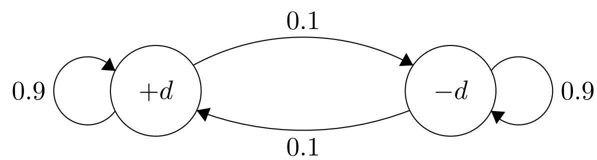
CS 188
Fall 2019

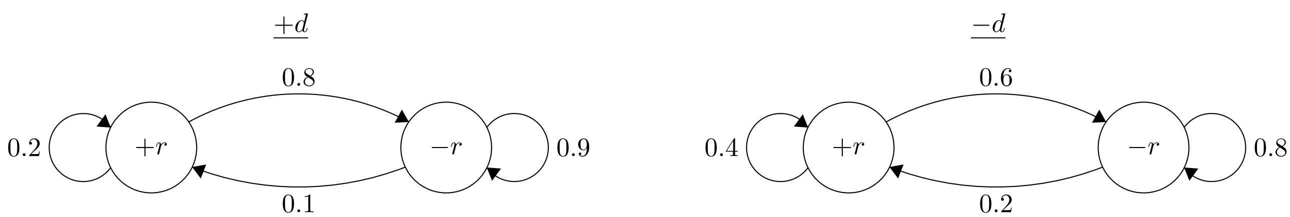
Final Review Bayes Nets Solutions

Q1. I Heard You Like Markov Chains

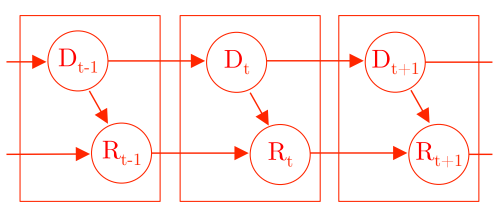
In California, whether it rains or not from each day to the next forms a Markov chain (note: this is a terrible model for real weather). However, sometimes California is in a drought and sometimes it is not. Whether California is in a drought from each day to the next itself forms a Markov chain, and the state of this Markov chain affects the transition probabilities in the rain-or-shine Markov chain. This is the state diagram for droughts:



These are the state diagrams for rain given that California is and is not in a drought, respectively:



(a) Draw a dynamic Bayes net which encodes this behavior. Use variables D_{t-1}, D_t, D_{t+1}, R_{t-1}, R_t, and R_{t+1}. Assume that on a given day, it is determined whether or not there is a drought before it is determined whether or not it rains that day.



(b) Draw the CPT for D_t in the above DBN. Fill in the actual numerical probabilities.

$P(D_t \mid D_{t-1})$		
$+d_{t-1}$	$+d_t$	0.9
$+d_{t-1}$	$-d_t$	0.1
$-d_{t-1}$	$+d_t$	0.1
$-d_{t-1}$	$-d_t$	0.9

(c) Draw the CPT for R_t in the above DBN. Fill in the actual numerical probabilities.

$P(R_t \mid R_{t-1}, D_t)$			
$+d_t$	$+r_{t-1}$	$+r_t$	0.2
$+d_t$	$+r_{t-1}$	$-r_t$	0.8
$+d_t$	$-r_{t-1}$	$+r_t$	0.1
$+d_t$	$-r_{t-1}$	$-r_t$	0.9
$-d_t$	$+r_{t-1}$	$+r_t$	0.4
$-d_t$	$+r_{t-1}$	$-r_t$	0.6
$-d_t$	$-r_{t-1}$	$+r_t$	0.2
$-d_t$	$-r_{t-1}$	$-r_t$	0.8

Suppose we are observing the weather on a day-to-day basis, but we cannot directly observe whether California is in a drought or not. We want to predict whether or not it will rain on day $t+1$ given observations of whether or not it rained on days 1 through t.

(d) First, we need to determine whether California will be in a drought on day $t+1$. Derive a formula for $P(D_{t+1}|r_{1:t})$ in terms of the given probabilities (the transition probabilities on the above state diagrams) and $P(D_t|r_{1:t})$ (that is, you can assume we've already computed the probability there is a drought today given the weather over time).

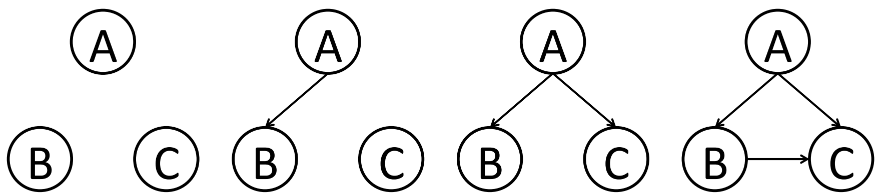
$$P(D_{t+1}|r_{1:t}) = \sum_{d_t} P(D_{t+1}|d_t)P(d_t|r_{1:t})$$

(e) Now derive a formula for $P(R_{t+1}|r_{1:t})$ in terms of $P(D_{t+1}|r_{1:t})$ and the given probabilities.

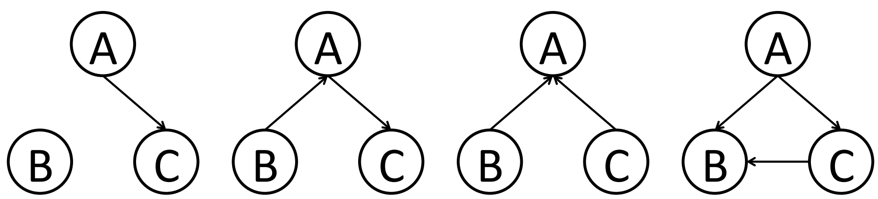
$$P(R_{t+1}|r_{1:t}) = \sum_{d_{t+1}} P(D_{t+1}|r_{1:t})P(R_{t+1}|r_t, d_{t+1})$$

Q2. Learning a Bayes' Net Structure

(a) You want to learn a Bayes' net over the random variables A, B, C. You decide you want to learn not only the Bayes' net parameters, but also the structure from the data. You are willing to consider the 8 structures shown below. First you use your training data to perform maximum likelihood estimation of the parameters of each of the Bayes' nets. Then for each of the learned Bayes' nets, you evaluate the likelihood of the training data (l^{train}), and the likelihood of your cross-validation data (l^{cross}). Both likelihoods are shown below each structure.



l^{train}	0.0001	0.0005	0.0015	0.0100
l^{cross}	0.0001	0.0004	0.0011	0.0009
	(a)	(b)	(c)	(d)



l^{train}	0.0008	0.0015	0.0020	0.0100
l^{cross}	0.0006	0.0011	0.0010	0.0009
	(e)	(f)	(g)	(h)

(i) Which Bayes' net structure will (on expectation) perform best on test-data? (If there is a tie, list all Bayes' nets that are tied for the top spot.) Justify your answer.

Bayes' nets (c) and (f) as they have the highest cross validation data likelihood.

(ii) Two pairs of the learned Bayes' nets have identical likelihoods. Explain why this is the case.

(c) and (f) have the same likelihoods, and (d) and (h) have the same likelihoods. When learning a Bayes' net with maximum likelihood, we end up selecting the distribution that maximizes the likelihood of the training data from the set of all distributions that can be represented by the Bayes' net structure. (c) and (f) have the same set of conditional independence assumptions, and hence can represent the same set of distributions. This means that they end up with the same distribution as the one that maximizes the training data likelihood, and therefore have identical training and cross validation likelihoods. Same holds true for (d) and (h).

(iii) For every two structures S_1 and S_2, where S_2 can be obtained from S_1 by adding one or more edges, l^{train} is higher for S_2 than for S_1. Explain why this is the case. When learning a Bayes' net with maximum likelihood, we end up selecting the distribution that maximizes the likelihood of the training data from the set of all distributions that can be represented by the Bayes' net structure. Adding an edge grows the set of distributions that can be represented by the Bayes' net, and can hence only increase the training data likelihood under the best distribution in this set.