CS 188 Spring 2014 — Introduction to Artificial Intelligence — Midterm 1

- You have approximately 2 hours and 50 minutes.
- The exam is closed book, closed notes except your one-page crib sheet.
- Mark your answers ON THE EXAM ITSELF. If you are not sure of your answer you may wish to provide a *brief* explanation. All short answer sections can be successfully answered in a few sentences AT MOST.

First name	
Last name	
SID	
edX username	
First and last name of student to your left	
First and last name of student to your right	

For staff use only:

Q1.	All Searches Lead to the Same Destination	/10
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Q1. [10 pts] All Searches Lead to the Same Destination

For all the questions below assume :

- All search algorithms are *graph* search (as opposed to tree search).
- $c_{ij} > 0$ is the cost to go from node i to node j.
- There is only one goal node (as opposed to a set of goal nodes).
- All ties are broken alphabetically.
- Assume heuristics are consistent.

Definition: Two search algorithms are defined to be ***equivalent*** if and only if they expand the same nodes in the same order and return the same path.

In this question we study what happens if we run uniform cost search with action costs d_{ij} that are potentially different from the search problem's actual action costs c_{ij}. Concretely, we will study how this might, or might not, result in running uniform cost search (with these new choices of action costs) being equivalent to another search algorithm.

(a) [2 pts] Mark *all* choices for costs d_{ij} that make running **Uniform Cost Search** algorithm with these costs d_{ij} *equivalent* to running **Breadth-First Search**.

○ $d_{ij} = 0$
○ $d_{ij} = \alpha$, $\alpha > 0$
○ $d_{ij} = \alpha$, $\alpha < 0$
○ $d_{ij} = 1$
○ $d_{ij} = -1$
○ None of the above

(b) [2 pts] Mark *all* choices for costs d_{ij} that make running **Uniform Cost Search** algorithm with these costs d_{ij} *equivalent* to running **Depth-First Search**.

○ $d_{ij} = 0$
○ $d_{ij} = \alpha$, $\alpha > 0$
○ $d_{ij} = \alpha$, $\alpha < 0$
○ $d_{ij} = 1$
○ $d_{ij} = -1$
○ None of the above

(c) [2 pts] Mark *all* choices for costs d_{ij} that make running **Uniform Cost Search** algorithm with these costs d_{ij} *equivalent* to running **Uniform Cost Search** with the original costs c_{ij}.

○ $d_{ij} = c_{ij}^2$

○ $d_{ij} = 1/c_{ij}$

○ $d_{ij} = \alpha\ c_{ij}, \qquad \alpha > 0$

○ $d_{ij} = c_{ij} + \alpha, \qquad \alpha > 0$

○ $d_{ij} = \alpha\ c_{ij} + \beta, \quad \alpha > 0,\ \beta > 0$

○ None of the above

(d) Let $h(n)$ be the value of the heuristic function at node n.

(i) [2 pts] Mark *all* choices for costs d_{ij} that make running **Uniform Cost Search** algorithm with these costs d_{ij} *equivalent* to running **Greedy Search** with the original costs c_{ij} and heuristic function h.

○ $d_{ij} = h(i) - h(j)$

○ $d_{ij} = h(j) - h(i)$

○ $d_{ij} = \alpha\ h(i), \qquad \alpha > 0$

○ $d_{ij} = \alpha\ h(j), \qquad \alpha > 0$

○ $d_{ij} = c_{ij} + h(j) + h(i)$

○ None of the above

(ii) [2 pts] Mark *all* choices for costs d_{ij} that make running **Uniform Cost Search** algorithm with these costs d_{ij} *equivalent* to running **A* Search** with the original costs c_{ij} and heuristic function h.

○ $d_{ij} = \alpha\ h(i), \quad \alpha > 0$

○ $d_{ij} = \alpha\ h(j), \quad \alpha > 0$

○ $d_{ij} = c_{ij} + h(i)$

○ $d_{ij} = c_{ij} + h(j)$

○ $d_{ij} = c_{ij} + h(i) - h(j)$

○ $d_{ij} = c_{ij} + h(j) - h(i)$

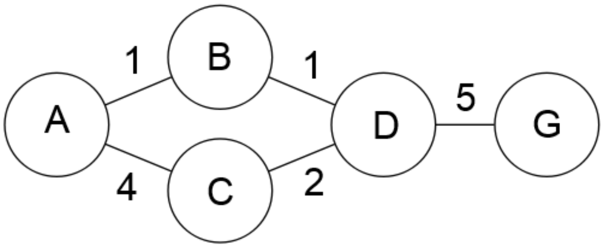
○ None of the above

Q2. [18 pts] Dynamic A* Search

After running A* graph search and finding an optimal path from start to goal, the cost of one of the edges, $X \to Y$, in the graph changes. Rather than re-running the entire search, you want to find a more efficient way of finding the optimal path for this new search problem.

You have access to the fringe, the closed set and the search tree as they were at the completion of the initial search. In addition, you have a *closed node map* that maps a state, s from the closed set to a list of nodes in the search tree ending in s which were not expanded because s was already in the closed set.

For example, after running A* search with the null heuristic on the following graph, the data structures would be as follows:



Fringe:	{}	Closed Node Map:	{A:[ ], B:[ ], C:[ ], D:[(A-C-D, 6)]}
Closed Set:	{A, B, C, D}	Search Tree:	{A : [(A-B, 1), (A-C, 4)], A-B : [(A-B-D, 2)], A-C : [ ], A-B-D : [(A-B-D-G, 7)], A-B-D-E : [ ]}

For a general graph, for each of the following scenarios, select the choice that finds the correct optimal path and cost *while expanding the fewest nodes.* Note that if you select the 4^{th} choice, you must fill in the change, and if you select the last choice, you must describe the set of nodes to add to the fringe.

In the answer choices below, if an option states some nodes will be added to the fringe, this also implies that the final state of each node gets cleared out of the closed set (indeed, otherwise it'd be rather useless to add something back into the fringe). You may assume that there are no ties in terms of path costs.

Following is a set of eight choices you should use to answer the questions on the following page.

i. The optimal path does not change, and the cost remains the same.
ii. The optimal path does not change, but the cost increases by n
iii. The optimal path does not change, but the cost decreases by n
iv. The optimal path does not change, but the cost changes by ______________________
v. The optimal path for the new search problem can be found by adding the subtree rooted at X that was expanded in the original search back onto the fringe and re-starting the search.
vi. The optimal path for the new search problem can be found by adding the subtree rooted at Y that was expanded in the original search back onto the fringe and re-starting the search.
vii. The optimal path for the new search problem can be found by adding all nodes for each state in the *closed node map* back onto the fringe and re-starting the search.
viii. The optimal path for the new search problem can be found by adding some other set of nodes back onto the fringe and re-starting the search. Describe the set below.

(a) [3 pts] Cost of $X \to Y$ is increased by $n, n > 0$, the edge is on the optimal path, and was explored by the first search.

○ i ○ ii ○ iii ○ iv, Change:
○ v ○ vi ○ vii ○ viii, Describe the set below:

(b) [3 pts] Cost of $X \to Y$ is decreased by $n, n > 0$, the edge is on the optimal path, and was explored by the first search.

○ i ○ ii ○ iii ○ iv, Change:
○ v ○ vi ○ vii ○ viii, Describe the set below:

(c) [3 pts] Cost of $X \to Y$ is increased by $n, n > 0$, the edge is not on the optimal path, and was explored by the first search.

○ i ○ ii ○ iii ○ iv, Change:
○ v ○ vi ○ vii ○ viii, Describe the set below:

(d) [3 pts] Cost of $X \to Y$ is decreased by $n, n > 0$, the edge is not on the optimal path, and was explored by the first search.

○ i ○ ii ○ iii ○ iv, Change:
○ v ○ vi ○ vii ○ viii, Describe the set below:

(e) [3 pts] Cost of $X \to Y$ is increased by $n, n > 0$, the edge is not on the optimal path, and was not explored by the first search.

○ i ○ ii ○ iii ○ iv, Change:
○ v ○ vi ○ vii ○ viii, Describe the set below:

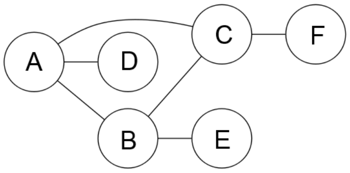
(f) [3 pts] Cost of $X \to Y$ is decreased by $n, n > 0$, the edge is not on the optimal path, and was not explored by the first search.

○ i ○ ii ○ iii ○ iv, Change:
○ v ○ vi ○ vii ○ viii, Describe the set below:

Q3. [16 pts] CSPs

(a) The graph below is a constraint graph for a CSP that has only binary constraints. Initially, no variables have been assigned.

For each of the following scenarios, mark all variables for which the specified filtering might result in their domain being changed.



(i) [1 pt] A value is assigned to A. Which domains might be changed as a result of running forward checking for A?

○ A ○ B ○ C ○ D ○ E ○ F

(ii) [1 pt] A value is assigned to A, and then forward checking is run for A. Then a value is assigned to B. Which domains might be changed as a result of running forward checking for B?

○ A ○ B ○ C ○ D ○ E ○ F

(iii) [1 pt] A value is assigned to A. Which domains might be changed as a result of enforcing arc consistency after this assignment?

○ A ○ B ○ C ○ D ○ E ○ F

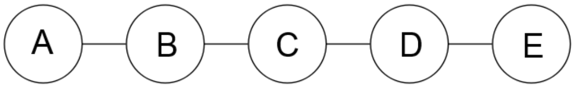
(iv) [1 pt] A value is assigned to A, and then arc consistency is enforced. Then a value is assigned to B. Which domains might be changed as a result of enforcing arc consistency after the assignment to B?

○ A ○ B ○ C ○ D ○ E ○ F

(b) You decide to try a new approach to using arc consistency in which you initially enforce arc consistency, and then enforce arc consistency every time you have assigned an even number of variables.

You have to backtrack if, after a value has been assigned to a variable, X, the recursion returns at X without a solution. Concretely, this means that for a single variable with d values remaining, it is possible to backtrack up to d times. For each of the following constraint graphs, if each variable has a domain of size d, how many times would you have to backtrack in the worst case for each of the specified orderings?

(i) [6 pts]

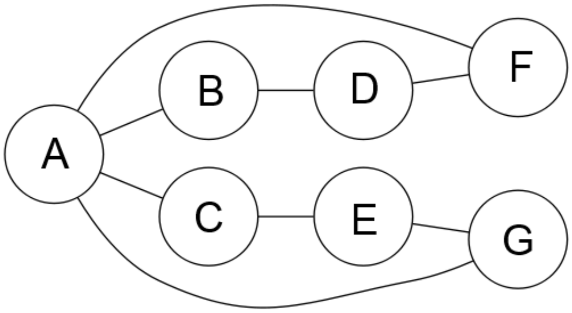


A-B-C-D-E: ______________________

A-E-B-D-C: ______________________

C-B-D-E-A: ______________________

(ii) [6 pts]



A-B-C-D-E-F-G: ______________________

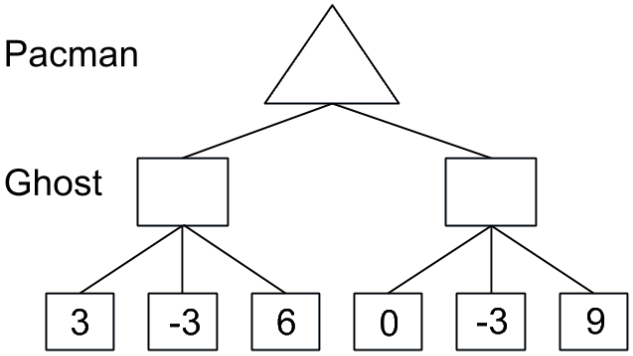
F-D-B-A-C-G-E: ______________________

C-A-F-E-B-G-D: ______________________

Q4. [15 pts] Variants of Trees

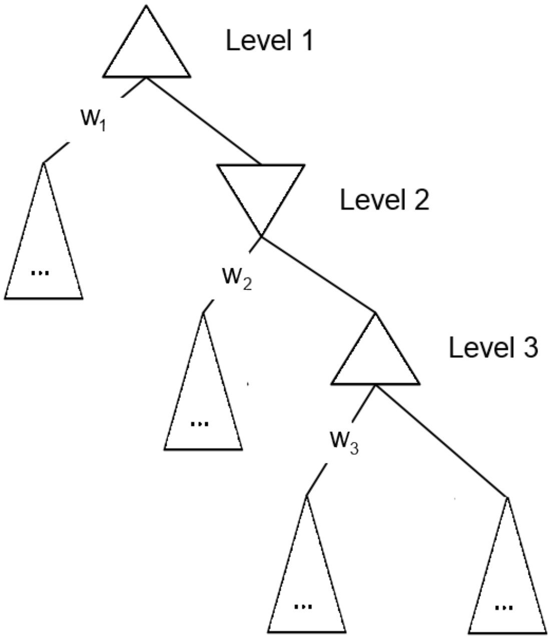
(a) Pacman is going to play against a careless ghost, which makes a move that is optimal for Pacman $\frac{1}{3}$ of the time, and makes a move that that minimizes Pacman's utility the other $\frac{2}{3}$ of the time.

(i) [2 pts] Fill in the correct utility values in the game tree below where Pacman is the maximizer:



(ii) [2 pts] Draw a complete game tree for the game above that contains only max nodes, min nodes, and chance nodes.

(b) Consider a modification of alpha-beta pruning where, rather than keeping track of a single value for α and β, you instead keep a list containing the best value, w_i, for the minimizer/maximizer (depending on the level) at each level up to and including the current level. Assume that the root node is always a max node. For example, consider the following game tree in which the first 3 levels are shown. When the considering the right child of the node at level 3, you have access to w_1, w_2, and w_3.



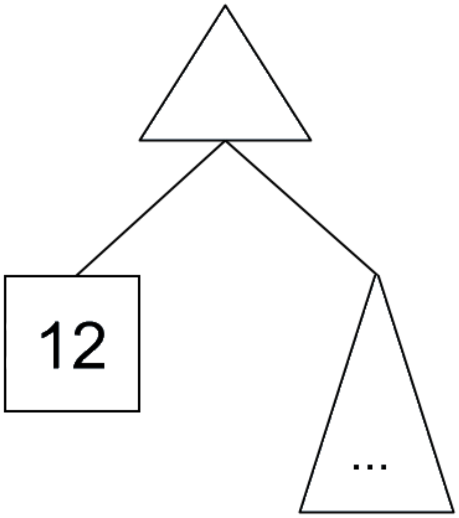
(i) [1 pt] Under this new scenario, what is the pruning condition for a max node at the n^{th} level of the tree (in terms of v and $w_1 ... w_n$)?

(ii) [1 pt] What is the pruning condition for a min node at the n^{th} level of the tree?

(iii) [2 pts] What is the relationship between α, β and the list of $w_1 ... w_n$ at a max node at the n^{th} level of the tree?

○ $\sum_i w_i = \alpha + \beta$
○ $\max_i w_i = \alpha, \quad \min_i w_i = \beta$
○ $\min_i w_i = \alpha, \quad \max_i w_i = \beta$
○ $w_n = \alpha, \quad w_{n-1} = \beta$
○ $w_{n-1} = \alpha, \quad w_n = \beta$
○ None of the above. The relationship is __________________________

(c) Pacman is in a dilemma. He is trying to maximize his overall utility in a game, which is modeled as the following game tree.



The left subtree contains a utility of 12. The right subtree contains an unknown utility value. An oracle has told you that the value of the right subtree is one of -3, -9, or 21. You know that each value is equally likely, but without exploring the subtree you do not know which one it is.

Now Pacman has 3 options:

1. Choose left;
2. Choose right;
3. Pay a cost of c = 1 to explore the right subtree, determine the exact utility it contains, and then make a decision.

(i) [3 pts] What is the expected utility for option 3?

(ii) [4 pts] For what values of c (for example, $c > 5$ or $-2 < c < 2$) should Pacman choose option 3? If option 3 is never optimal regardless of the value for c, write None.

Q5. [10 pts] More Games and Utilities

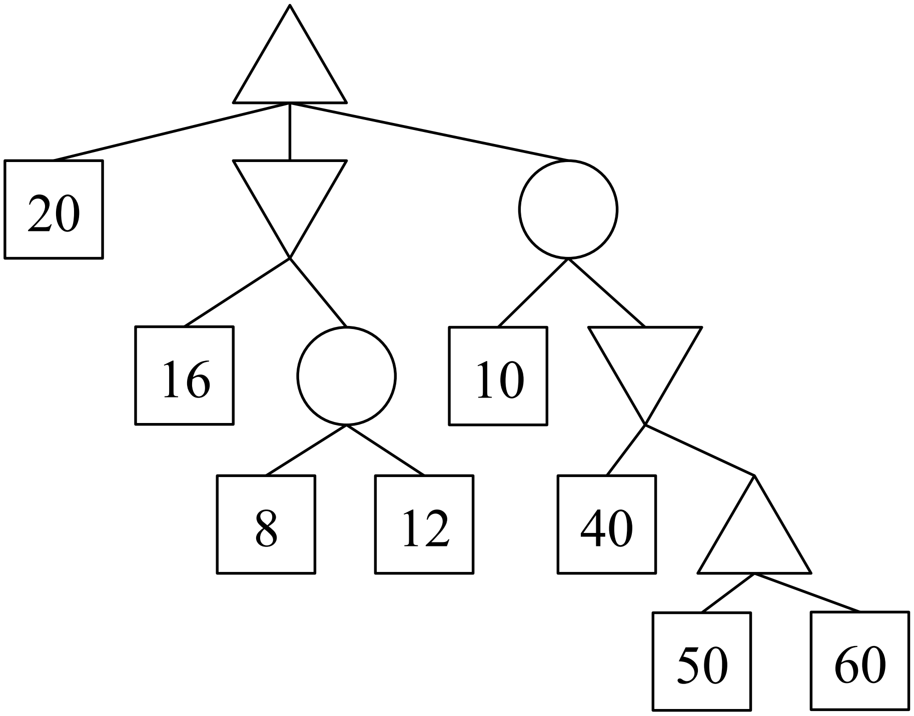
(a) Games. Consider the game tree below, which contains maximizer nodes, minimizer nodes, and chance nodes. For the chance nodes the probability of each outcome is equally likely.

(i) [3 pts] Fill in the values of each of the nodes.

(ii) [4 pts] Is pruning possible?

○ No. Brief justification: ________________________

○ Yes. Cross out the branches that can be pruned.



(b) Utilities. Pacman's utility function is $U(\$x) = \sqrt{x}$. He is faced with the following lottery: $[0.5, \$36 \ ; \ 0.5, \$64]$. Compute the following quantities:

(i) [1 pt] What is Pacman's expected utility?

$EU([0.5, \$36 \ ; \ 0.5, \ \$64]) =$

(ii) [1 pt] What is Equivalent Monetary Value for this lottery?

$EMV([0.5, \$36 \ ; \ 0.5, \$64]) =$

(iii) [1 pt] What is the maximum amount Pacman would be willing to pay for an insurance that guarantees he gets \$64 in exchange for giving his lottery to the insurance company?

Q6. [9 pts] Faulty Keyboard

Your new wireless keyboard works great, but when the battery runs out it starts behaving strangely. All keys other than $\{a, b, c\}$ stop working. The $\{a, b, c\}$ keys work as intended with probability 0.8, but could also produce either of the other two different characters with probability 0.1 each. For example, pressing a will produce 'A' with probability 0.8, 'B' with probability 0.1, and 'C' with probability 0.1. After 11 key presses the battery is completely drained and no new characters can be produced.

The naïve state space for this problem consists of three states $\{A, B, C\}$ and three actions $\{a, b, c\}$.

For each of the objectives below specify a reward function on the naïve state space such that the optimal policy in the resulting MDP optimizes the specified objective. The reward can only depend on current state, current action, and next state; it cannot depend on time.

If no such reward function exist, then specify how the naïve state space needs to be expanded and a reward function on this expanded state space such that the optimal policy in the resulting MDP optimizes the specified objective. The size of the extended state space needs to be minimal. You may assume that $\gamma = 1$.

(a) [3 pts] Produce as many B's as possible before the first C.

○ $R(s, a, s') =$ ____

○ The state-space needs to be extended to include: ____________________________

Reward function over extended state space: ____________________________

(b) [3 pts] Produce a string where the number of B's and the number of C's are as close as possible.

○ $R(s, a, s') =$ ____

○ The state-space needs to be extended to include: ____________________________

Reward function over extended state space: ____________________________

(c) [3 pts] Produce a string where the first character re-appears in the string as many times as possible.

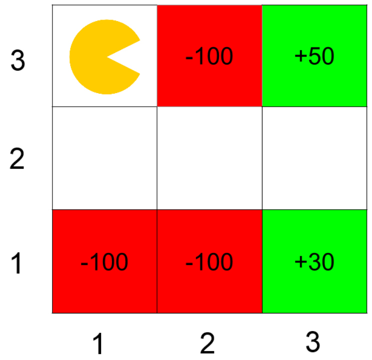
○ $R(s, a, s') =$ ____

○ The state-space needs to be extended to include: ____________________________

Reward function over extended state space: ____________________________

Q7. [8 pts] Deep inside Q-learning

Consider the grid-world given below and an agent who is trying to learn the optimal policy. Rewards are only awarded for taking the *Exit* action from one of the shaded states. Taking this action moves the agent to the Done state, and the MDP terminates. Assume $\gamma = 1$ and $\alpha = 0.5$ for all calculations. All equations need to explicitly mention γ and α if necessary.



(a) [3 pts] The agent starts from the top left corner and you are given the following episodes from runs of the agent through this grid-world. Each line in an Episode is a tuple containing (s, a, s', r).

Episode 1	Episode 2	Episode 3	Episode 4	Episode 5
(1,3), S, (1,2), 0	(1,3), S, (1,2), 0	(1,3), S, (1,2), 0	(1,3), S, (1,2), 0	(1,3), S, (1,2), 0
(1,2), E, (2,2), 0	(1,2), E, (2,2), 0	(1,2), E, (2,2), 0	(1,2), E, (2,2), 0	(1,2), E, (2,2), 0
(2,2), E, (3,2), 0	(2,2), S, (2,1), 0	(2,2), E, (3,2), 0	(2,2), E, (3,2), 0	(2,2), E, (3,2), 0
(3,2), N, (3,3), 0	(2,1), Exit, D, -100	(3,2), S, (3,1), 0	(3,2), N, (3,3), 0	(3,2), S, (3,1), 0
(3,3), Exit, D, +50		(3,1), Exit, D, +30	(3,3), Exit, D, +50	(3,1), Exit, D, +30

Fill in the following Q-values obtained from direct evaluation from the samples:

$Q((3,2), \text{N}) = \underline{\hspace{2cm}}$ $\qquad$ $Q((3,2), \text{S}) = \underline{\hspace{2cm}}$ $\qquad$ $Q((2,2), \text{E}) = \underline{\hspace{2cm}}$

(b) [3 pts] Q-learning is an online algorithm to learn optimal Q-values in an MDP with unknown rewards and transition function. The update equation is:

$$Q(s_t, a_t) = (1 - \alpha)Q(s_t, a_t) + \alpha(r_t + \gamma \max_{a'} Q(s_{t+1}, a'))$$

where γ is the discount factor, α is the learning rate and the sequence of observations are $(\cdots, s_t, a_t, s_{t+1}, r_t, \cdots)$. Given the episodes in (a), fill in the time at which the following Q values first become non-zero. Your answer should be of the form (**episode#,iter#**) where **iter#** is the Q-learning update iteration in that episode. If the specified Q value never becomes non-zero, write *never*.

$Q((1,2), \text{E}) = \underline{\hspace{2cm}}$ $\qquad$ $Q((2,2), \text{E}) = \underline{\hspace{2cm}}$ $\qquad$ $Q((3,2), \text{S}) = \underline{\hspace{2cm}}$

(c) [2 pts] In Q-learning, we look at a window of (s_t, a_t, s_{t+1}, r_t) to update our Q-values. One can think of using an update rule that uses a larger window to update these values. Give an update rule for $Q(s_t, a_t)$ given the window $(s_t, a_t, r_t, s_{t+1}, a_{t+1}, r_{t+1}, s_{t+2})$.

$Q(s_t, a_t) =$

$Q(s_t, a_t) =$

$Q(s_t, a_t) =$

Q8. [14 pts] Bellman Equations for the Post-Decision State

Consider an infinite-horizon, discounted MDP (S, A, T, R, γ). Suppose that the transition probabilities and the reward function have the following form:

$$T(s, a, s') = P(s'|f(s, a))$$
$$R(s, a, s') = R(s, a)$$

Here, f is some deterministic function mapping $S \times A \to Y$, where Y is a set of states called *post-decision states*. We will use the letter y to denote an element of Y, i.e., a post-decision state. In words, the state transitions consist of two steps: a deterministic step that depends on the action, and a stochastic step that does not depend on the action. The sequence of states (s_t), actions (a_t), post-decision-states (y_t), and rewards (r_t) is illustrated below.

$$\begin{tikzcd}
(s_0, a_0) \arrow[r, "f"] \arrow[dr, dotted] & y_0 \arrow[r, "P"] & (s_1, a_1) \arrow[r, "f"] \arrow[dr, dotted] & y_1 \arrow[r, "P"] & (s_2, a_2) \arrow[r, "f"] \arrow[dr, dotted] & \cdots \\
 & r_0 & & r_1 & & r_2
\end{tikzcd}$$

You have learned about $V^\pi(s)$, which is the expected discounted sum of rewards, starting from state s, when acting according to policy π.

$$V^\pi(s_0) = E\left[R(s_0, a_0) + \gamma R(s_1, a_1) + \gamma^2 R(s_2, a_2) + \dots\right]$$
$$\text{given } a_t = \pi(s_t) \text{ for } t = 0, 1, 2, \dots$$

$V^*(s)$ is the value function of the optimal policy, $V^*(s) = \max_\pi V^\pi(s)$.

This question will explore the concept of computing value functions on the post-decision-states y. [1]

$$W^\pi(y_0) = E\left[R(s_1, a_1) + \gamma R(s_2, a_2) + \gamma^2 R(s_3, a_3) + \dots\right]$$

We define $W^*(y) = \max_\pi W^\pi(y)$.

(a) [2 pts] Write W^* in terms of V^*.
$W^*(y) =$

○ $\sum_{s'} P(s' \mid y) V^*(s')$
○ $\sum_{s'} P(s' \mid y)[V^*(s') + \max_a R(s', a)]$
○ $\sum_{s'} P(s' \mid y)[V^*(s') + \gamma \max_a R(s', a)]$
○ $\sum_{s'} P(s' \mid y)[\gamma V^*(s') + \max_a R(s', a)]$
○ None of the above

(b) [2 pts] Write V^* in terms of W^*.
$V^*(s) =$

○ $\max_a [W^*(f(s, a))]$
○ $\max_a [R(s, a) + W^*(f(s, a))]$
○ $\max_a [R(s, a) + \gamma W^*(f(s, a))]$
○ $\max_a [\gamma R(s, a) + W^*(f(s, a))]$
○ None of the above

[1] In some applications, it is easier to learn an approximate W function than V or Q. For example, to use reinforcement learning to play Tetris, a natural approach is to learn the value of the block pile *after* you've placed your block, rather than the value of the pair (current block, block pile). TD-Gammon, a computer program developed in the early 90s, was trained by reinforcement learning to play backgammon as well as the top human experts. TD-Gammon learned an approximate W function.

(c) [4 pts] Recall that the optimal value function V^* satisfies the Bellman equation:

$$V^*(s) = \max_a \sum_{s'} T(s,a,s') \left(R(s,a) + \gamma V^*(s') \right),$$

which can also be used as an update equation to compute V^*.

Provide the equivalent of the Bellman equation for W^*.

$W^*(y) =$____________________________

(d) [3 pts] Fill in the blanks to give a policy iteration algorithm, which is guaranteed return the optimal policy π^*.

- Initialize policy $\pi^{(1)}$ arbitrarily.
- For $i = 1, 2, 3, \ldots$
  - Compute $W^{\pi^{(i)}}(y)$ for all $y \in Y$.
  - Compute a new policy $\pi^{(i+1)}$, where $\pi^{(i+1)}(s) = \arg\max_a$ ___(1)___ for all $s \in S$.
  - If ___(2)___ for all $s \in S$, **return** $\pi^{(i)}$.

Fill in your answers for blanks (1) and (2) below.

(1)
○ $W^{\pi^{(i)}}(f(s,a))$
○ $R(s,a) + W^{\pi^{(i)}}(f(s,a))$
○ $R(s,a) + \gamma W^{\pi^{(i)}}(f(s,a))$
○ $\gamma R(s,a) + W^{\pi^{(i)}}(f(s,a))$
○ None of the above

(2) ____

(e) [3 pts] In problems where f is known but $P(s'|y)$ is not necessarily known, one can devise reinforcement learning algorithms based on the W^* and W^π functions. Suppose that an the agent goes through the following sequence of states, actions and post-decision states: $s_t, a_t, y_t, s_{t+1}, a_{t+1}, y_{t+1}$. Let $\alpha \in (0,1)$ be the learning rate parameter.

Write an update equation analogous to Q-learning that enables one to estimate W^*

$W(y_t) \leftarrow (1-\alpha)W(y_t) + \alpha\,($____$)$