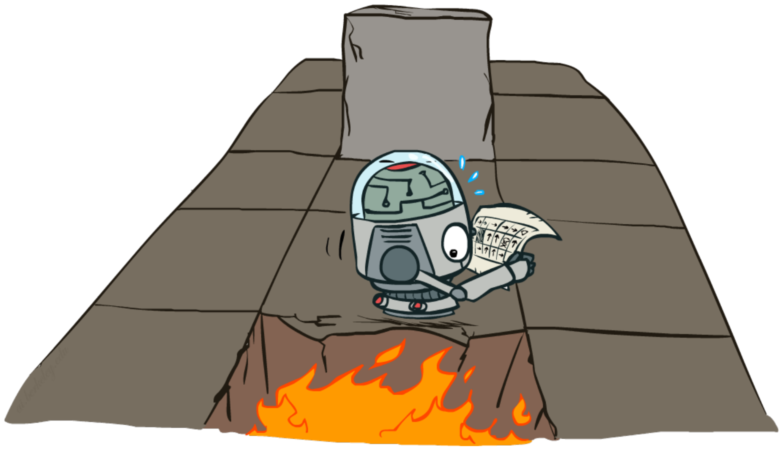
# CS 188: Artificial Intelligence

## Markov Decision Processes II



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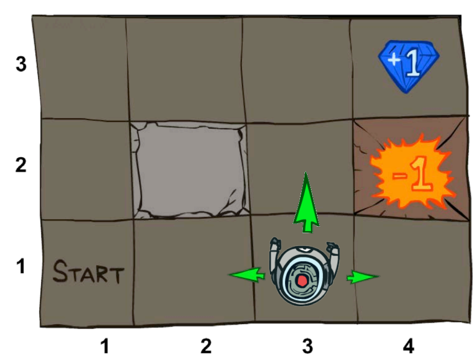
[These slides were created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley.  All CS188 materials are available at http://ai.berkeley.edu.]

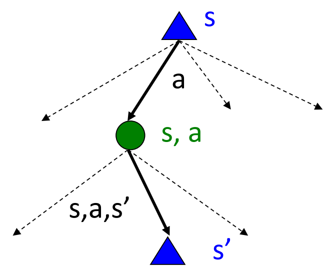
# Example: Grid World

- A maze-like problem
  - The agent lives in a grid
  - Walls block the agent's path
- Noisy movement: actions do not always go as planned
  - 80% of the time, the action North takes the agent North
  - 10% of the time, North takes the agent West; 10% East
  - If there is a wall in the direction the agent would have been taken, the agent stays put
- The agent receives rewards each time step
  - Small "living" reward each step (can be negative)
  - Big rewards come at the end (good or bad)
- Goal: maximize sum of (discounted) rewards



# Recap: MDPs

- Markov decision processes:
  - States S
  - Actions A
  - Transitions P(s'|s,a) (or T(s,a,s'))
  - Rewards R(s,a,s') (and discount $\gamma$)
  - Start state $s_0$




- Quantities:
  - Policy = map of states to actions
  - Utility = sum of discounted rewards
  - Values = expected future utility from a state (max node)
  - Q-Values = expected future utility from a q-state (chance node)

# Optimal Quantities

- The value (utility) of a state s:

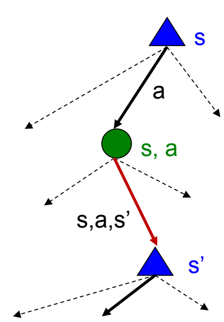
  $V^*(s)$ = expected utility starting in s and acting optimally

- The value (utility) of a q-state (s,a):

  $Q^*(s,a)$ = expected utility starting out having taken action a from state s and (thereafter) acting optimally

- The optimal policy:

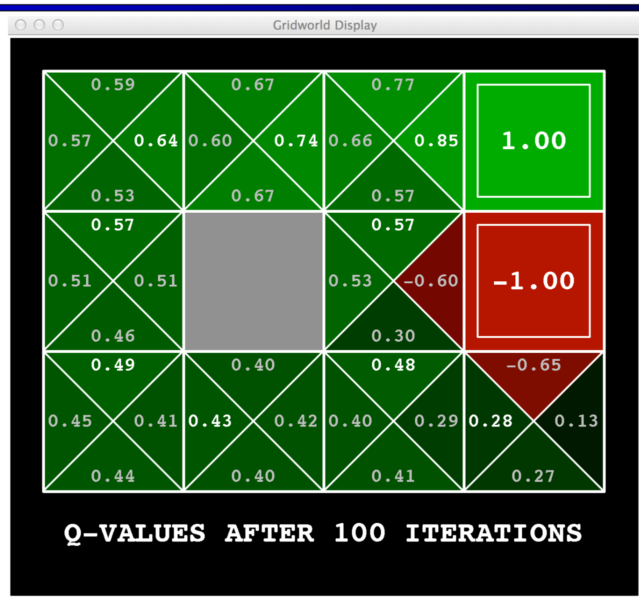
  $\pi^*(s)$ = optimal action from state s



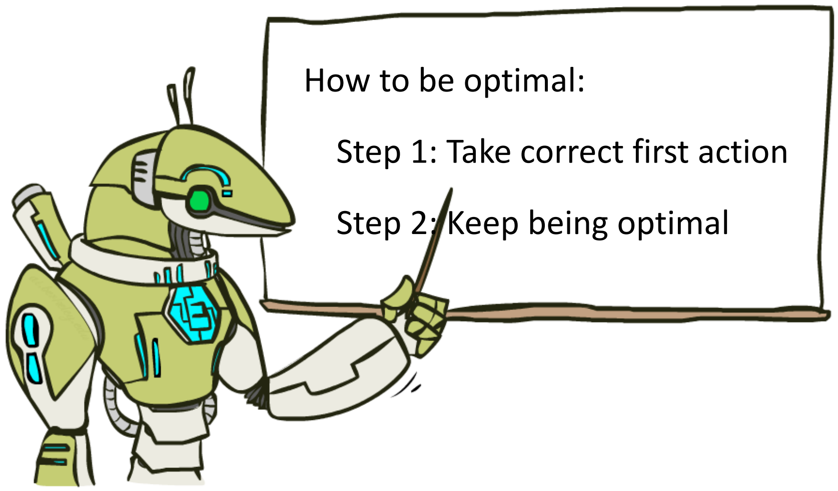

[Demo: gridworld values (L9D1)]

# Gridworld Values V*


# Gridworld: Q*



# The Bellman Equations



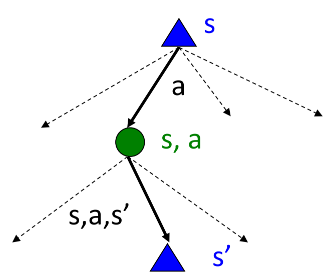

# The Bellman Equations

- Definition of "optimal utility" via expectimax recurrence gives a simple one-step lookahead relationship amongst optimal utility values

$$V^*(s) = \max_a Q^*(s,a)$$

$$Q^*(s,a) = \sum_{s'} T(s,a,s') \left[ R(s,a,s') + \gamma V^*(s') \right]$$

$$V^*(s) = \max_a \sum_{s'} T(s,a,s') \left[ R(s,a,s') + \gamma V^*(s') \right]$$

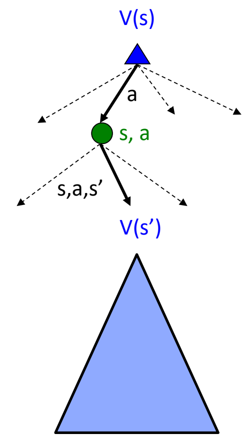



- These are the Bellman equations, and they characterize optimal values in a way we'll use over and over

# Value Iteration

- Bellman equations characterize the optimal values:

$$V^*(s) = \max_a \sum_{s'} T(s,a,s') \left[ R(s,a,s') + \gamma V^*(s') \right]$$



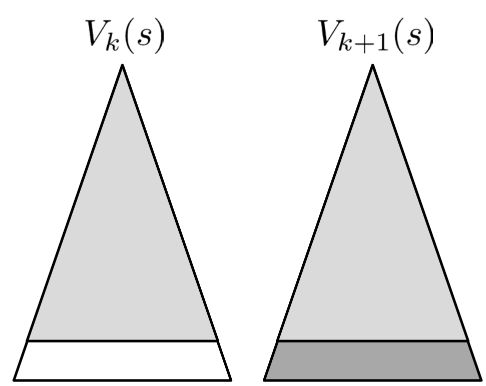

- Value iteration computes them:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[ R(s,a,s') + \gamma V_k(s') \right]$$

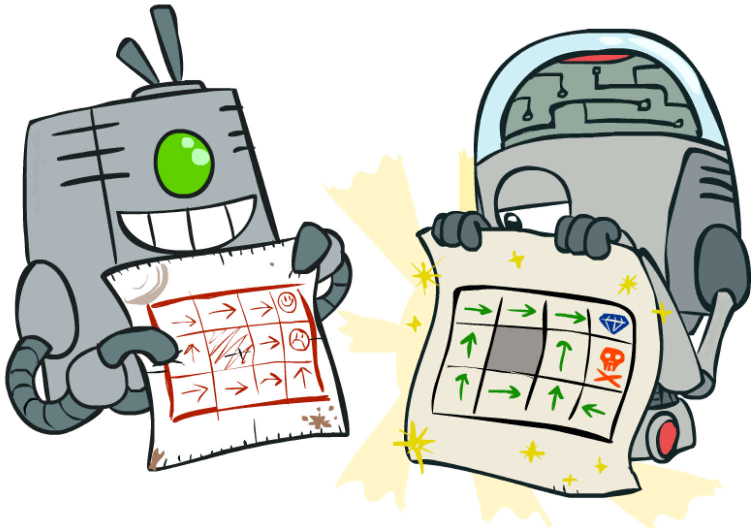
- Value iteration is just a fixed point solution method
  - … though the $V_k$ vectors are also interpretable as time-limited values

# Convergence*

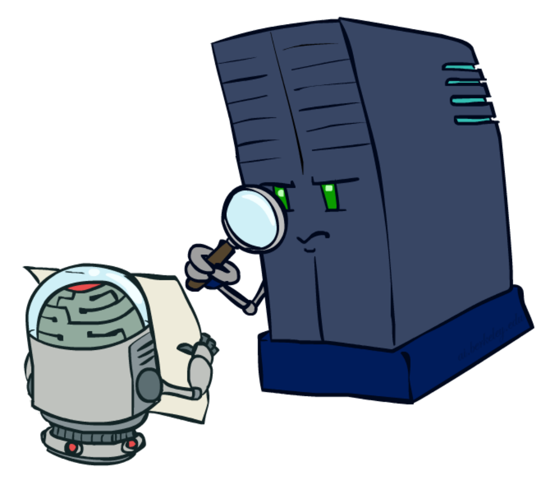
- How do we know the $V_k$ vectors are going to converge?
- Case 1: If the tree has maximum depth M, then $V_M$ holds the actual untruncated values
- Case 2: If the discount is less than 1
  - Sketch: For any state $V_k$ and $V_{k+1}$ can be viewed as depth k+1 expectimax results in nearly identical search trees
  - The difference is that on the bottom layer, $V_{k+1}$ has actual rewards while $V_k$ has zeros
  - That last layer is at best all $R_{MAX}$
  - It is at worst $R_{MIN}$
  - But everything is discounted by $\gamma^k$ that far out
  - So $V_k$ and $V_{k+1}$ are at most $\gamma^k$ max|R| different
  - So as k increases, the values converge



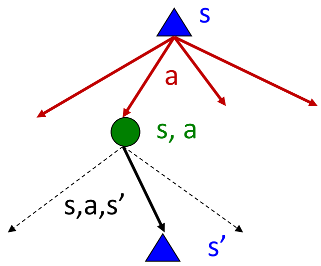
# Policy Methods

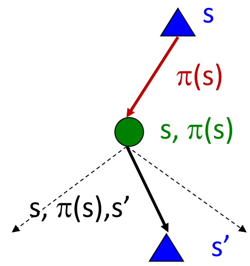


# Policy Evaluation



# Fixed Policies



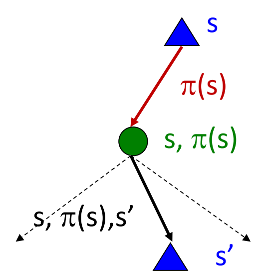




- Expectimax trees max over all actions to compute the optimal values
- If we fixed some policy $\pi(s)$, then the tree would be simpler – only one action per state
  - … though the tree's value would depend on which policy we fixed

# Utilities for a Fixed Policy

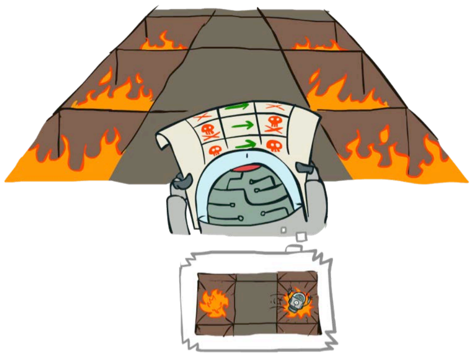
- Another basic operation: compute the utility of a state s under a fixed (generally non-optimal) policy
- Define the utility of a state s, under a fixed policy $\pi$:
  $V^{\pi}(s)$ = expected total discounted rewards starting in s and following $\pi$
- Recursive relation (one-step look-ahead / Bellman equation):

$$V^{\pi}(s) = \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V^{\pi}(s')]$$

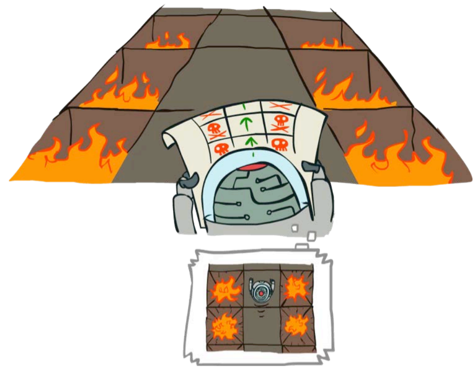


# Example: Policy Evaluation

Always Go Right



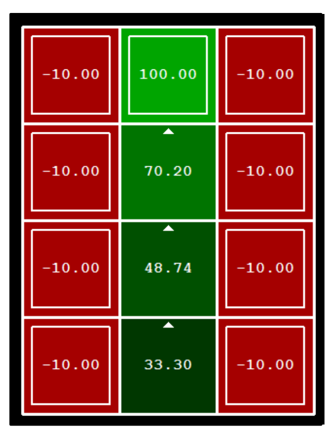
Always Go Forward



# Example: Policy Evaluation

Always Go Right

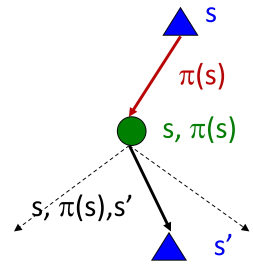

Always Go Forward



# Policy Evaluation

- How do we calculate the V's for a fixed policy $\pi$?

- Idea 1: Turn recursive Bellman equations into updates (like value iteration)



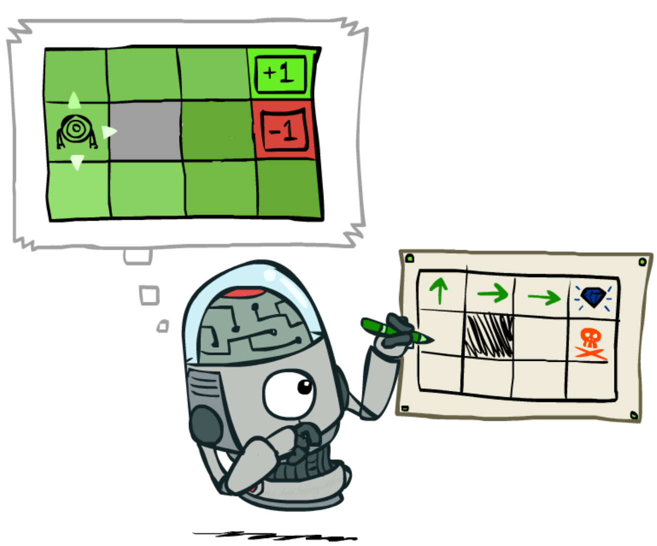

$$V_0^\pi(s) = 0$$

$$V_{k+1}^\pi(s) \leftarrow \sum_{s'} T(s,\pi(s),s')[R(s,\pi(s),s') + \gamma V_k^\pi(s')]$$

- Efficiency: $O(S^2)$ per iteration

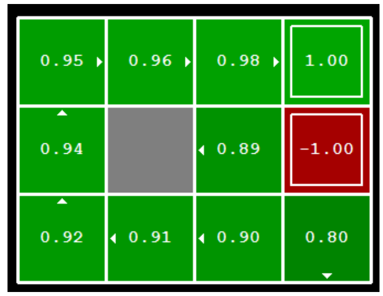
- Idea 2: Without the maxes, the Bellman equations are just a linear system
  - Solve with Matlab (or your favorite linear system solver)

# Policy Extraction



# Computing Actions from Values

- Let's imagine we have the optimal values V*(s)
- How should we act?
  - It's not obvious!
- We need to do a mini-expectimax (one step)



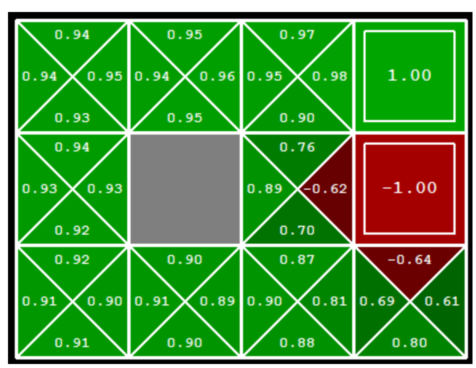

$$\pi^*(s) = \arg\max_a \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V^*(s')]$$

- This is called policy extraction, since it gets the policy implied by the values

# Computing Actions from Q-Values

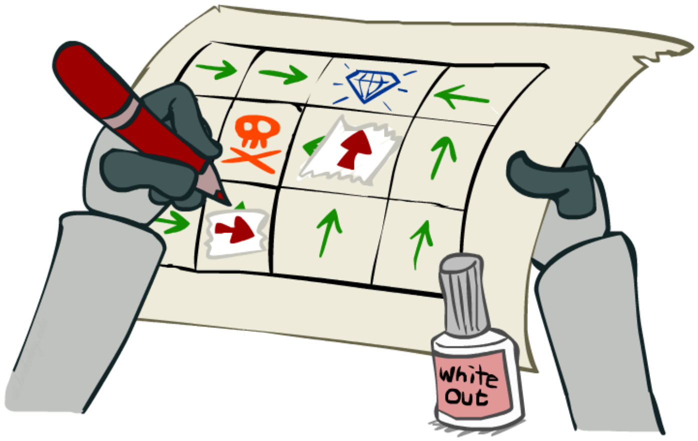
- Let's imagine we have the optimal q-values:
- How should we act?
  - Completely trivial to decide!

$$\pi^*(s) = \arg\max_a Q^*(s,a)$$




- Important lesson: actions are easier to select from q-values than values!

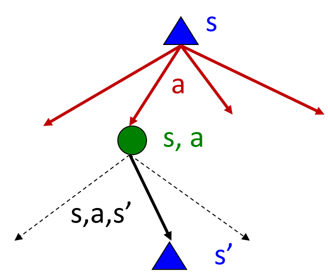
# Policy Iteration




# Problems with Value Iteration

- Value iteration repeats the Bellman updates:

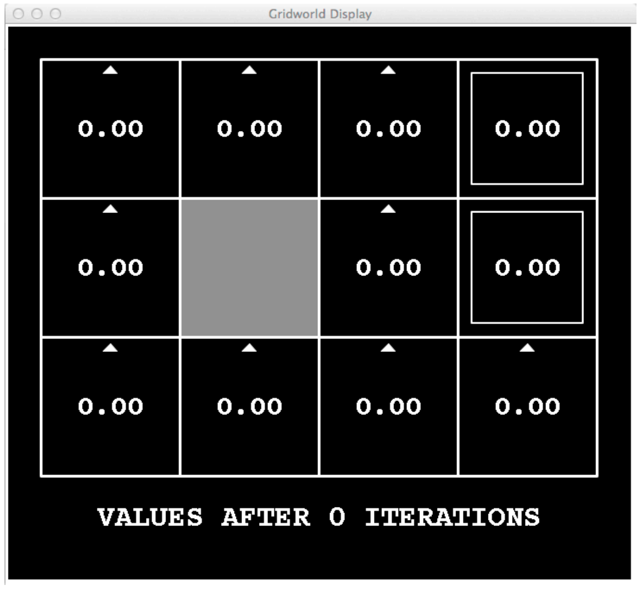
$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$




- Problem 1: It's slow – $O(S^2A)$ per iteration
- Problem 2: The "max" at each state rarely changes
- Problem 3: The policy often converges long before the values

[Demo: value iteration (L9D2)]

# k=0

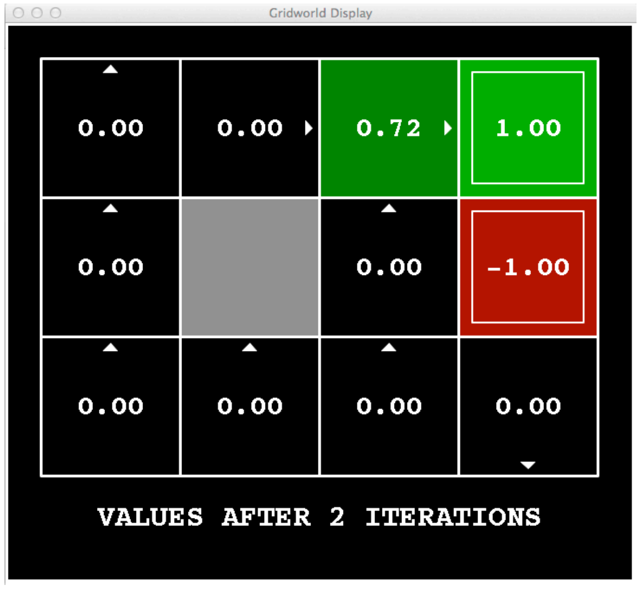



Noise = 0.2
Discount = 0.9
Living reward = 0

# k=1


Noise = 0.2
Discount = 0.9
Living reward = 0

# k=2

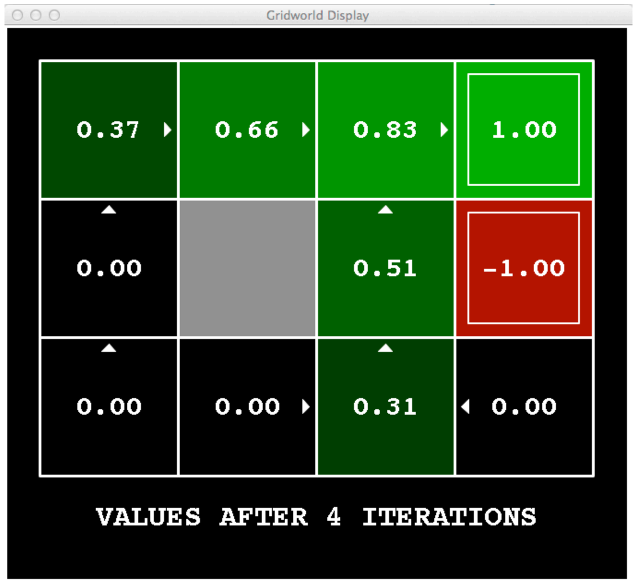



Noise = 0.2
Discount = 0.9
Living reward = 0

# k=3


Noise = 0.2
Discount = 0.9
Living reward = 0

# k=4

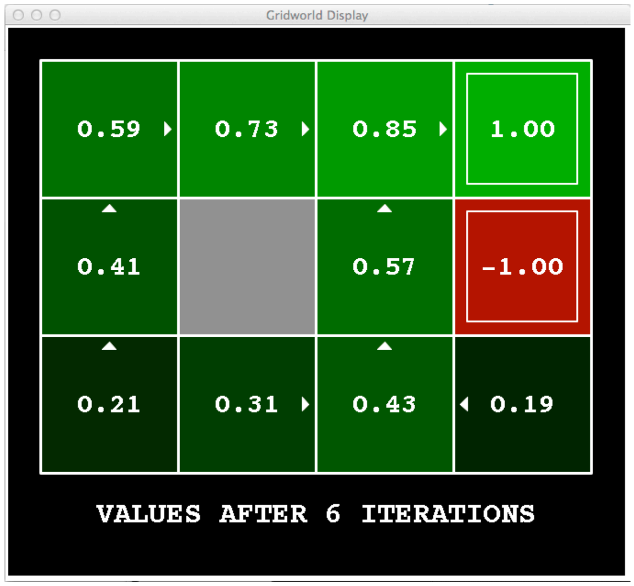



Noise = 0.2
Discount = 0.9
Living reward = 0

# k=5


Noise = 0.2
Discount = 0.9
Living reward = 0

# k=6

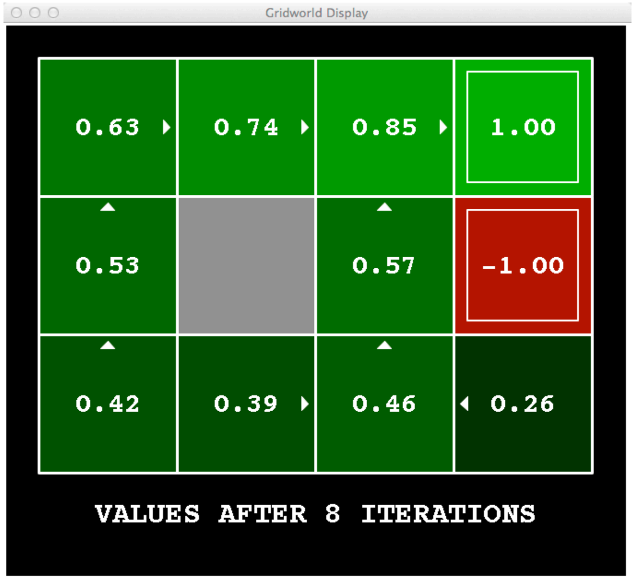



Noise = 0.2
Discount = 0.9
Living reward = 0

# k=7


Noise = 0.2
Discount = 0.9
Living reward = 0

# k=8

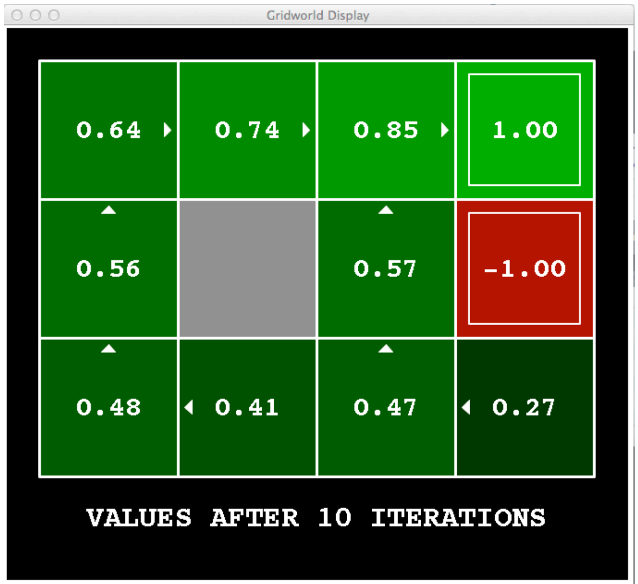



Noise = 0.2
Discount = 0.9
Living reward = 0

# k=9


Noise = 0.2
Discount = 0.9
Living reward = 0

# k=10




Noise = 0.2
Discount = 0.9
Living reward = 0

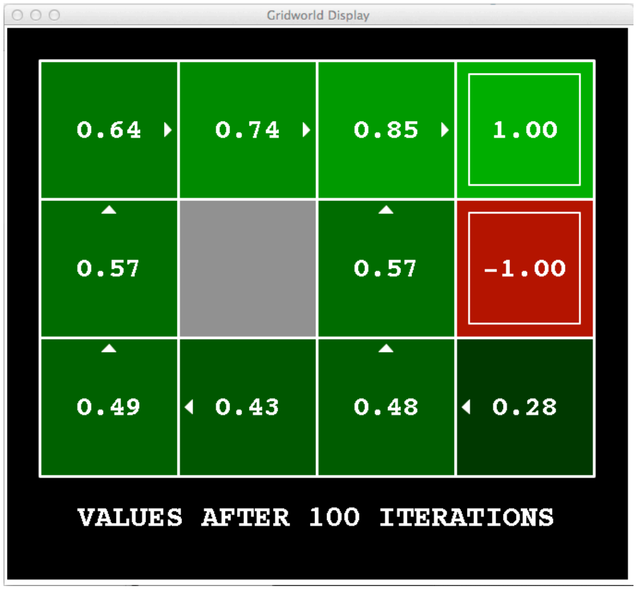
# k=11


Noise = 0.2
Discount = 0.9
Living reward = 0

# k=12


Noise = 0.2
Discount = 0.9
Living reward = 0

# k=100




Noise = 0.2
Discount = 0.9
Living reward = 0

# Policy Iteration

- Alternative approach for optimal values:
  - Step 1: Policy evaluation: calculate utilities for some fixed policy (not optimal utilities!) until convergence
  - Step 2: Policy improvement: update policy using one-step look-ahead with resulting converged (but not optimal!) utilities as future values
  - Repeat steps until policy converges
- This is policy iteration
  - It's still optimal!
  - Can converge (much) faster under some conditions

# Policy Iteration

- Evaluation: For fixed current policy $\pi$, find values with policy evaluation:
  - Iterate until values converge:

$$V_{k+1}^{\pi_i}(s) \leftarrow \sum_{s'} T(s, \pi_i(s), s') \left[R(s, \pi_i(s), s') + \gamma V_k^{\pi_i}(s')\right]$$

- Improvement: For fixed values, get a better policy using policy extraction
  - One-step look-ahead:

$$\pi_{i+1}(s) = \arg\max_a \sum_{s'} T(s, a, s') \left[R(s, a, s') + \gamma V^{\pi_i}(s')\right]$$

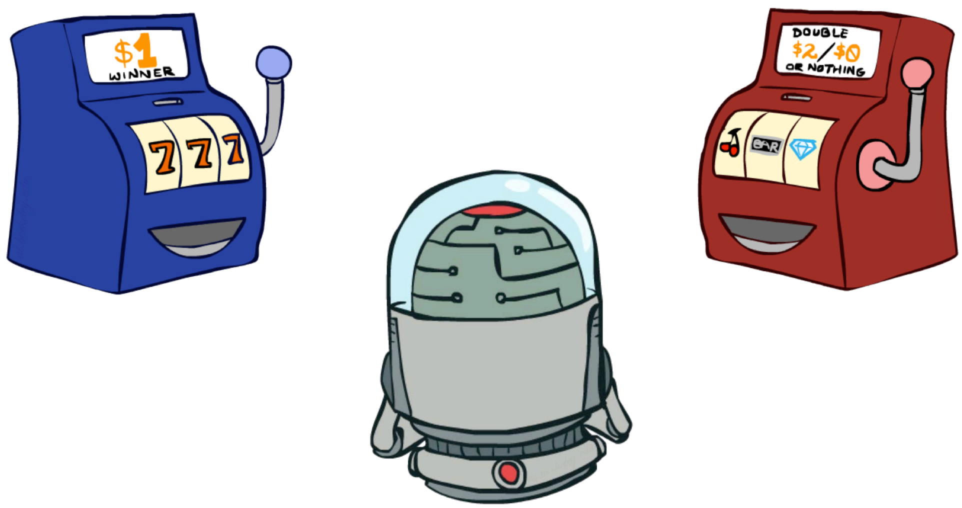
# Comparison

- Both value iteration and policy iteration compute the same thing (all optimal values)
- In value iteration:
  - Every iteration updates both the values and (implicitly) the policy
  - We don't track the policy, but taking the max over actions implicitly recomputes it
- In policy iteration:
  - We do several passes that update utilities with fixed policy (each pass is fast because we consider only one action, not all of them)
  - After the policy is evaluated, a new policy is chosen (slow like a value iteration pass)
  - The new policy will be better (or we're done)
- Both are dynamic programs for solving MDPs

# Summary: MDP Algorithms

- So you want to….
  - Compute optimal values: use value iteration or policy iteration
  - Compute values for a particular policy: use policy evaluation
  - Turn your values into a policy: use policy extraction (one-step lookahead)
- These all look the same!
  - They basically are – they are all variations of Bellman updates
  - They all use one-step lookahead expectimax fragments
  - They differ only in whether we plug in a fixed policy or max over actions

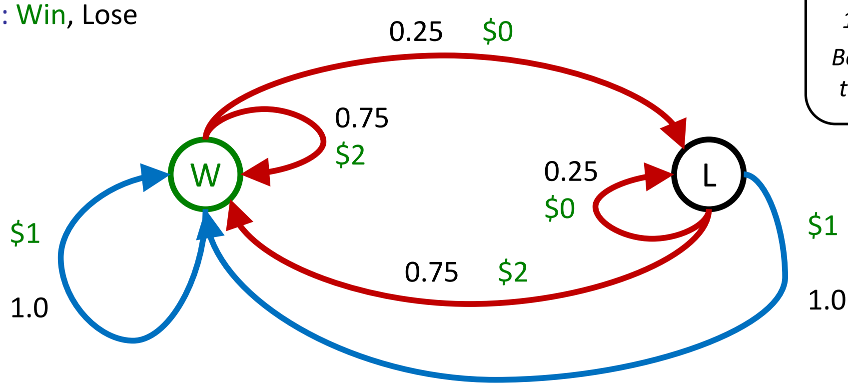
# Double Bandits




# Double-Bandit MDP

- Actions: *Blue*, *Red*
- States: Win, Lose

*No discount*
*100 time steps*
*Both states have the same value*

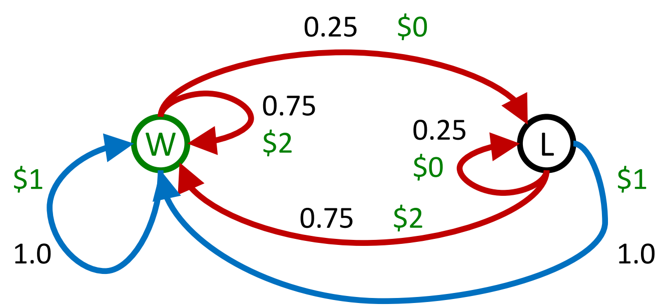


# Offline Planning

- Solving MDPs is offline planning
  - You determine all quantities through computation
  - You need to know the details of the MDP
  - You do not actually play the game!

*No discount*
*100 time steps*
*Both states have the same value*

| | Value |
|---|---|
| Play Red | 150 |
| Play Blue | 100 |



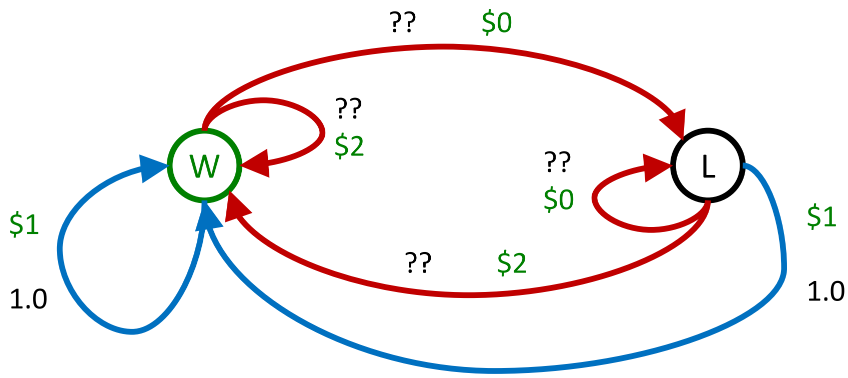

# Let's Play!



$2 $2 $0 $2 $2
$2 $2 $0 $0 $0

# Online Planning

- Rules changed!  Red's win chance is different.




# Let's Play!



$0 $0 $0 $2 $0
$2 $0 $0 $0 $0

# What Just Happened?


- That wasn't planning, it was learning!
  - Specifically, reinforcement learning
  - There was an MDP, but you couldn't solve it with just computation
  - You needed to actually act to figure it out
- Important ideas in reinforcement learning that came up
  - Exploration: you have to try unknown actions to get information
  - Exploitation: eventually, you have to use what you know
  - Regret: even if you learn intelligently, you make mistakes
  - Sampling: because of chance, you have to try things repeatedly
  - Difficulty: learning can be much harder than solving a known MDP

# Next Time: Reinforcement Learning!