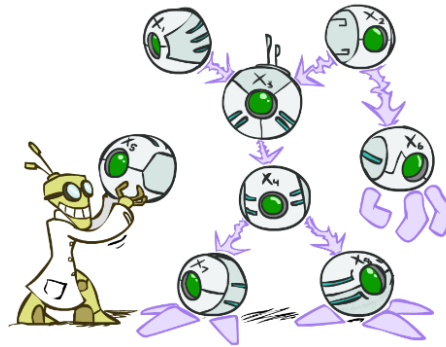


CS 188: Artificial Intelligence

Bayes' Nets



Instructors: Dan Klein and Pieter Abbeel --- University of California, Berkeley

[These slides were created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley. All CS188 materials are available at <http://ai.berkeley.edu>.]

Probabilistic Models

- Models describe how (a portion of) the world works

- Models are always simplifications**

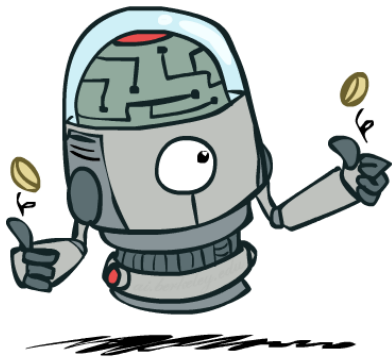
- May not account for every variable
- May not account for all interactions between variables
- "All models are wrong; but some are useful."
 - George E. P. Box



- What do we do with probabilistic models?**

- We (or our agents) need to reason about unknown variables, given evidence
- Example: explanation (diagnostic reasoning)
- Example: prediction (causal reasoning)
- Example: value of information

Independence



Independence

- Two variables are *independent* if:

$$\forall x, y : P(x, y) = P(x)P(y)$$

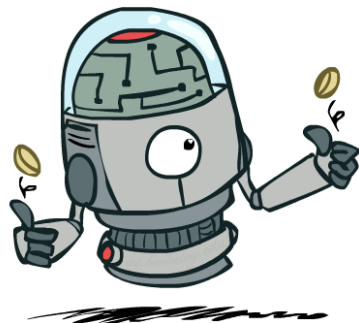
- This says that their joint distribution *factors* into a product two simpler distributions
- Another form:

$$\forall x, y : P(x|y) = P(x)$$

- We write: $X \perp\!\!\!\perp Y$

- Independence is a simplifying *modeling assumption*

- Empirical* joint distributions: at best “close” to independent
- What could we assume for {Weather, Traffic, Cavity, Toothache}?



Example: Independence?

$P_1(T, W)$			$P(T)$		$P_2(T, W)$		
T	W	P	T	P	T	W	P
hot	sun	0.4	hot	0.5	hot	sun	0.3
hot	rain	0.1	cold	0.5	hot	rain	0.2
cold	sun	0.2			cold	sun	0.3
cold	rain	0.3			cold	rain	0.2

$P(W)$	
W	P
sun	0.6
rain	0.4

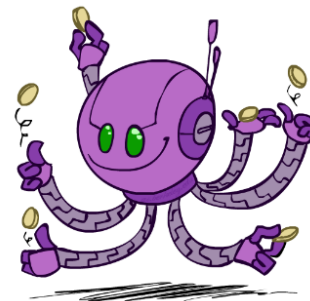
Example: Independence

- N fair, independent coin flips:

$P(X_1)$		$P(X_2)$		\dots		$P(X_n)$	
H	0.5	H	0.5			H	0.5
T	0.5	T	0.5			T	0.5

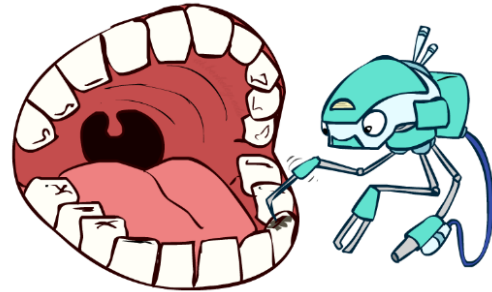
2^n {

$P(X_1, X_2, \dots, X_n)$



Conditional Independence

- $P(\text{Toothache}, \text{Cavity}, \text{Catch})$
- If I have a cavity, the probability that the probe catches in it doesn't depend on whether I have a toothache:
 - $P(+\text{catch} \mid +\text{toothache}, +\text{cavity}) = P(+\text{catch} \mid +\text{cavity})$
- The same independence holds if I don't have a cavity:
 - $P(+\text{catch} \mid +\text{toothache}, -\text{cavity}) = P(+\text{catch} \mid -\text{cavity})$
- Catch is *conditionally independent* of Toothache given Cavity:
 - $P(\text{Catch} \mid \text{Toothache}, \text{Cavity}) = P(\text{Catch} \mid \text{Cavity})$
- Equivalent statements:
 - $P(\text{Toothache} \mid \text{Catch}, \text{Cavity}) = P(\text{Toothache} \mid \text{Cavity})$
 - $P(\text{Toothache}, \text{Catch} \mid \text{Cavity}) = P(\text{Toothache} \mid \text{Cavity}) P(\text{Catch} \mid \text{Cavity})$
 - One can be derived from the other easily



Conditional Independence

- Unconditional (absolute) independence very rare (why?)
- *Conditional independence* is our most basic and robust form of knowledge about uncertain environments.
- X is conditionally independent of Y given Z

$$X \perp\!\!\!\perp Y \mid Z$$

if and only if:

$$\forall x, y, z : P(x, y \mid z) = P(x \mid z) P(y \mid z)$$

or, equivalently, if and only if

$$\forall x, y, z : P(x \mid z, y) = P(x \mid z)$$

Conditional Independence

- What about this domain:

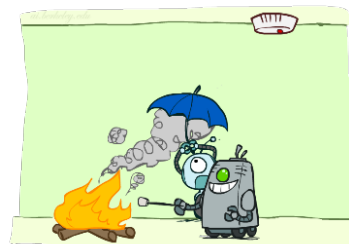
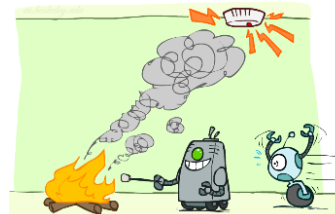
- Traffic
- Umbrella
- Raining



Conditional Independence

- What about this domain:

- Fire
- Smoke
- Alarm



Conditional Independence and the Chain Rule

- Chain rule: $P(X_1, X_2, \dots, X_n) = P(X_1)P(X_2|X_1)P(X_3|X_1, X_2) \dots$

- Trivial decomposition:

$$P(\text{Traffic, Rain, Umbrella}) = P(\text{Rain})P(\text{Traffic}|\text{Rain})P(\text{Umbrella}|\text{Rain, Traffic})$$

- With assumption of conditional independence:

$$P(\text{Traffic, Rain, Umbrella}) = P(\text{Rain})P(\text{Traffic}|\text{Rain})P(\text{Umbrella}|\text{Rain})$$



- Bayes' nets / graphical models help us express conditional independence assumptions

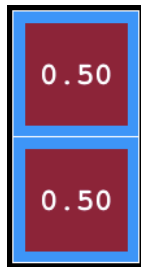
Ghostbusters Chain Rule

- Each sensor depends only on where the ghost is

$$P(T, B, G) = P(G) P(T|G) P(B|G)$$

- That means, the two sensors are conditionally independent, given the ghost position

- T: Top square is red
B: Bottom square is red
G: Ghost is in the top

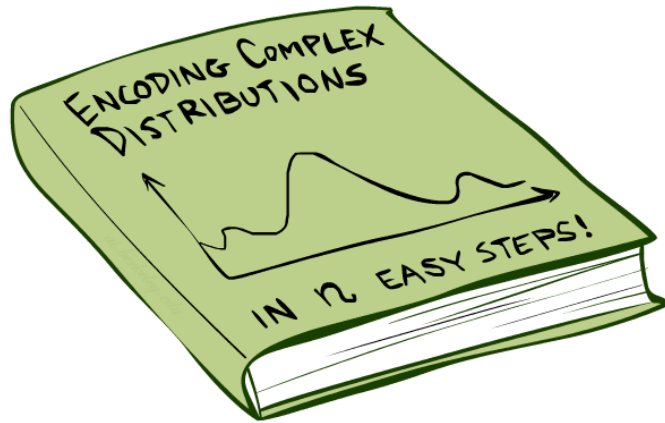


- Givens:
 $P(+g) = 0.5$
 $P(-g) = 0.5$
 $P(+t | +g) = 0.8$
 $P(+t | -g) = 0.4$
 $P(+b | +g) = 0.4$
 $P(+b | -g) = 0.8$

T	B	G	P(T,B,G)
+t	+b	+g	0.16
+t	+b	-g	0.16
+t	-b	+g	0.24
+t	-b	-g	0.04
-t	+b	+g	0.04
-t	+b	-g	0.24
-t	-b	+g	0.06
-t	-b	-g	0.06

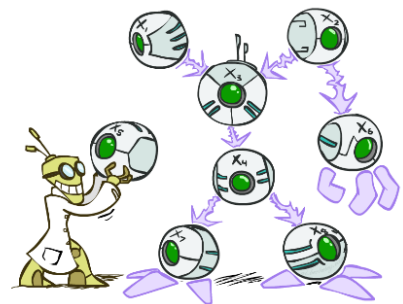


Bayes' Nets: Big Picture



Bayes' Nets: Big Picture

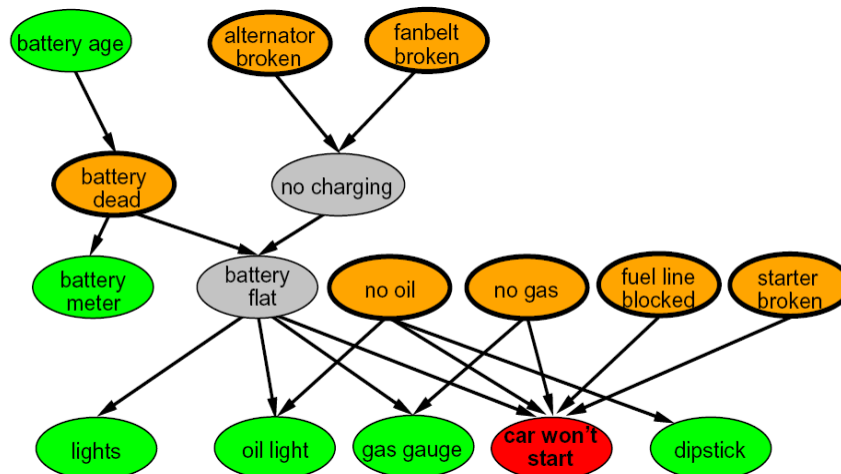
- Two problems with using full joint distribution tables as our probabilistic models:
 - Unless there are only a few variables, the joint is WAY too big to represent explicitly
 - Hard to learn (estimate) anything empirically about more than a few variables at a time
- Bayes' nets: a technique for describing complex joint distributions (models) using simple, local distributions (conditional probabilities)
 - More properly called **graphical models**
 - We describe how variables locally interact
 - Local interactions chain together to give global, indirect interactions
 - For about 10 min, we'll be vague about how these interactions are specified



Example Bayes' Net: Insurance



Example Bayes' Net: Car



Graphical Model Notation

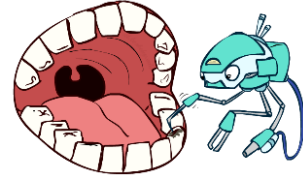
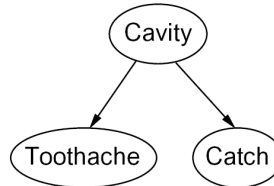
- Nodes: variables (with domains)

- Can be assigned (observed) or unassigned (unobserved)



- Arcs: interactions

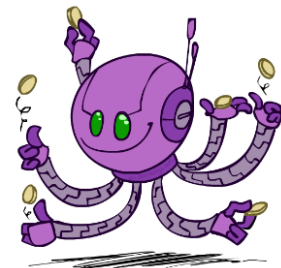
- Similar to CSP constraints
- Indicate "direct influence" between variables
- Formally: encode conditional independence (more later)



- For now: imagine that arrows mean direct causation (in general, they don't!)

Example: Coin Flips

- N independent coin flips

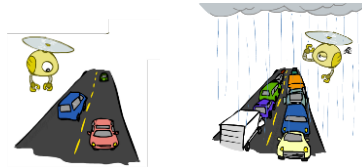


- No interactions between variables: **absolute independence**

Example: Traffic

- Variables:

- R: It rains
- T: There is traffic



- Model 1: independence



- Model 2: rain causes traffic



- Why is an agent using model 2 better?

Example: Traffic II

- Let's build a causal graphical model!

- Variables

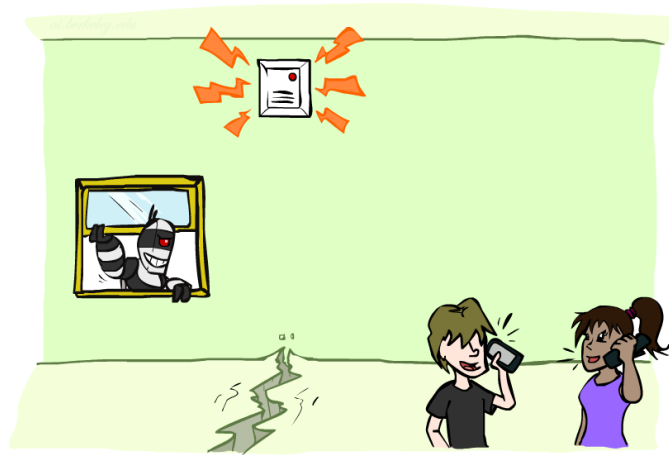
- T: Traffic
- R: It rains
- L: Low pressure
- D: Roof drips
- B: Ballgame
- C: Cavity



Example: Alarm Network

- Variables

- B: Burglary
- A: Alarm goes off
- M: Mary calls
- J: John calls
- E: Earthquake!



Bayes' Net Semantics



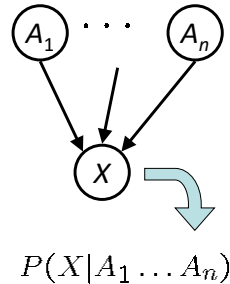
Bayes' Net Semantics



- A set of nodes, one per variable X
- A directed, acyclic graph
- A conditional distribution for each node
 - A collection of distributions over X , one for each combination of parents' values

$$P(X|a_1 \dots a_n)$$

- CPT: conditional probability table
- Description of a noisy "causal" process



A Bayes net = Topology (graph) + Local Conditional Probabilities

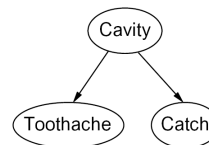
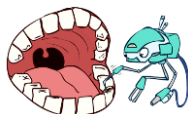
Probabilities in BNs



- Bayes' nets **implicitly** encode joint distributions
 - As a product of local conditional distributions
 - To see what probability a BN gives to a full assignment, multiply all the relevant conditionals together:

$$P(x_1, x_2, \dots, x_n) = \prod_{i=1}^n P(x_i | \text{parents}(X_i))$$

- Example:



$$P(+cavity, +catch, -toothache)$$

Probabilities in BNs



- Why are we guaranteed that setting

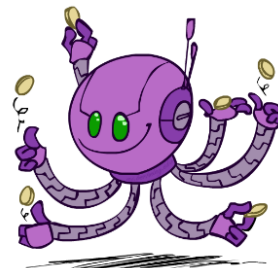
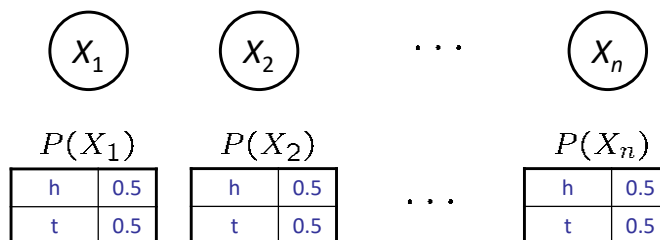
$$P(x_1, x_2, \dots, x_n) = \prod_{i=1}^n P(x_i | \text{parents}(X_i))$$

results in a proper joint distribution?

- Chain rule (valid for all distributions): $P(x_1, x_2, \dots, x_n) = \prod_{i=1}^n P(x_i | x_1 \dots x_{i-1})$
- Assume conditional independences: $P(x_i | x_1, \dots, x_{i-1}) = P(x_i | \text{parents}(X_i))$
- Consequence: $P(x_1, x_2, \dots, x_n) = \prod_{i=1}^n P(x_i | \text{parents}(X_i))$

- Not every BN can represent every joint distribution
 - The topology enforces certain conditional independencies

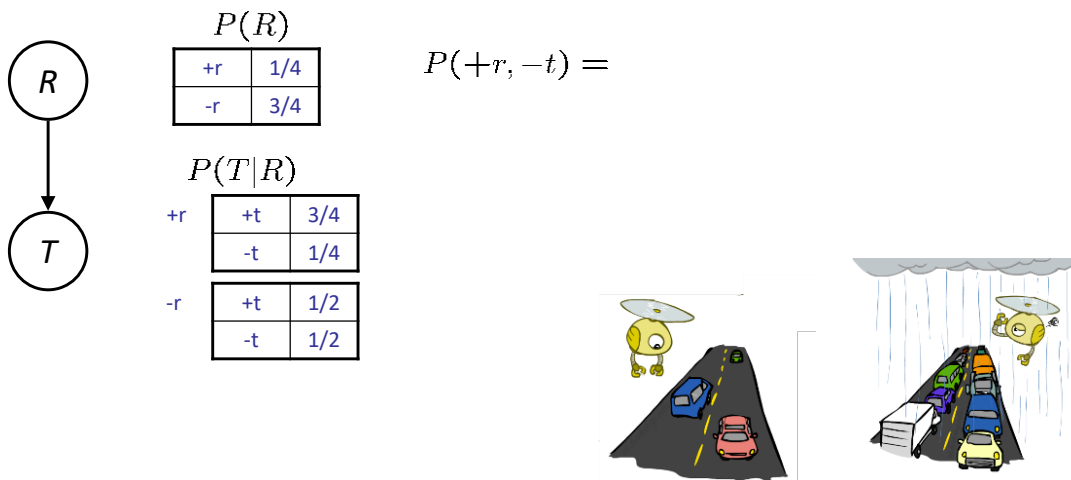
Example: Coin Flips



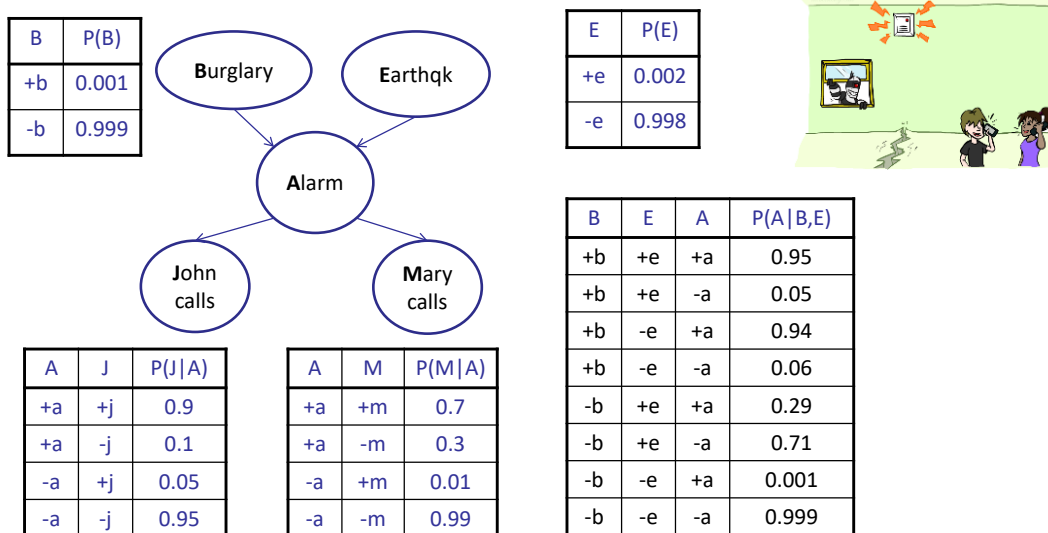
$$P(h, h, t, h) =$$

Only distributions whose variables are absolutely independent can be represented by a Bayes' net with no arcs.

Example: Traffic

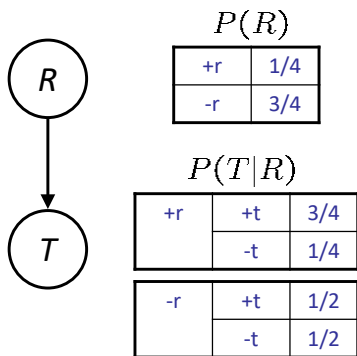


Example: Alarm Network



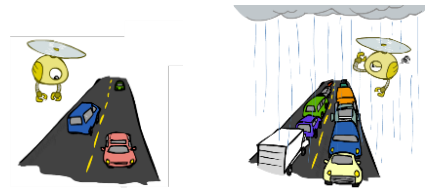
Example: Traffic

■ Causal direction



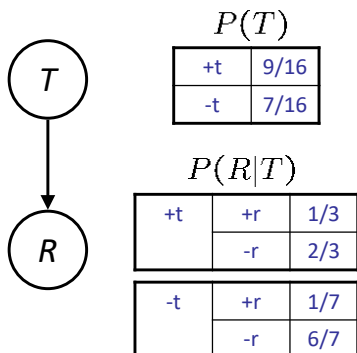
$P(T, R)$

+r	+t	3/16
+r	-t	1/16
-r	+t	6/16
-r	-t	6/16



Example: Reverse Traffic

■ Reverse causality?



$P(T, R)$

+r	+t	3/16
+r	-t	1/16
-r	+t	6/16
-r	-t	6/16



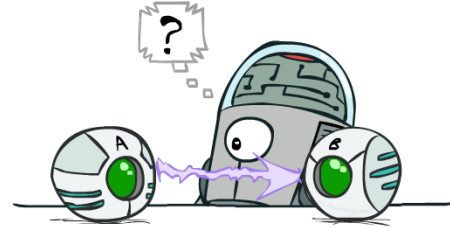
Causality?

- When Bayes' nets reflect the true causal patterns:

- Often simpler (nodes have fewer parents)
- Often easier to think about
- Often easier to elicit from experts

- BNs need not actually be causal

- Sometimes no causal net exists over the domain (especially if variables are missing)
- E.g. consider the variables *Traffic* and *Drips*
- End up with arrows that reflect correlation, not causation



- What do the arrows really mean?

- Topology may happen to encode causal structure
- **Topology really encodes conditional independence**

$$P(x_i | x_1, \dots, x_{i-1}) = P(x_i | \text{parents}(X_i))$$

Bayes' Nets

- So far: how a Bayes' net encodes a joint distribution

- Next: how to answer queries about that distribution

- Today:
 - First assembled BNs using an intuitive notion of conditional independence as causality
 - Then saw that key property is conditional independence
- Main goal: answer queries about conditional independence and influence

- After that: how to answer numerical queries (inference)

