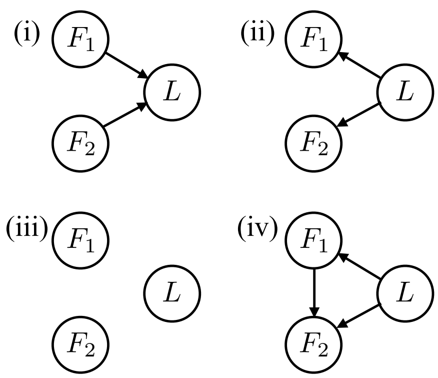
CS 188
Fall 2019

Final Review ML Solutions

Q1. ML: Maximum Likelihood



Training Data

$(L = 1,\ \ F_1 = 1,\ \ F_2 = 1)$
$(L = 1,\ \ F_1 = 1,\ \ F_2 = 1)$
$(L = 0,\ \ F_1 = 1,\ \ F_2 = 1)$
$(L = 1,\ \ F_1 = 0,\ \ F_2 = 0)$
$(L = 0,\ \ F_1 = 0,\ \ F_2 = 0)$
$(L = 0,\ \ F_1 = 0,\ \ F_2 = 0)$
$(L = 0,\ \ F_1 = 0,\ \ F_2 = 1)$

You've decided to use a model-based approach to classification of text documents. Your goal is to build a classifier that can determine whether or not a document is about cats. You're taking a minimalist approach and you're only characterizing the input documents in terms of two binary features: F_1 and F_2. Both of these features have domain $\{0, 1\}$. The thing you're trying to predict is the label, L, which is also binary valued. When $L = 1$, the document is about cats. When $L = 0$, the document is not.

The particular meaning of the two features F_1 and F_2 is not important for your current purposes. You are only trying to decide on a particular Bayes' net structure for your classifier. You've got your hands on some training data (shown above) and you're trying to figure out which of several potential Bayes' nets (also shown above) might yield a decent classifier when trained on that training data.

(a) Which of the Bayes' nets, once learned from the training data with maximum likelihood estimation, would assign non-zero probability to the following query: $P(L = 1|F_1 = 0, F_2 = 0)$? Fill in all that apply.

● (i) ● (ii) ● (iii) ● (iv)

(b) Which of the Bayes' nets, once learned from the training data with maximum likelihood estimation, would assign non-zero probability to the following query: $P(L = 1|F_1 = 0, F_2 = 1)$? Fill in all that apply.

○ (i) ● (ii) ● (iii) ○ (iv)

(c) Which of the Bayes' nets, once learned from the training data with Laplace smoothing using $k = 1$, would assign non-zero probability to the following query: $P(L = 1|F_1 = 0, F_2 = 1)$? Fill in all that apply.

● (i) ● (ii) ● (iii) ● (iv)

(d) What probability does Bayes' net (i), once learned from the training data with Laplace smoothing using $k = 1$, assign to the query $P(L = 1|F_1 = 0, F_2 = 1)$?

$\frac{1}{3}$

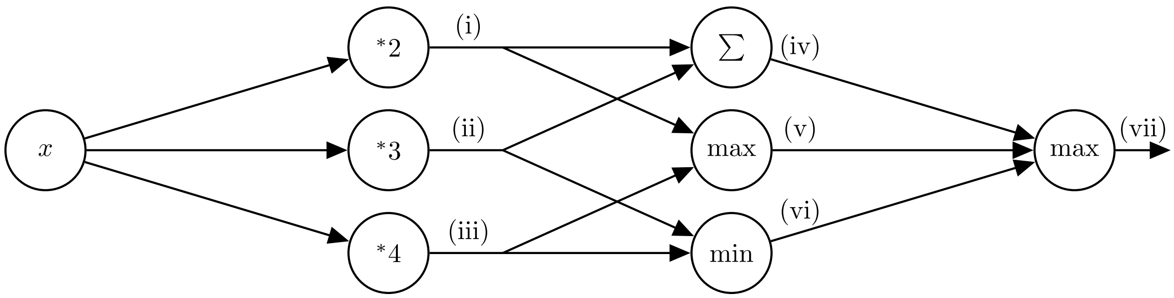
(e) As $k \to \infty$ (the constant used for Laplace smoothing), what does the probability that Bayes' net (i) assigns to the query $P(L = 1|F_1 = 0, F_2 = 1)$ converge to?

$\frac{1}{2}$

Q2. Deep Learning

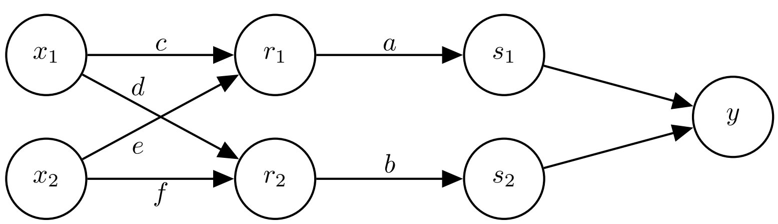
(a) Perform forward propagation on the neural network below for $x = 1$ by filling in the values in the table. Note that (i), . . . , (vii) are outputs after performing the appropriate operation as indicated in the node.

(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
2	3	4	5	4	3	5



(b) Below is a neural network with weights a, b, c, d, e, f. The inputs are x_1 and x_2.
The first hidden layer computes $r_1 = \max(c \cdot x_1 + e \cdot x_2, 0)$ and $r_2 = \max(d \cdot x_1 + f \cdot x_2, 0)$.
The second hidden layer computes $s_1 = \frac{1}{1+\exp(-a \cdot r_1)}$ and $s_2 = \frac{1}{1+\exp(-b \cdot r_2)}$.
The output layer computes $y = s_1 + s_2$. Note that the weights a, b, c, d, e, f are indicated along the edges of the neural network here.

Suppose the network has inputs $x_1 = 1, x_2 = -1$.
The weight values are $a = 1, b = 1, c = 4, d = 1, e = 2, f = 2$.
Forward propagation then computes $r_1 = 2, r_2 = 0, s_1 = 0.9, s_2 = 0.5, y = 1.4$. Note: some values are rounded.



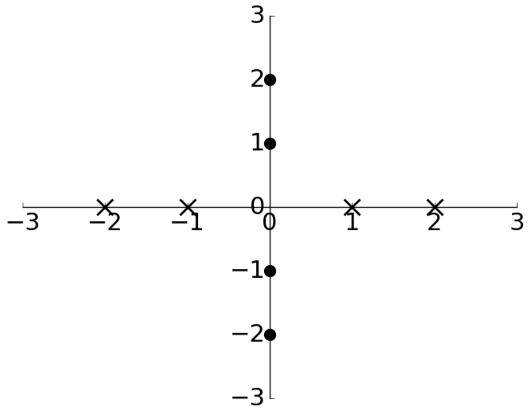
Using the values computed from forward propagation, use backpropagation to numerically calculate the following partial derivatives. Write your answers as a single number (not an expression). You do not need a calculator. Use scratch paper if needed.
Hint: For $g(z) = \frac{1}{1+\exp(-z)}$, the derivative is $\frac{\partial g}{\partial z} = g(z)(1 - g(z))$.

$\frac{\partial y}{\partial a}$	$\frac{\partial y}{\partial b}$	$\frac{\partial y}{\partial c}$	$\frac{\partial y}{\partial d}$	$\frac{\partial y}{\partial e}$	$\frac{\partial y}{\partial f}$
0.18	0	0.09	0	−0.09	0

$$\begin{aligned}
\frac{\partial y}{\partial a} &= \frac{\partial y}{\partial s_1}\frac{\partial s_1}{\partial a} \\
&= 1 \cdot \frac{\partial g(a \cdot r_1)}{\partial a} \\
&= r_1 \cdot g(a \cdot r_1)(1 - g(a \cdot r_1)) \\
&= r_1 \cdot s_1(1 - s_1) \\
&= 2 \cdot 0.9 \cdot (1 - 0.9) \\
&= 0.18 \\
\frac{\partial y}{\partial b} &= \frac{\partial y}{\partial s_2}\frac{\partial s_2}{\partial b} \\
&= 1 \cdot \frac{\partial g(b \cdot r_2)}{\partial b} \\
&= r_2 \cdot g(b \cdot r_2)(1 - g(b \cdot r_2)) \\
&= r_2 \cdot s_2(1 - s_2) \\
&= 0 \cdot 0.5(1 - 0.5) \\
&= 0 \\
\frac{\partial y}{\partial c} &= \frac{\partial y}{\partial s_1}\frac{\partial s_1}{\partial r_1}\frac{\partial r_1}{\partial c} \\
&= 1 \cdot [a \cdot g(a \cdot r_1)(1 - g(a \cdot r_1))] \cdot x_1 \\
&= [a \cdot s_1(1 - s_1)] \cdot x_1 \\
&= [1 \cdot 0.9(1 - 0.9)] \cdot 1 \\
&= 0.09 \\
\frac{\partial y}{\partial d} &= \frac{\partial y}{\partial s_2}\frac{\partial s_2}{\partial r_2}\frac{\partial r_2}{\partial d} \\
&= \frac{\partial y}{\partial s_2}\frac{\partial s_2}{\partial r_2} \cdot 0 \\
&= 0 \\
\frac{\partial y}{\partial e} &= \frac{\partial y}{\partial s_1}\frac{\partial s_1}{\partial r_1}\frac{\partial r_1}{\partial e} \\
&= 1 \cdot [a \cdot g(a \cdot r_1)(1 - g(a \cdot r_1))] \cdot x_2 \\
&= [a \cdot s_1(1 - s_1)] \cdot x_2 \\
&= [1 \cdot 0.9(1 - 0.9)] \cdot -1 \\
&= -0.09 \\
\frac{\partial y}{\partial f} &= \frac{\partial y}{\partial s_2}\frac{\partial s_2}{\partial r_2}\frac{\partial r_2}{\partial f} \\
&= \frac{\partial y}{\partial s_2}\frac{\partial s_2}{\partial r_2} \cdot 0 \\
&= 0
\end{aligned}$$

(c) Below are two plots with horizontal axis x_1 and vertical axis x_2 containing data labelled $\times$ and $\bullet$. For each plot, we wish to find a function $f(x_1, x_2)$ such that $f(x_1, x_2) \geq 0$ for all data labelled $\times$ and $f(x_1, x_2) < 0$ for all data labelled $\bullet$.

Below each plot is the function $f(x_1, x_2)$ for that specific plot. Complete the expressions such that all the data is labelled correctly. If not possible, mark "No valid combination".



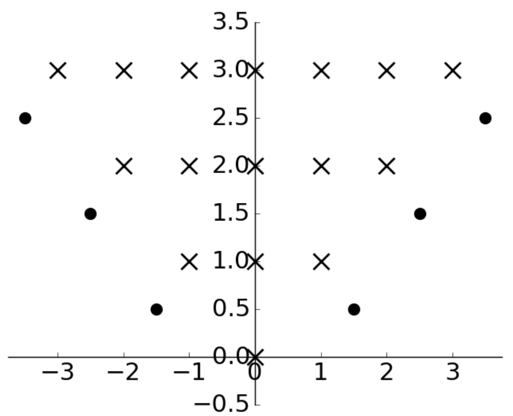
$f(x_1, x_2) = \max(\underline{\textbf{(i)}} + \underline{\textbf{(ii)}}, \underline{\textbf{(iii)}} + \underline{\textbf{(iv)}}) + \underline{\textbf{(v)}}$

(i)	● x_1	○ $-x_1$	○ 0
(ii)	○ x_2	○ $-x_2$	● 0
(iii)	○ x_1	● $-x_1$	○ 0
(iv)	○ x_2	○ $-x_2$	● 0
(v)	○ 1	● -1	○ 0

○ No valid combination

There are two possible solutions:

$$f(x_1, x_2) = \max(x_1, -x_1) - 1$$
$$f(x_1, x_2) = \max(-x_1, x_1) - 1$$



$f(x_1, x_2) = \underline{\textbf{(vi)}} - \max(\underline{\textbf{(vii)}} + \underline{\textbf{(viii)}}, \underline{\textbf{(ix)}} + \underline{\textbf{(x)}})$

(vi)	● x_2	○ $-x_2$	○ 0
(vii)	● x_1	○ $-x_1$	○ 0
(viii)	○ x_2	○ $-x_2$	● 0
(ix)	○ x_1	● $-x_1$	○ 0
(x)	○ x_2	○ $-x_2$	● 0

○ No valid combination

There are four possible solutions:

$$f(x_1, x_2) = x_2 - \max(x_1, -x_1)$$
$$f(x_1, x_2) = x_2 - \max(-x_1, x_1)$$
$$f(x_1, x_2) = -\max(x_1 - x_2, -x_1 - x_2)$$
$$f(x_2, x_2) = -\max(-x_1 - x_2, x_1 - x_2)$$