CS 188
Fall 2019

Exam Prep 6 Solutions

Q1. Probability and Bayes Nets

(a) A, B, and C are random variables with binary domains. How many entries are in the following probability tables and what is the sum of the values in each table? Write a "?" in the box if there is not enough information given.

Table	Size	Sum
$P(A, B\|C)$	8	2
$P(A\| +b, +c)$	2	1
$P(+a\|B)$	2	?

(b) Circle true if the following probability equalities are valid and circle false if they are invalid (leave it blank if you don't wish to risk a guess). Each True/False question is worth 1 points. Leaving a question blank is worth 0 points. **Answering incorrectly is worth -1 points.**

No independence assumptions are made.

(i) [*true* or *false*] $P(A, B) = P(A|B)P(A)$
False. $P(A, B) = P(A|B)P(B)$ would be a valid example.

(ii) [*true* or *false*] $P(A|B)P(C|B) = P(A, C|B)$
False. This assumes that A and C are conditionally independent given B.

(iii) [*true* or *false*] $P(B, C) = \sum_{a \in A} P(B, C|A)$
False. $P(B, C) = \sum_{a \in A} P(A, B, C)$ would be a valid example.

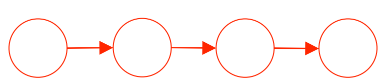
(iv) [*true* or *false*] $P(A, B, C, D) = P(C)P(D|C)P(A|C, D)P(B|A, C, D)$
True. This is a valid application of the chain rule.

(c) Space Complexity of Bayes Nets

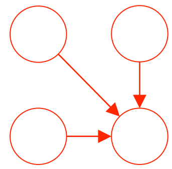
Consider a joint distribution over N variables. Let k be the domain size for all of these variables, and let d be the maximum indegree of any node in a Bayes net that encodes this distribution.

(i) What is the space complexity of storing the entire joint distribution? Give an answer of the form $O(\cdot)$.
$O(k^N)$ was the intended answer. Because of the potentially misleading wording, we also allowed $O(Nk^{d+1})$, one possible bound on the space complexity of storing the Bayes net ($O((N-d)k^{d+1})$ is an asymptotically tighter bound, but this requires considerably more effort to prove).

(ii) Draw an example of a Bayes net over four binary variables such that it takes less space to store the Bayes net than to store the joint distribution.
A simple Markov chain works. Size $2 + 4 + 4 + 4 = 14$, which is less than $2^4 = 16$. Less edges, less inbound edges (v-shape), or no edges would work too.



(iii) Draw an example of a Bayes net over four binary variables such that it takes more space to store the Bayes net than to store the joint distribution.
Size $2 + 2 + 2 + 2^4 = 22$, which is more than $2^4 = 16$. Other configurations could work too, especially any with a node with indegree 3.



Q2. Probability

(a) For the following questions, you will be given a set of probability tables and a set of conditional independence assumptions. Given these tables and independence assumptions, write an expression for the requested probability tables. Keep in mind that your expressions cannot contain any probabilities other than the given probability tables. If it is not possible, mark "Not possible."

(i) Using probability tables $\mathbf{P(A), P(A \mid C), P(B \mid C), P(C \mid A, B)}$ and no conditional independence assumptions, write an expression to calculate the table $\mathbf{P(A, B \mid C)}$.

$\mathbf{P(A, B \mid C)} =$ __________________ ● Not possible.

(ii) Using probability tables $\mathbf{P(A), P(A \mid C), P(B \mid A), P(C \mid A, B)}$ and no conditional independence assumptions, write an expression to calculate the table $\mathbf{P(B \mid A, C)}$.

$\mathbf{P(B \mid A, C)} = \frac{P(A)\ P(B|A)\ P(C|A,B)}{\sum_b P(A)\ P(B|A)\ P(C|A,B)}$ ○ Not possible.

(iii) Using probability tables $\mathbf{P(A \mid B), P(B), P(B \mid A, C), P(C \mid A)}$ and conditional independence assumption $\mathbf{A \perp\!\!\!\perp B}$, write an expression to calculate the table $\mathbf{P(C)}$.

$\mathbf{P(C)} = \sum_a P(A \mid B)\ P(C \mid A)$ ○ Not possible.

(iv) Using probability tables $\mathbf{P(A \mid B, C), P(B), P(B \mid A, C), P(C \mid B, A)}$ and conditional independence assumption $\mathbf{A \perp\!\!\!\perp B \mid C}$, write an expression for $\mathbf{P(A, B, C)}$.

$\mathbf{P(A, B, C)} =$ __________________ ● Not possible.

(b) For each of the following equations, select the *minimal set* of conditional independence assumptions necessary for the equation to be true.

(i) $\mathbf{P(A, C) = P(A \mid B)\ P(C)}$

- ■ $A \perp\!\!\!\perp B$
- ☐ $A \perp\!\!\!\perp B \mid C$
- ■ $A \perp\!\!\!\perp C$
- ☐ $A \perp\!\!\!\perp C \mid B$
- ☐ $B \perp\!\!\!\perp C$
- ☐ $B \perp\!\!\!\perp C \mid A$
- ☐ No independence assumptions needed.

Alternatively, $A \perp\!\!\!\perp B \mid C$ and $A \perp\!\!\!\perp C \mid B$ can be selected.

(ii) $\mathbf{P(A \mid B, C)} = \frac{\mathbf{P(A)\ P(B|A)\ P(C|A)}}{\mathbf{P(B|C)\ P(C)}}$

- ☐ $A \perp\!\!\!\perp B$
- ☐ $A \perp\!\!\!\perp B \mid C$
- ☐ $A \perp\!\!\!\perp C$
- ☐ $A \perp\!\!\!\perp C \mid B$
- ☐ $B \perp\!\!\!\perp C$
- ■ $B \perp\!\!\!\perp C \mid A$
- ☐ No independence assumptions needed.

(iii) $\mathbf{P(A, B) = \sum_c P(A \mid B, c)\ P(B \mid c)\ P(c)}$

- ☐ $A \perp\!\!\!\perp B$
- ☐ $A \perp\!\!\!\perp B \mid C$
- ☐ $A \perp\!\!\!\perp C$
- ☐ $A \perp\!\!\!\perp C \mid B$
- ☐ $B \perp\!\!\!\perp C$
- ☐ $B \perp\!\!\!\perp C \mid A$
- ■ No independence assumptions needed.

(iv) $\mathbf{P}(\mathbf{A}, \mathbf{B} \mid \mathbf{C}, \mathbf{D}) = \mathbf{P}(\mathbf{A} \mid \mathbf{C}, \mathbf{D})\ \mathbf{P}(\mathbf{B} \mid \mathbf{A}, \mathbf{C}, \mathbf{D})$

- [ ] $A \perp\!\!\!\perp B$
- [ ] $A \perp\!\!\!\perp B \mid C$
- [ ] $A \perp\!\!\!\perp B \mid D$
- [ ] $C \perp\!\!\!\perp D$
- [ ] $C \perp\!\!\!\perp D \mid A$
- [ ] $C \perp\!\!\!\perp D \mid B$
- [x] No independence assumptions needed.

(c) **(i)** Mark **all** expressions that are equal to $\mathbf{P}(\mathbf{A} \mid \mathbf{B})$, **given no independence assumptions**.

- [ ] $\sum_c P(A \mid B, c)$
- [x] $\sum_c P(A, c \mid B)$
- [ ] $\frac{P(B|A)\ P(A|C)}{\sum_c P(B,c)}$
- [x] $\frac{\sum_c P(A,B,c)}{\sum_c P(B,c)}$
- [ ] $\frac{P(A,C|B)}{P(C|B)}$
- [ ] $\frac{P(A|C,B)\ P(C|A,B)}{P(C|B)}$
- [ ] None of the provided options.

(ii) Mark **all** expressions that are equal to $\mathbf{P}(\mathbf{A}, \mathbf{B}, \mathbf{C})$, given that $\mathbf{A} \perp\!\!\!\perp \mathbf{B}$.

- [ ] $P(A \mid C)\ P(C \mid B)\ P(B)$
- [x] $P(A)\ P(B)\ P(C \mid A, B)$
- [ ] $P(C)\ P(A \mid C)\ P(B \mid C)$
- [ ] $P(A)\ P(C \mid A)\ P(B \mid C)$
- [x] $P(A)\ P(B \mid A)\ P(C \mid A, B)$
- [x] $P(A, C)\ P(B \mid A, C)$
- [ ] None of the provided options.

(iii) Mark **all** expressions that are equal to $\mathbf{P}(\mathbf{A}, \mathbf{B} \mid \mathbf{C})$, given that $\mathbf{A} \perp\!\!\!\perp \mathbf{B} \mid \mathbf{C}$.

- [x] $P(A \mid C)\ P(B \mid C)$
- [ ] $\frac{P(A)\ P(B|A)\ P(C|A,B)}{\sum_c P(A,B,c)}$
- [ ] $P(A \mid B)\ P(B \mid C)$
- [ ] $\frac{P(C)\ P(B|C)\ P(A|C)}{P(C|A,B)}$
- [ ] $\frac{\sum_c P(A,B,c)}{P(C)}$
- [x] $\frac{P(C,A|B)\ P(B)}{P(C)}$
- [ ] None of the provided options.