is this a problem?  
Is it a problem?



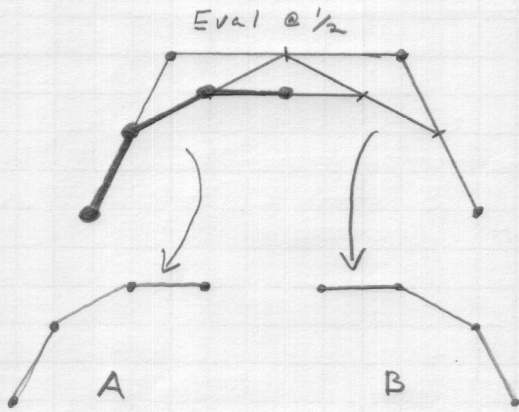
One way to avoid cracks -- split edges (& triangulate)



\* Note, you may also care about the aspect ratio of the triangles.

## Bézier Sub Division

- Based on De Casteljau Eval.



Split curve into two new Bézier curves

At some point control poly is nearly flat or small  
 $\rightarrow$  then just draw control polygon

Express as a Matrix:

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 \\ \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} \end{bmatrix} \cdot \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix}$$

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix} = \begin{bmatrix} \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} \\ 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix}$$