Exam Prep 11

Q1. Perceptron

We would like to use a perceptron to train a classifier for datasets with 2 features per point and labels +1 or -1.

Consider the following labeled training data:

Features (x_1, x_2)	Label y^*
(-1,2)	1
(3,-1)	-1
(1,2)	-1
(3,1)	1

(a) Our two perceptron weights have been initialized to $w_1 = 2$ and $w_2 = -2$. After processing the first point with the perceptron algorithm, what will be the updated values for these weights?

(b) After how many steps will the perceptron algorithm converge? Write "never" if it will never converge.

Note: one steps means processing one point. Points are processed in order and then repeated, until convergence.

(c) Instead of the standard perceptron algorithm, we decide to treat the perceptron as a single node neural network and update the weights using gradient descent on the loss function.

The loss function for one data point is $Loss(y, y^*) = (y - y^*)^2$, where y^* is the training label for a given point and y is the output of our single node network for that point.

(i) Given a general activation function $g(z)$ and its derivative $g'(z)$, what is the derivative of the loss function with respect to w_1 in terms of g, g', y^*, x_1, x_2, w_1, and w_2?

$\frac{\partial Loss}{\partial w_1} =$

(ii) For this question, the specific activation function that we will use is:

$$g(z) = 1 \text{ if } z \geq 0 \text{ and } = -1 \text{ if } z < 0$$

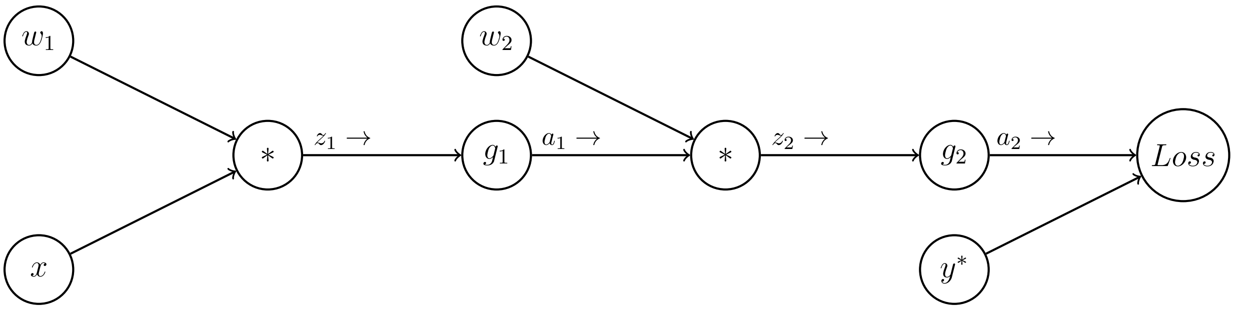
Given the following gradient descent equation to update the weights given a single data point. With initial weights of $w_1 = 2$ and $w_2 = -2$, what are the updated weights after processing the first point?

Gradient descent update equation: $w_i = w_i - \alpha \frac{\partial Loss}{\partial w_1}$

(iii) What is the most critical problem with this gradient descent training process with that activation function?

2 Neural Nets

Consider the following computation graph for a simple neural network for binary classification. Here x is a single real-valued input feature with an associated class y^* (0 or 1). There are two weight parameters w_1 and w_2, and non-linearity functions g_1 and g_2 (to be defined later, below). The network will output a value a_2 between 0 and 1, representing the probability of being in class 1. We will be using a loss function $Loss$ (to be defined later, below), to compare the prediction a_2 with the true class y^*.



1. Perform the forward pass on this network, writing the output values for each node z_1, a_1, z_2 and a_2 in terms of the node's input values:

2. Compute the loss $Loss(a_2, y^*)$ in terms of the input x, weights w_i, and activation functions g_i:

3. Now we will work through parts of the backward pass, incrementally. Use the chain rule to derive $\frac{\partial Loss}{\partial w_2}$. Write your expression as a product of partial derivatives at each node: i.e. the partial derivative of the node's output with respect to its inputs. (Hint: the series of expressions you wrote in part 1 will be helpful; you may use any of those variables.)

4. Suppose the loss function is quadratic, $Loss(a_2, y^*) = \frac{1}{2}(a_2 - y^*)^2$, and g_1 and g_2 are both sigmoid functions $g(z) = \frac{1}{1+e^{-z}}$ (note: it's typically better to use a different type of loss, *cross-entropy*, for classification problems, but we'll use this to make the math easier).

   Using the chain rule from Part 3, and the fact that $\frac{\partial g(z)}{\partial z} = g(z)(1 - g(z))$ for the sigmoid function, write $\frac{\partial Loss}{\partial w_2}$ in terms of the values from the forward pass, y^*, a_1, and a_2:

5. Now use the chain rule to derive $\frac{\partial Loss}{\partial w_1}$ as a product of partial derivatives at each node used in the chain rule:

6. Finally, write $\frac{\partial Loss}{\partial w_1}$ in terms of x, y^*, w_i, a_i, z_i:

7. What is the gradient descent update for w_1 with step-size α in terms of the values computed above?