

CS 184: Problems and Questions on Rendering

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Problems

1. Define the terms Radiance and Irradiance, and give the units for each. Write down the formula (integral) for irradiance at a point in terms of the illumination $L(\omega)$ incident from all directions ω . Write down the local reflectance equation, i.e. express the net reflected radiance in a given direction as an integral over the incident illumination.
2. Make appropriate approximations to derive the radiosity equation from the full rendering equation.
3. Match the surface material to the formula (and goniometric diagram shown in class). Also, give an example of a real material that reasonably closely approximates the mathematical description. Not all materials need have a corresponding diagram. The materials are *ideal mirror*, *dark glossy*, *ideal diffuse*, *retroreflective*. The formulae for the BRDF f_r are $k_a(\vec{R} \cdot \vec{V})$, $k_b(\vec{R} \cdot \vec{V})^4$, $k_c/(\vec{N} \cdot \vec{V})$, $k_d\delta(\vec{R})$, k_e .
4. Consider the Cornell Box (as in the radiosity lecture, assume for now that this is essentially a room with only the walls, ceiling and floor. Assume for now, there are no small boxes or other furniture in the room, and that all surfaces are Lambertian. The box also has a small rectangular white light source at the center of the ceiling.) Assume we make careful measurements of the light source intensity and dimensions of the room, as well as the material properties of the walls, floor and ceiling. We then use these as inputs to our simple OpenGL renderer. Assuming we have been completely accurate, will the computer-generated picture be identical to a photograph of the same scene from the same location? If so, why? If not, what will be the differences? Ignore gamma correction and other nonlinear transfer issues. Now, answer this question again with the two small boxes added, i.e. the floor has two smaller boxes sitting on it. You may assume we have accurately measured geometric and material properties of the smaller boxes also.
5. Consider a simplified skylight model, so the radiance along any direction is given by $A + B \sin \alpha$ where A and B are positive constants, and α is the elevation angle (i.e. the angle to the horizontal, being 0 degrees toward the horizontal and 90 degrees toward the zenith or top of the sky). That is, the radiance is more higher up in the sky. The lighting is isotropic; there is no variation with azimuthal angle (ϕ). Assume for this problem that there is no occlusion by trees, buildings etc., the sky hemisphere is the only source of illumination [no ground lighting, direct sunlight etc.], the surfaces are Lambertian with albedo 1, and the sky can be assumed to be a distant source. What is the irradiance on the ground, assumed to be a horizontal surface?. Now, assume we have a sphere suspended (or on the ground, if that makes things more logical for you). Remember the assumptions, i.e. lighting only from the (distant) sky, no occlusions etc. Which point on the sphere will be brightest? What will be the reflected radiance at this point? Which point will be the dimmest? What will be the reflected radiance at that point? Qualitatively, how will the brightness on the sphere vary as a function of location (parameterized by spherical coordinates for instance)? Extra credit for deriving an analytic quantitative formula for the brightness of the sphere as a function of the surface normal.

Answers

1. Radiance and Irradiance For the final, you should be prepared to know basic definitions and units.

- *Radiance* is the power per unit projected area perpendicular to the ray per unit solid angle in the direction of the ray. Its units are $Wm^{-2}sr^{-1}$.
- *Irradiance* is the radiant power per unit area. Its units are Wm^{-2} .

The irradiance is a cosine-weighted integral of the incoming radiance over the visible hemisphere,

$$E = \int L(\omega)(\omega \cdot n) d\omega, \quad (1)$$

where L is the radiance, E is the irradiance, n is the surface normal, and the integral is over the hemisphere. If we use spherical angles θ and ϕ ,

$$E = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} L(\theta, \phi) \cos \theta \sin \theta d\theta d\phi, \quad (2)$$

where the $\sin \theta d\theta d\phi$ is the solid angle measure.

For the reflectance equation, we must also consider the BRDF f of the surface, that is a function of incident and outgoing directions,

$$L_r(\omega_r) = \int_{\Omega} L_i(\omega_i) f(\omega_i, \omega_r) (\omega_i \cdot n) d\omega_i, \quad (3)$$

where the terms are respectively the reflected radiance, the incident radiance, BRDF and cosine term.

2. Radiosity Equation We first generalize the reflectance equation above to the rendering equation,

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} L_i(x, \omega_i) f(x, \omega_i, \omega_r) (\omega_i \cdot n) d\omega_i, \quad (4)$$

where we've added the spatial coordinate and an emission term. Switching to the area form, we instead get

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\text{Surfaces } x'} L_r(x', -\omega_i) f(x, \omega_i, \omega_r) G(x, x') V(x, x') dA(x'), \quad (5)$$

where we integrate over an area measure on x' and G and V are geometry and visibility terms respectively.

For radiosity from diffuse surfaces, we drop the angular dependence,

$$L_r(x) = L_e(x) + f(x) \int_{\text{Surfaces } x'} L_r(x') G(x, x') V(x, x') dA(x'), \quad (6)$$

where the BRDF is also taken outside the integral. Now, switching radiance to radiosity B (corresponding to outgoing irradiance), and replacing the BRDF by $f = \rho/\pi$ where ρ is the albedo in the range from 0 to 1, we derive the radiosity equation,

$$B(x) = E(x) + \rho(x) \int_S L_r(x') G(x, x') V(x, x') dA(x'). \quad (7)$$

Note that $E(x)$ here is the emission (and not irradiance), and we integrate over surfaces S ,

3. Materials An ideal mirror is something like a normal reflective mirror, and the BRDF corresponds to $k_d \delta(\vec{R})$. A dark glossy surface is close to glossy plastic with a formula like $k_b (\vec{R} \cdot \vec{V})^4$. An ideal diffuse surface is Lambertian, close to wall paint or ideally, spectralon, with BRDF a constant k_e . A retroreflective surface is something like a highway reflector, that reflects light back toward the viewer and would have a formula using $\delta(\vec{L})$ instead of \vec{R} as in an ordinary reflector.

4. Cornell Box The missing feature will be global illumination (effects like color bleeding). Furthermore, area light sources are not supported in OpenGL, and approximating them with a point light will lead to other minor differences. After the smaller boxes are added, we will also not get shadowing effects in OpenGL.

5. Irradiance The irradiance is given by

$$E = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} L(\theta, \phi) \cos \theta \sin \theta d\theta, \quad (8)$$

where θ stands for the angle with respect to the zenith, or normal to the ground plane. In this coordinate frame, the elevation angle $= \pi/2 - \theta$, and so the incident radiance $L = A + B \cos \theta$. Removing the azimuthal dependence in the above integral, we obtain

$$E = 2\pi \int_0^{\pi/2} (A + B \cos \theta) \cos \theta \sin \theta d\theta. \quad (9)$$

To evaluate this integral, we put $u = \cos \theta$, to obtain

$$E = 2\pi \int_0^1 Au + Bu^2 du = \pi \left(A + \frac{2B}{3} \right). \quad (10)$$

The brightest point is clearly the top of the sphere, or the point facing directly upward, which receives the same irradiance as the ground. Since the albedo is 1, the BRDF is $1/\pi$ and the reflected radiance is $A + 2B/3$. The dimmest point will be at the bottom of the sphere, which will be completely dark (since it sees only the lower hemisphere that is dark, since we assume no interreflection from the ground). The radiance will decrease from top to bottom, remaining azimuthally symmetric.

Finding an analytic formula for the radiance as a function of angle is an advanced topic, well outside the scope of this course. One can try to use the spherical harmonic irradiance formula in my thesis.