Foundations of Computer Graphics (Spring 2012)

CS 184, Lecture 21: Radiometry

http://inst.eecs.berkeley.edu/~cs184

Many slides courtesy Pat Hanrahan

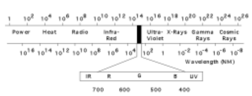
Overview

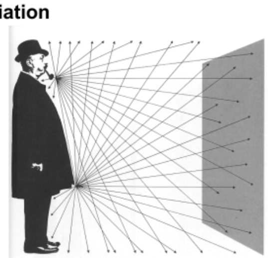
- Lighting and shading key in computer graphics
- HW 2 etc. ad-hoc shading models, no units
- Really, want to match physical light reflection
- Next 3 lectures look at this formally
- Today: physical measurement of light: radiometry
- Formal reflection equation, reflectance models
- Global Illumination (later)

Light

Visible electromagnetic radiation

Power spectrum





From London and Upton

Polarization

Photon (quantum effects)

Wave (interference, diffraction)

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Radiometry and Photometry

- Physical measurement of electromagnetic energy
- *Measure spatial (and angular) properties of light*
  - *Radiant Power*
  - Radiant Intensity
  - Irradiance
    - Inverse square and cosine law
  - Radiance
  - Radiant Exitance (Radiosity)
- Reflection functions: Bi-Directional Reflectance Distribution Function or BRDF
- Reflection Equation

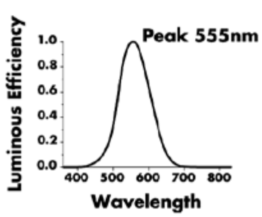
Radiant Energy and Power

Power: Watts (radiometry)

vs. Lumens (photometry)

Φ

- **Spectral efficacy**
- **Energy efficiency**



Energy: Joules vs. Talbot

- **Exposure**
  - **Film response**
  - **Skin - sunburn**

Luminance

$$Y = \int V(\lambda) L(\lambda) d\lambda$$

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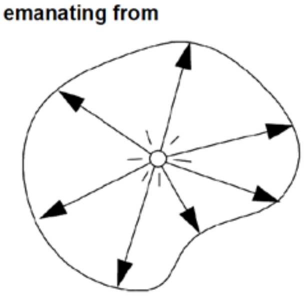
Radiant Intensity

Definition: The *radiant* (*luminous*) *intensity* is the power per unit solid angle emanating from a point source.

$$I(\omega) \equiv \frac{d\Phi}{d\omega}$$



$$\left[\frac{W}{sr}\right]\left[\frac{lm}{sr} = cd = candela\right]$$

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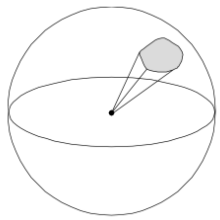
Angles and Solid Angles

- **Angle** $\theta = \frac{l}{r}$

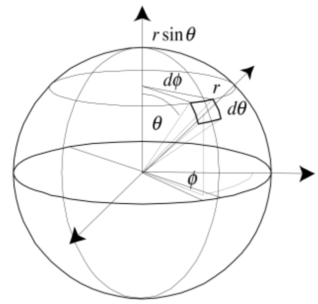
  $\Rightarrow$ circle has 2π radians

- **Solid angle** $\Omega = \frac{A}{R^2}$

  $\Rightarrow$ sphere has 4π steradians

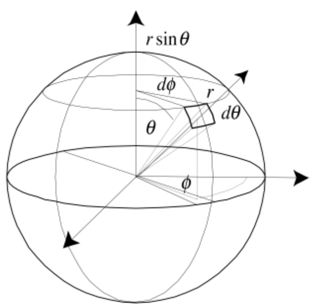


Differential Solid Angles



$$dA = (r\,d\theta)(r\sin\theta\,d\phi)$$
$$= r^2 \sin\theta\,d\theta\,d\phi$$

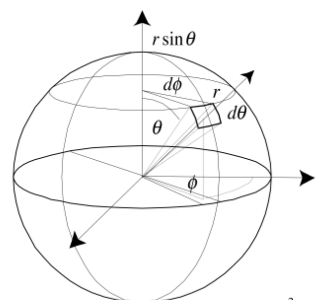
Differential Solid Angles



$$dA = (r\,d\theta)(r\sin\theta\,d\phi)$$
$$= r^2 \sin\theta\,d\theta\,d\phi$$

$$d\omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$$

Differential Solid Angles

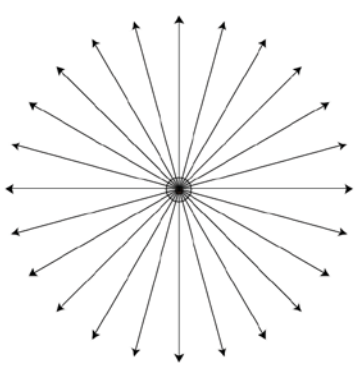


$$d\omega = \sin\theta\,d\theta\,d\phi$$

$$\Omega = \int_{S^2} d\omega$$
$$= \int_0^{\pi}\int_0^{2\pi} \sin\theta\,d\theta\,d\phi$$
$$= \int_{-1}^{1}\int_0^{2\pi} d\cos\theta\;d\phi$$
$$= 4\pi$$

Sphere S^2

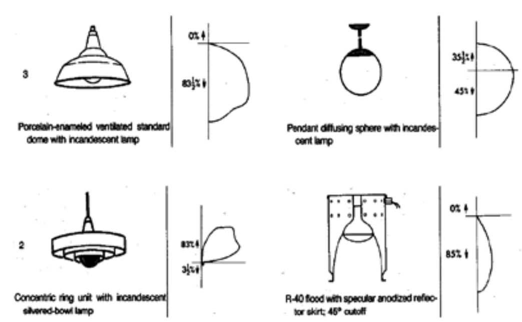
Isotropic Point Source



$$\Phi = \int_{S^2} I\,d\omega$$
$$= 4\pi I$$

$$I = \frac{\Phi}{4\pi}$$

Light Source Goniometric Diagrams



Radiometry and Photometry

- Physical measurement of electromagnetic energy
- Measure spatial (and angular) properties of light
  - Radiant Power
  - Radiant Intensity
  - *Irradiance*
    - *Inverse square and cosine law*
  - Radiance
  - Radiant Exitance (Radiosity)
- Reflection functions: Bi-Directional Reflectance Distribution Function or BRDF
- Reflection Equation

Irradiance

Definition: The *irradiance* (*illuminance*) is the power per unit area incident on a surface.

$$E(x) \equiv \frac{d\Phi_i}{dA}$$

$$\left[\frac{W}{m^2}\right] \left[\frac{lm}{m^2} = lux\right]$$

Sometimes referred to as the radiant (luminous) incidence.

Beam Power in Terms of Irradiance

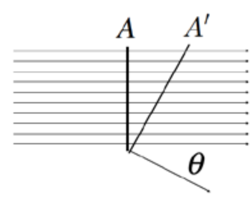
$$\Phi = EA$$

$$E = \frac{\Phi}{A}$$

Lambert's Cosine Law

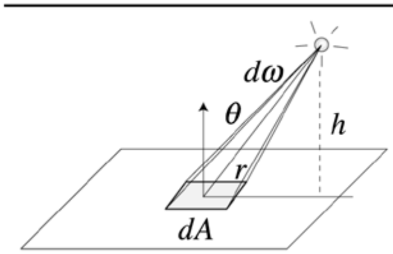
$$A = A' \cos\theta$$

$$\Phi = \Phi'$$



$$E' = \frac{\Phi'}{A'} = \frac{\Phi}{A}\cos\theta = E\cos\theta$$

Irradiance: Isotropic Point Source

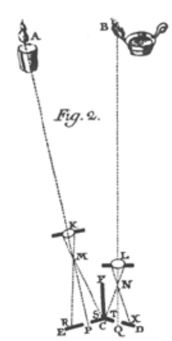


$$I = \frac{\Phi}{4\pi}$$

$$I\,d\omega = \frac{\Phi}{4\pi}\frac{\cos\theta}{r^2}dA = E\,dA$$

$$E = \frac{\Phi}{4\pi}\frac{\cos\theta}{r^2}$$

The Invention of Photometry



Bouguer's classic experiment

- **Compare a light source and a candle**
- **Move until they both appear equally bright**
- **Intensity is proportional to ratio of distances squared**

Definition of a candela

- **Originally a "standard" candle**
- **Currently 550 nm laser w/ 1/683 W/sr**
- **1 of 6 fundamental SI units**

Typical Values of Illuminance [lm/m²]

Sunlight plus skylight	**100,000 lux**
Sunlight plus skylight (overcast)	**10,000**
Interior near window (daylight)	**1,000**
Artificial light (minimum)	**100**
Moonlight (full)	**0.02**
Starlight	**0.0003**

Radiometry and Photometry

- Physical measurement of electromagnetic energy
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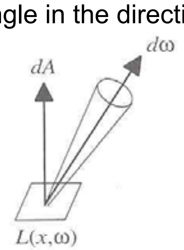
  - Radiant Power
  - Radiant Intensity
  - Irradiance
    - Inverse square and cosine law
  - ***Radiance***
  - Radiant Exitance (Radiosity)
- Reflection functions: Bi-Directional Reflectance Distribution Function or BRDF
- Reflection Equation

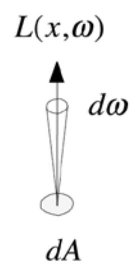
Radiance

- Power per unit projected area perpendicular to the ray per unit solid angle in the direction of the ray
- Symbol: $L(x,\omega)$ ($W/m^2\ sr$)
- Flux given by
  $d\Phi = L(x,\omega)\cos\theta\, d\omega\, dA$



Area Lights – Surface Radiance

Definition: The surface *radiance* (*luminance*) is the intensity per unit area leaving a surface



$$L(x,\omega) \equiv \frac{dI(x,\omega)}{dA} = \frac{d^2\Phi(x,\omega)}{d\omega dA}$$

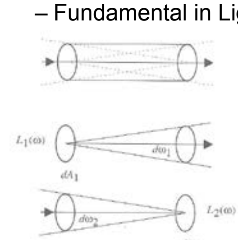
$$\left[\frac{W}{sr\,m^2}\right]\left[\frac{cd}{m^2} = \frac{lm}{sr\,m^2} = nit\right]$$

Typical Values of Luminance [cd/m²]

Surface of the sun	**2,000,000,000 nit**
Sunlight clouds	**30,000**
Clear sky	**3,000**
Overcast sky	**300**
Moon	**0.03**

Radiance properties

- Radiance constant as propagates along ray
  - Derived from conservation of flux
  - Fundamental in Light Transport.



$$d\Phi_1 = L_1 d\omega_1 dA_1 = L_2 d\omega_2 dA_2 = d\Phi_2$$

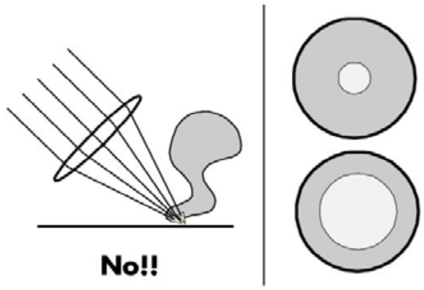
$$d\omega_1 = dA_2/r^2 \qquad d\omega_2 = dA_1/r^2$$

$$d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2$$

$$\therefore L_1 = L_2$$

Quiz

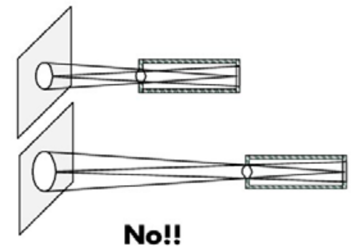
Does radiance increase under a magnifying glass?



No!!

Quiz

Does the brightness that a wall appears to the eye depend on the distance of the viewer to the wall?



No!!

Radiance properties

- Sensor response proportional to radiance (constant of proportionality is throughput)
  - Far surface: See more, but subtend smaller angle
  - Wall equally bright across viewing distances

Consequences
  - Radiance associated with rays in a ray tracer
  - Other radiometric quants derived from radiance

Irradiance, Radiosity

- Irradiance E is radiant power per unit area
- Integrate incoming radiance over hemisphere
  - Projected solid angle ($\cos\theta \, d\omega$)
  - Uniform illumination: Irradiance = π [CW 24,25]
  - Units: W/m^2
- Radiant Exitance (radiosity)
  - Power per unit area leaving surface (like irradiance)

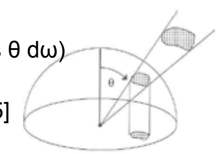


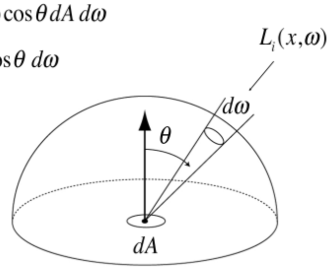
Figure 2.8: Projection of differential area.

Irradiance from the Environment

$$d^2\Phi_i(x,\omega) = L_i(x,\omega)\cos\theta \, dA \, d\omega$$

$$dE(x,\omega) = L_i(x,\omega)\cos\theta \, d\omega$$

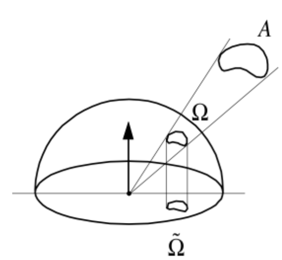
Light meter



$$E(x) = \int_{H^2} L_i(x,\omega)\cos\theta \, d\omega$$

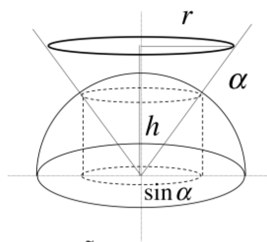
Uniform Area Source

$$\begin{aligned} E(x) &= \int_{H^2} L\cos\theta \, d\omega \\ &= L\int_{\Omega} \cos\theta \, d\omega \\ &= L\tilde{\Omega} \end{aligned}$$



Uniform Disk Source

Geometric Derivation



$$\tilde{\Omega} = \pi \sin^2 \alpha$$

Algebraic Derivation

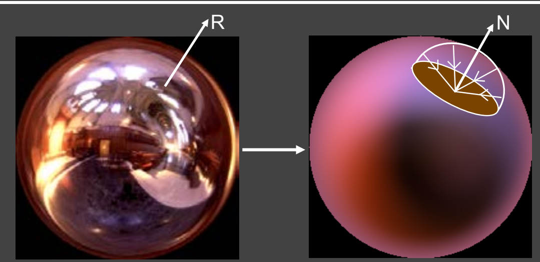
$$\tilde{\Omega} = \int_1^{\cos\alpha} \int_0^{2\pi} \cos\theta \, d\phi \, d\cos\theta$$

$$= 2\pi \left. \frac{\cos^2\theta}{2} \right|_1^{\cos\alpha}$$

$$= \pi \sin^2 \alpha$$

$$= \pi \frac{r^2}{r^2 + h^2}$$

Irradiance Environment Maps



Incident Radiance (Illumination Environment Map)

Irradiance Environment Map

Radiant Exitance

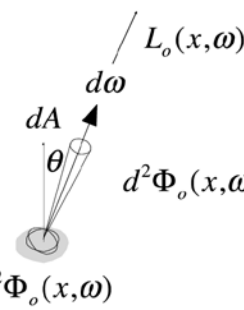
Definition: The *radiant* (*luminous*) *exitance* is the energy per unit area leaving a surface.

$$M(x) \equiv \frac{d\Phi_o}{dA}$$

$$\left[\frac{W}{m^2} \right] \left[\frac{lm}{m^2} = lux \right]$$

In computer graphics, this quantity is often referred to as the *radiosity (B)*

Directional Power Leaving a Surface



$$d^2\Phi_o(x,\omega) = L_o(x,\omega) \cos\theta \, dA \, d\omega$$

Note cosine

Radiometry and Photometry

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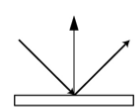
- *Reflection functions: Bi-Directional Reflectance Distribution Function or BRDF*
- Reflection Equation

Types of Reflection Functions

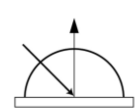
Ideal Specular

- **Reflection Law**
- **Mirror**



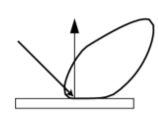
Ideal Diffuse

- **Lambert's Law**
- **Matte**



Specular

- **Glossy**
- **Directional diffuse**



Materials

From Apodaca and Gritz, *Advanced RenderMan*

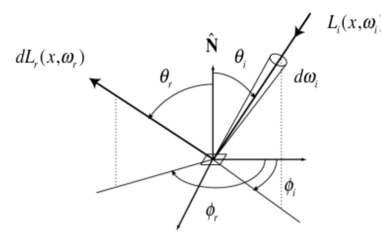
Spheres [Matusik et al.]

Building up the BRDF

- Bi-Directional Reflectance Distribution Function [Nicodemus 77]
- Function based on incident, view direction
- Relates incoming light energy to outgoing
- Unifying framework for many materials

The BRDF

Bidirectional Reflectance-Distribution Function



$$f_r(\omega_i \to \omega_r) \equiv \frac{dL_r(\omega_i \to \omega_r)}{dE_i} \quad \left[\frac{1}{sr}\right]$$

BRDF

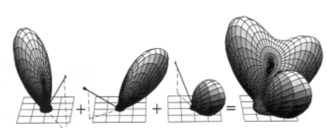
- Reflected Radiance proportional Irradiance
- Constant proportionality: BRDF
- Ratio of outgoing light (radiance) to incoming light (irradiance)
  - Bidirectional Reflection Distribution Function
  - (4 Vars) units 1/sr

$$f(\omega_i,\omega_r) = \frac{L_r(\omega_r)}{L_i(\omega_i)\cos\theta_i d\omega_i}$$

$$L_r(\omega_r) = L_i(\omega_i) f(\omega_i,\omega_r)\cos\theta_i d\omega_i$$

Properties of BRDF's

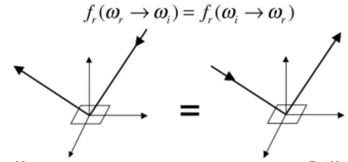
1. Linearity



From Sillion, Arvo, Westin, Greenberg

2. Reciprocity principle

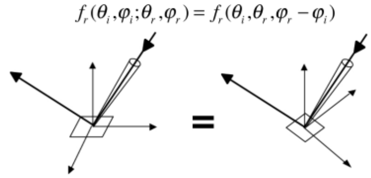
$$f_r(\omega_r \to \omega_i) = f_r(\omega_i \to \omega_r)$$



Properties of BRDF's

3. Isotropic vs. anisotropic

$$f_r(\theta_i,\varphi_i;\theta_r,\varphi_r) = f_r(\theta_i,\theta_r,\varphi_r-\varphi_i)$$



Reciprocity *and* isotropy

$$f_r(\theta_i,\theta_r,\varphi_r-\varphi_i) = f_r(\theta_r,\theta_i,\varphi_i-\varphi_r) = f_r(\theta_i,\theta_r,|\varphi_r-\varphi_i|)$$

4. Energy conservation

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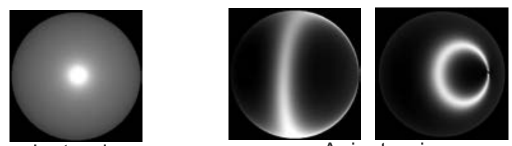
Isotropic vs Anisotropic

- Isotropic: Most materials (you can rotate about normal without changing reflections)
- Anisotropic: brushed metal etc. preferred tangential direction



Isotropic — Anisotropic

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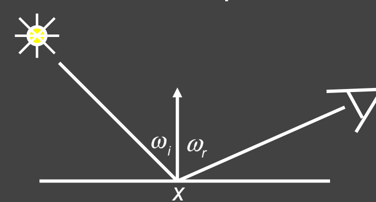
- Reflection functions: Bi-Directional Reflectance Distribution Function or BRDF
- *Reflection Equation (and simple BRDF models)*

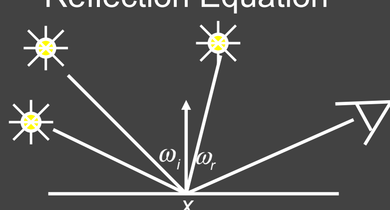
Reflection Equation



$$L_r(x,\omega_r) = L_e(x,\omega_r) + L_i(x,\omega_i) f(x,\omega_i,\omega_r)(\omega_i \bullet n)$$

Reflected Light (Output Image)	Emission	Incident Light (from light source)	BRDF	Cosine of Incident angle

Reflection Equation

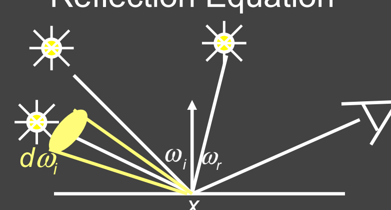


Sum over all light sources

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \sum L_i(x,\omega_i) f(x,\omega_i,\omega_r)(\omega_i \bullet n)$$

Reflected Light (Output Image)	Emission	Incident Light (from light source)	BRDF	Cosine of Incident angle

Reflection Equation



Replace sum with integral

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_i(x,\omega_i) f(x,\omega_i,\omega_r) \cos\theta_i \, d\omega_i$$

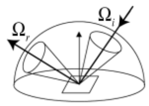
Reflected Light (Output Image)	Emission	Incident Light (from light source)	BRDF	Cosine of Incident angle

Energy Conservation

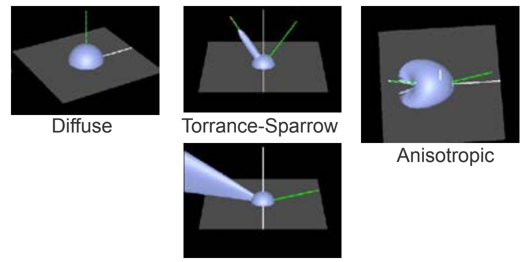
$$\frac{d\Phi_r}{d\Phi_i} = \frac{\int_{\Omega_r} L_r(\omega_r)\cos\theta_r\, d\omega_r}{\int_{\Omega_i} L_i(\omega_i)\cos\theta_i\, d\omega_i}$$

$$= \frac{\int_{\Omega_r}\int_{\Omega_i} f_r(\omega_i \to \omega_r) L_i(\omega_i)\cos\theta_i\, d\omega_i \cos\theta_r\, d\omega_r}{\int_{\Omega_i} L_i(\omega_i)\cos\theta_i\, d\omega_i}$$

$$\leq 1$$



BRDF Viewer plots



bv written by Szymon Rusinkiewicz

Ideal Diffuse Reflection

Assume light is equally likely to be reflected in any output direction (independent of input direction).

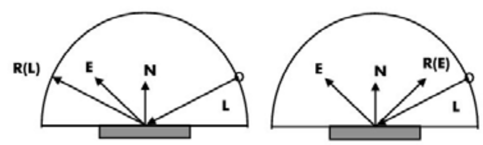
$$L_{r,d}(\omega_r) = \int f_{r,d} L_i(\omega_i)\cos\theta_i\, d\omega_i$$
$$= f_{r,d}\int L_i(\omega_i)\cos\theta_i\, d\omega_i$$
$$= f_{r,d} E$$

$$M = \int L_r(\omega_r)\cos\theta_r\, d\omega_r = L_r\int\cos\theta_r\, d\omega_r = \pi L_r$$

$$\rho_d = \frac{M}{E} = \frac{\pi L_r}{E} = \frac{\pi f_{r,d} E}{E} = \pi f_{r,d} \quad\Rightarrow\quad f_{r,d} = \frac{\rho_d}{\pi}$$

Lambert's Cosine Law $M = \rho_d E = \rho_d E_s \cos\theta_s$

Phong Model



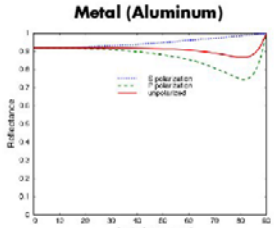
$(\hat{\mathbf{E}}\bullet\mathbf{R}(\hat{\mathbf{L}}))^s$ $(\hat{\mathbf{L}}\bullet\mathbf{R}(\hat{\mathbf{E}}))^s$

Reciprocity: $(\hat{\mathbf{E}}\bullet\mathbf{R}(\hat{\mathbf{L}}))^s = (\hat{\mathbf{L}}\bullet\mathbf{R}(\hat{\mathbf{E}}))^s$

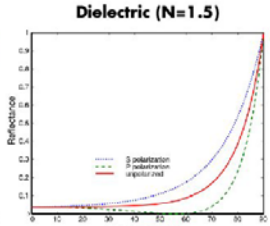
Distributed light source!

Fresnel Reflectance

Metal (Aluminum)



Dielectric (N=1.5)



Gold F(0)=0.82
Silver F(0)=0.95

Glass n=1.5 F(0)=0.04
Diamond n=2.4 F(0)=0.15

Schlick Approximation $F(\theta) = F(0) + (1 - F(0))(1-\cos\theta)^5$

Experiment

Reflections from a shiny floor

From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97

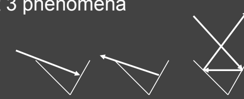
Analytical BRDF: TS example

- One famous analytically derived BRDF is the Torrance-Sparrow model
- T-S is used to model specular surface, like Phong
  - more accurate than Phong
  - has more parameters that can be set to match different materials
  - derived based on assumptions of underlying geometry. (instead of ‘because it works well’)

Torrance-Sparrow

- Assume the surface is made up grooves at microscopic level.

- Assume the faces of these grooves (called microfacets) are perfect reflectors.
- Take into account 3 phenomena



Torrance-Sparrow Result

Fresnel term: allows for wavelength dependency

Geometric Attenuation: reduces the output based on the amount of shadowing or masking that occurs.

$$f = \frac{F(\theta_i)G(\omega_i,\omega_r)D(\theta_h)}{4\cos(\theta_i)\cos(\theta_r)}$$

Distribution: distribution function determines what percentage of microfacets are oriented to reflect in the viewer direction.

How much of the macroscopic surface is visible to the light source

How much of the macroscopic surface is visible to the viewer

Other BRDF models

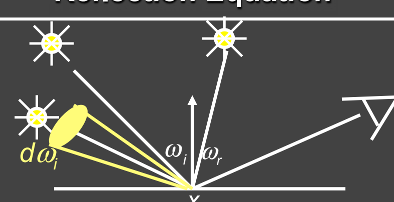
- Empirical: Measure and build a 4D table
- Anisotropic models for hair, brushed steel
- Cartoon shaders, funky BRDFs
- Capturing spatial variation
- Very active area of research

Environment Maps

- Light as a function of direction, from entire environment
- Captured by photographing a chrome steel or mirror sphere
- Accurate only for one point, but distant lighting same at other scene locations (typically use only one env. map)

Blinn and Newell 1976, Miller and Hoffman, 1984
Later, Greene 86, Cabral et al. 87

Reflection Equation



Replace sum with integral

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_\Omega L_i(x,\omega_i) f(x,\omega_i,\omega_r)\cos\theta_i d\omega_i$$

Reflected Light (Output Image) | Emission | *Environment Map (continuous)* | BRDF | Cosine of Incident angle

Environment Maps

- Environment maps widely used as lighting representation
- Many modern methods deal with offline and real-time rendering with environment maps
- Image-based complex lighting + complex BRDFs

Demo