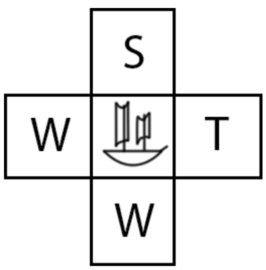
CS 188
Fall 2019

Exam Prep 8 Solutions

Q1. Argg! Sampling for the Legendary Treasure

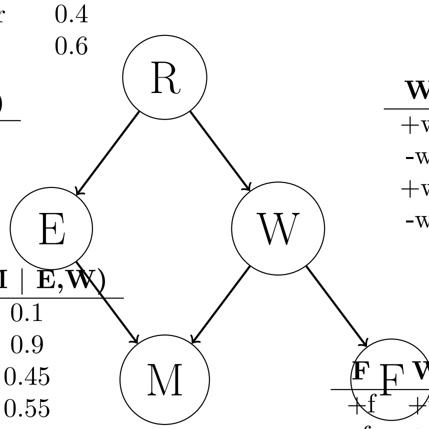
Little did you know that Michael and John are actually infamous pirates. One day, they go treasure hunting in the Ocean of Bayes, where rumor says a great treasure lies in wait for explorers who dare navigate in the rough waters. After navigating about the ocean, they are within grasp of the treasure. Their current configuration is represented by the boat in the figure below. They can only make one move, and must choose from the actions: (North, South, East, West). Stopping is not allowed. They will land in either a whirlpool (W), an island with a small treasure (S), or an island with the legendary treasure (T). The utilities of the three types of locations are shown below:



State	**U(State)**
T (Legendary Treasure)	100
S (Small Treasure)	25
W (Whirlpool)	-50

The success of their action depends on the random variable **Movement (M)**, which takes on one of two values: (+m, -m). The Movement random variable has many relationships with other variables: Presence of Enemy Pirates (E), Rain (R), Strong Waves (W), and Presence of Fishermen (F). The Bayes' net graph that represents these relationships is shown below:

R	**P(R)**
+r	0.4
-r	0.6



E	**R**	**P(E | R)**
+e	+r	0.3
-e	+r	0.7
+e	-r	0.6
-e	-r	0.4

W	**R**	**P(W | R)**
+w	+r	0.9
-w	+r	0.1
+w	-r	0.2
-w	-r	0.8

M	**E**	**W**	**P(M | E,W)**
+m	+e	+w	0.1
-m	+e	+w	0.9
+m	+e	-w	0.45
-m	+e	-w	0.55
+m	-e	+w	0.35
-m	-e	+w	0.65
+m	-e	-w	0.9
-m	-e	-w	0.1

F	**W**	**P(F | W)**
+f	+w	0.15
-f	+w	0.85
+f	-w	0.75
-f	-w	0.25

In the following questions we will follow a two-step process:
– (1) Michael and John observed the random variables $R = -r$ and $F = +f$. We then determine the distribution for $P(M| -r, +f)$ via sampling.

– (2) Based on the estimate for $P(M| -r, +f)$, after committing to an action, landing in the intended location of an action successfully occurs with probability $P(M = +m| -r, +f)$. The other three possible landing positions occur with probability $\frac{P(M=-m|-r,+f)}{3}$ each. Use this transition distribution to calculate the optimal action(s) to take and the expected utility of those actions.

(a) (i) **Rejection Sampling**: You want to estimate $P(M = +m|-r, +f)$ by rejection sampling. Below is a list of samples that were generated using prior sampling. Cross out those that would be rejected by rejection sampling.

$+r$	$+e$	$+w$	$-m$	$-f$		$-r$	$-e$	$+w$	$-m$	$+f$
$-r$	$-e$	$+w$	$-m$	$-f$		$+r$	$-e$	$+w$	$+m$	$-f$
$-r$	$+e$	$-w$	$-m$	$+f$		$-r$	$-e$	$-w$	$+m$	$+f$
$+r$	$-e$	$-w$	$+m$	$-f$		$+r$	$-e$	$-w$	$+m$	$+f$
$-r$	$-e$	$-w$	$-m$	$+f$		$-r$	$+e$	$+w$	$-m$	$+f$
$-r$	$+e$	$-w$	$-m$	$+f$		$-r$	$+e$	$-w$	$-m$	$+f$

All samples without the conditioning $-r, +f$ are rejected.

(ii) What is the approximation for $P(M = +m|-r, +f)$ using the remaining samples?
$\frac{1}{7}$, the fraction of accepted samples with $+m$ instantiated.

(iii) What are the optimal action(s) for Michael and John based on this estimate of $P(M = +m|-r, +f)$?
South, West. As $p(+m|-r, +f) = \frac{1}{7}$, $p(-m|-r, +f) = \frac{6}{7}$. Michael and John will succeed in the selected action $\frac{1}{7}$ of the time, or take one of the other 3 actions with equal probability of $\frac{2}{7}$. In this case, $p(+m|-r, +f)$ is so low that deciding to head in the direction of the whirlpool actually decreases the chances of landing in it.

(iv) What is the expected utility for the optimal action(s) based on this estimate of $P(M = +m|-r, +f)$?
$\frac{1}{7} * (-50) + \frac{2}{7} * (-50) + \frac{2}{7} * (25) + \frac{2}{7} * (100) = \frac{100}{7}$, the weighted sum of all four outcomes.

(b) (i) **Likelihood Weighting:** Suppose instead that you perform likelihood weighting on the following samples to get the estimate for $P(M = +m|-r, +f)$. You receive 4 samples consistent with the evidence.

Sample	**Weight**
$-r \quad -e \quad +w \quad +m \quad +f$	$P(-r)P(+f|+w) = 0.6 * 0.15 = 0.09$
$-r \quad -e \quad -w \quad +m \quad +f$	$P(-r)P(+f|-w) = 0.6 * 0.75 = 0.45$
$-r \quad -e \quad +w \quad -m \quad +f$	$P(-r)P(+f|+w) = 0.6 * 0.15 = 0.09$
$-r \quad +e \quad -w \quad -m \quad +f$	$P(-r)P(+f|-w) = 0.6 * 0.75 = 0.45$

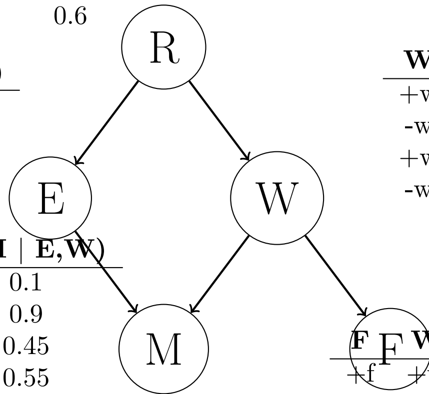
(ii) What is the approximation for $P(M = +m|-r, +f)$ using the samples above?
$\frac{0.09+0.45}{0.09+0.45+0.09+0.45} = \frac{1}{2}$

(iii) What are the optimal action(s) for Michael and John based on this estimate of $P(M = +m|-r, +f)$?
East

(iv) What is the expected utility for the optimal action(s) based on this estimate of $P(M = +m|-r, +f)$?
$\frac{1}{6} * (-50) + \frac{1}{6} * (-50) + \frac{1}{6} * (25) + \frac{1}{2} * (100) = \frac{75}{2}$

Here is a copy of the Bayes' Net, repeated for your convenience.

R	**P(R)**
+r	0.4
-r	0.6



E	**R**	**P(E | R)**
+e	+r	0.3
-e	+r	0.7
+e	-r	0.6
-e	-r	0.4

W	**R**	**P(W | R)**
+w	+r	0.9
-w	+r	0.1
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-w	-r	0.8

M	**E**	**W**	**P(M | E,W)**
+m	+e	+w	0.1
-m	+e	+w	0.9
+m	+e	-w	0.45
-m	+e	-w	0.55
+m	-e	+w	0.35
-m	-e	+w	0.65
+m	-e	-w	0.9
-m	-e	-w	0.1

F	**W**	**P(F | W)**
+f	+w	0.15
-f	+w	0.85
+f	-w	0.75
-f	-w	0.25

(c) **(i)** **Gibbs Sampling**. Now, we tackle the same problem, this time using Gibbs' sampling. We start out with initializing our evidence: $R = -r$, $F = +f$. Furthermore, we start with this random sample:

$-r \ \ +e \ \ -w \ \ +m \ \ +f.$

We select variable E to resample. Calculate the numerical value for:
$P(E = +e|R = -r, W = -w, M = +m, F = +f)$.

$P(E = +e|R = -r, W = -w, M = +m, F = +f) = \frac{P(+e|-r)P(+m|+e,-w)}{P(+e|-r)P(+m|+e,-w)+P(-e|-r)P(+m|-e,-w)}$

$= \frac{0.6*0.45}{0.6*0.45+0.4*0.9} = \frac{3}{7}$

We resample for a long time until we end up with the sample:

$-r \ \ -e \ \ +w \ \ +m \ \ +f.$

Michael and John are happy for fixing this one sample, but they do not have enough time left to compute another sample before making a move. They will let this one sample approximate the distribution: $P(M = +m| - r, +f)$.

(ii) What is the approximation for $P(M = +m| - r, +f)$, using this one sample?
1

(iii) What are the optimal action(s) for Michael and John based on this estimate of $P(M = +m|-r, +f)$?
East

(iv) What is the expected utility for the optimal action(s) based on this estimate of $P(M = +m|-r, +f)$?
100

Q2. Utilities

Davis is on his way to a final exam planning meeting. He is already running late (the meeting is starting now) and he's trying to determine whether he should wait for the bus or just walk.

It takes 20 minutes to get to Cory Hall by walking, and only 5 minutes to get to there by bus. The bus will either come in 10, 20, or 30 minutes, each with probability 1/3.

(a) Davis hates being late; his utility for being late as a function of t, the number of minutes late he is, is

$$U_D(t) = \begin{cases} 0 & : t \leq 0 \\ -2^{t/5} & : t > 0 \end{cases}$$

What is the expected utility of each action? Should he wait for the bus or walk?

$$\begin{aligned} EU(walk) &= -2^{20/5} = -16 \\ EU(bus) &= \frac{1}{3}(-2^{(10+5)/5} - 2^{(20+5)/5} - 2^{(30+5)/5}) \\ &= \frac{1}{3}(-8 - 32 - 128) = -168/3 = -56 \end{aligned}$$

Davis should walk.

(b) Pat is running late too. However, Pat reasons that once he's late, it doesn't matter how late he is. Therefore, his utility function is

$$U_P(t) = \begin{cases} 0 & : t \leq 0 \\ -10 & : t > 0 \end{cases}$$

Moreover, Pat prefers riding the bus because it is more comfortable, so riding the bus incurs a utility bonus of 5.

If Pat is deciding whether to take the bus or walk when the meeting is just starting, what are his expected utilities for each action? Should he take the bus or walk?

$$\begin{aligned} EU(walk) &= -10 \\ EU(bus) &= -10 + 5 = -5 \end{aligned}$$

Pat should take the bus.

(c) Give an example of a decreasing utility function in terms of time such that it will favor decisions that always minimize expected time to get to the meeting.

$U(t) = -t$. Any decreasing linear function of t is correct.

(d) Give an example of a decreasing utility function in terms of time such that it will be risk-seeking; that is, a lottery with expected time of arrival t will be preferred to a guarantee of arrival time t.

$U(t) = -\sqrt{t}$. Any decreasing function with a positive second derivative (concave up) is correct.