## CS188 Fall 2018 Section 2: Graph Search + CSPs

## 1 Search and Heuristics

Imagine a car-like agent wishes to exit a maze like the one shown below:


The agent is directional and at all times faces some direction $d \in(N, S, E, W)$. With a single action, the agent can either move forward at an adjustable velocity $v$ or turn. The turning actions are left and right, which change the agent's direction by 90 degrees. Turning is only permitted when the velocity is zero (and leaves it at zero). The moving actions are fast and slow. Fast increments the velocity by 1 and slow decrements the velocity by 1 ; in both cases the agent then moves a number of squares equal to its NEW adjusted velocity. Any action that would result in a collision with a wall crashes the agent and is illegal. Any action that would reduce $v$ below 0 or above a maximum speed $V_{\max }$ is also illegal. The agent's goal is to find a plan which parks it (stationary) on the exit square using as few actions (time steps) as possible.

As an example: if the agent shown were initially stationary, it might first turn to the east using (right), then move one square east using fast, then two more squares east using fast again. The agent will of course have to slow to turn.

1. If the grid is $M$ by $N$, what is the size of the state space? Justify your answer. You should assume that all configurations are reachable from the start state.
2. Is the Manhattan distance from the agent's location to the exit's location admissible? Why or why not?
3. State and justify a non-trivial admissible heuristic for this problem which is not the Manhattan distance to the exit.
4. If we used an inadmissible heuristic in A* graph search, would the search be complete? Would it be optimal?
5. If we used an admissible heuristic in A* graph search, is it guaranteed to return an optimal solution? What if the heuristic was consistent?
6. Give a general advantage that an inadmissible heuristic might have over an admissible one.

## 2 Course Scheduling

You are in charge of scheduling for computer science classes that meet Mondays, Wednesdays and Fridays. There are 5 classes that meet on these days and 3 professors who will be teaching these classes. You are constrained by the fact that each professor can only teach one class at a time.

The classes are:

1. Class 1 - Intro to Programming: meets from 8:00-9:00am
2. Class 2 - Intro to Artificial Intelligence: meets from 8:30-9:30am
3. Class 3 - Natural Language Processing: meets from 9:00-10:00am
4. Class 4 - Computer Vision: meets from 9:00-10:00am
5. Class 5 - Machine Learning: meets from 10:30-11:30am

The professors are:

1. Professor A, who is qualified to teach Classes 1,2 , and 5 .
2. Professor B, who is qualified to teach Classes 3,4 , and 5 .
3. Professor C, who is qualified to teach Classes 1,3 , and 4.
4. Formulate this problem as a CSP problem in which there is one variable per class, stating the domains (after enforcing unary constraints), and binary constraints. Constraints should be specified formally and precisely, but may be implicit rather than explicit.
5. Draw the constraint graph associated with your CSP.

## 3 (Optional) CSPs: Trapped Pacman

Pacman is trapped! He is surrounded by mysterious corridors, each of which leads to either a pit (P), a ghost (G), or an exit (E). In order to escape, he needs to figure out which corridors, if any, lead to an exit and freedom, rather than the certain doom of a pit or a ghost.

The one sign of what lies behind the corridors is the wind: a pit produces a strong breeze $(\mathrm{S})$ and an exit produces a weak breeze (W), while a ghost doesn't produce any breeze at all. Unfortuantely, Pacman cannot measure the the strength of the breeze at a specific corridor. Instead, he can stand between two adjacent corridors and feel the max of the two breezes. For example, if he stands between a pit and an exit he will sense a strong ( S ) breeze, while if he stands between an exit and a ghost, he will sense a weak (W) breeze. The measurements for all intersections are shown in the figure below.

Also, while the total number of exits might be zero, one, or more, Pacman knows that two neighboring squares will not both be exits.


Pacman models this problem using variables $X_{i}$ for each corridor $i$ and domains $\mathrm{P}, \mathrm{G}$, and E.

1. State the binary and/or unary constraints for this CSP (either implicitly or explicitly).
2. Cross out the values from the domains of the variables that will be deleted in enforcing arc consistency.

| $X_{1}$ | P | G | E |
| :---: | :---: | :---: | :---: |
| $X_{2}$ | P | G | E |
| $X_{3}$ | P | G | E |
| $X_{4}$ | P | G | E |
| $X_{5}$ | P | G | E |
| $X_{6}$ | P | G | E |

3. According to MRV, which variable or variables could the solver assign first?
4. Assume that Pacman knows that $X_{6}=G$. List all the solutions of this CSP or write none if no solutions exist.

The CSP described above has a circular structure with 6 variables. Now consider a CSP forming a circular structure that has $n$ variables $(n>2)$, as shown below. Also assume that the domain of each variable has cardinality $d$.

5. Explain precisely how to solve this general class of circle-structured CSPs efficiently (i.e. in time linear in the number of variables), using methods covered in class. Your answer should be at most two sentences.
6. 2 If standard backtracking search were run on a circle-structured graph, enforcing arc consistency at every step, what, if anything, can be said about the worst-case backtracking behavior (e.g. number of times the search could backtrack)?

