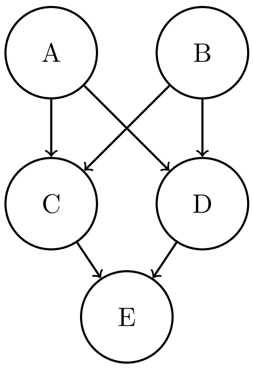
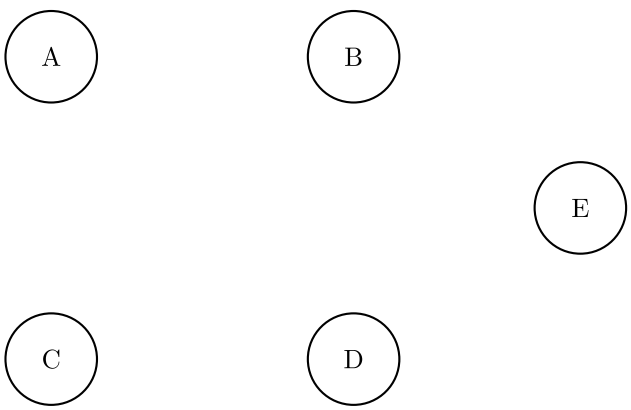
Exam Prep 7

Q1. Bayes Nets and Joint Distributions

(a) Write down the joint probability distribution associated with the following Bayes Net. Express the answer as a product of terms representing individual conditional probabilities tables associated with this Bayes Net:



(b) Draw the Bayes net associated with the following joint distribution:
$P(A) \cdot P(B) \cdot P(C|A, B) \cdot P(D|C) \cdot P(E|B, C)$



(c) Do the following products of factors correspond to a valid joint distribution over the variables A, B, C, D? (Circle TRUEFALSE.)

(i) TRUE FALSE $P(A) \cdot P(B) \cdot P(C|A) \cdot P(C|B) \cdot P(D|C)$

(ii) TRUE FALSE $P(A) \cdot P(B|A) \cdot P(C) \cdot P(D|B, C)$

(iii) TRUE FALSE $P(A) \cdot P(B|A) \cdot P(C) \cdot P(C|A) \cdot P(D)$

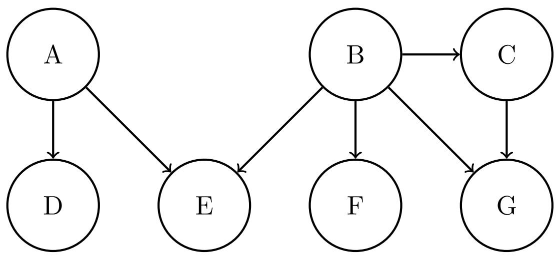
(iv) TRUE FALSE $P(A|B) \cdot P(B|C) \cdot P(C|D) \cdot P(D|A)$

(d) What factor can be multiplied with the following factors to form a valid joint distribution? (Write "none" if the given set of factors can't be turned into a joint by the inclusion of exactly one more factor.)

(i) $P(A) \cdot P(B|A) \cdot P(C|A) \cdot P(E|B, C, D)$

(ii) $P(D) \cdot P(B) \cdot P(C|D, B) \cdot P(E|C, D, A)$

(e) Answer the next questions based off of the Bayes Net below:
All variables have domains of {-1, 0, 1}



(i) Before eliminating any variables or including any evidence, how many entries does the factor at G have?

(ii) Now we observe $e = 1$ and want to query $P(D|e = 1)$, and you get to pick the first variable to be eliminated.

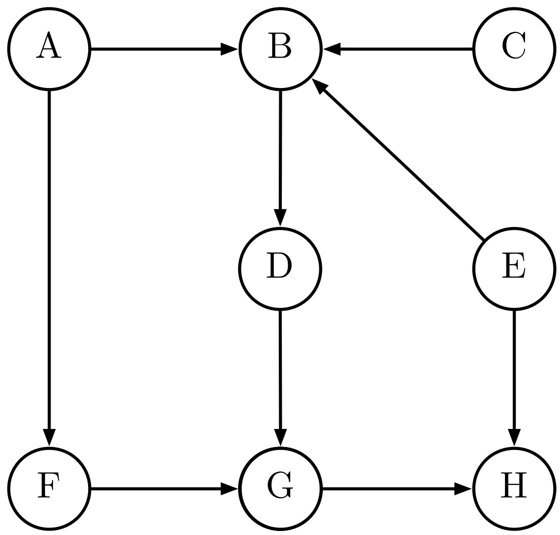
- Which choice would create the **largest** factor f_1?

- Which choice would create the **smallest** factor f_1?

Q2. Bayes' Nets Representation

(a) Graph Structure: Conditional Independence

Consider the Bayes' net given below.



Remember that $X \perp\!\!\!\perp Y$ reads as "X is independent of Y given nothing", and $X \perp\!\!\!\perp Y|\{Z,W\}$ reads as "X is independent of Y given Z and W."

For each expression, fill in the corresponding circle to indicate whether it is True or False.

(i) True False It is guaranteed that $A \perp\!\!\!\perp B$

(ii) True False It is guaranteed that $A \perp\!\!\!\perp C$

(iii) True False It is guaranteed that $A \perp\!\!\!\perp D \mid \{B, H\}$

(iv) True False It is guaranteed that $A \perp\!\!\!\perp E|F$

(v) True False It is guaranteed that $G \perp\!\!\!\perp E|B$

(vi) True False It is guaranteed that $F \perp\!\!\!\perp C|D$

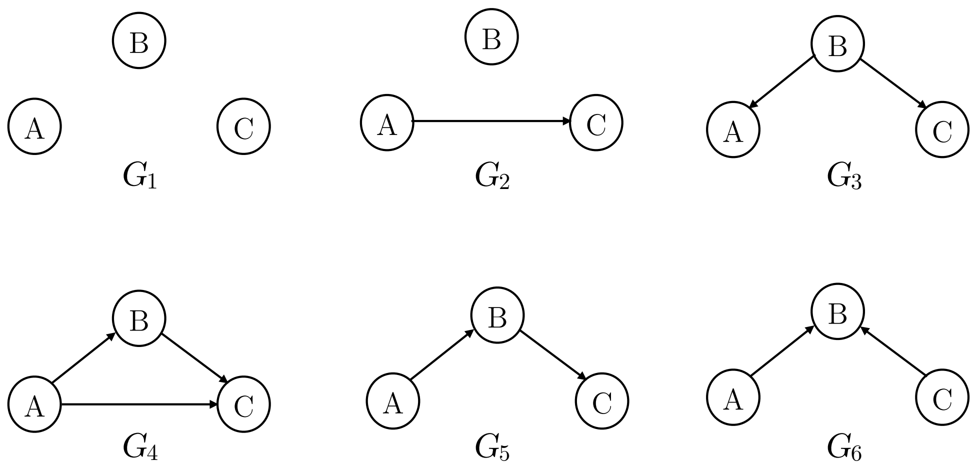
(vii) True False It is guaranteed that $E \perp\!\!\!\perp D|B$

(viii) True False It is guaranteeed that $C \perp\!\!\!\perp H|G$

(b) Graph structure: Representational Power

Recall that any directed acyclic graph G has an associated family of probability distributions, which consists of all probability distributions that can be represented by a Bayes' net with structure G.

For the following questions, consider the following six directed acyclic graphs:



(i) Assume all we know about the joint distribution $P(A, B, C)$ is that it can be represented by the product $P(A|B, C)P(B|C)P(C)$. Mark each graph for which the associated family of probability distributions is guaranteed to include $P(A, B, C)$.

☐ G_1 ☐ G_2 ☐ G_3

☐ G_4 ☐ G_5 ☐ G_6

(ii) Now assume all we know about the joint distribution $P(A, B, C)$ is that it can be represented by the product $P(C|B)P(B|A)P(A)$. Mark each graph for which the associated family of probability distributions is guaranteed to include $P(A, B, C)$.

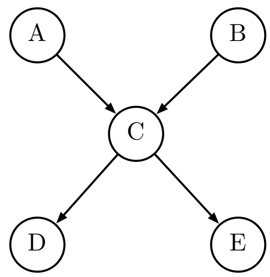
☐ G_1 ☐ G_2 ☐ G_3

☐ G_4 ☐ G_5 ☐ G_6

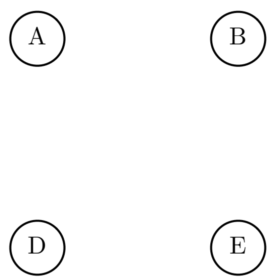
(c) Marginalization and Conditioning

Consider a Bayes' net over the random variables A, B, C, D, E with the structure shown below, with full joint distribution $P(A, B, C, D, E)$.

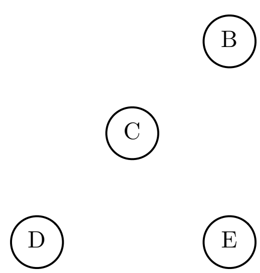
The following three questions describe different, unrelated situations (your answers to one question should not influence your answer to other questions).



(i) Consider the marginal distribution $P(A, B, D, E) = \sum_c P(A, B, c, D, E)$, where C was eliminated. On the diagram below, draw the minimal number of arrows that results in a Bayes' net structure that is able to represent this marginal distribution. If no arrows are needed write "No arrows needed."



(ii) Assume we are given an observation: $A = a$. On the diagram below, draw the minimal number of arrows that results in a Bayes' net structure that is able to represent the conditional distribution $P(B, C, D, E \mid A = a)$. If no arrows are needed write "No arrows needed."



(iii) Assume we are given two observations: $D = d, E = e$. On the diagram below, draw the minimal number of arrows that results in a Bayes' net structure that is able to represent the conditional distribution $P(A, B, C \mid D = d, E = e)$. If no arrows are needed write "No arrows needed."