Q1. CSPs: Potluck Pandemonium

The potluck is coming up and the staff haven't figured out what to bring yet! They've pooled their resources and determined that they can bring some subset of the following items.

1. Pho
2. Apricots
3. Frozen Yogurt
4. Fried Rice
5. Apple Pie
6. Animal Crackers

There are five people on the course staff: Taylor, Jonathan, Faraz, Brian, and Alvin. Each of them will only bring one item to the potluck.

i. If (F)araz brings the same item as someone else, it cannot be (B)rian.

ii. (A)lvin has pho-phobia so he won't bring Pho, but he'll be okay if someone else brings it.

iii. (B)rian is no longer allowed near a stove, so he can only bring items 2, 3, or 6.

iv. (F)araz literally can't even; he won't bring items 2, 4, or 6.

v. (J)onathan was busy, so he didn't see the last third of the list. Therefore, he will only bring item 1, 2, 3, or 4.

vi. (T)aylor will only bring an item that is before an item that (J)onathan brings.

vii. (T)aylor is allergic to animal crackers, so he won't bring item 6. (If someone else brings it, he'll just stay away from that table.)

viii. (F)araz and (J)onathan will only bring items that have the same first letter (e.g. Frozen Yogurt and Fried Rice).

ix. (B)rian will only bring an item that is after an item that (A)lvin brings on the list.

x. (J)onathan and (T)aylor want to be unique; they won't bring the same item as anyone else.

This page is repeated as the second-to-last page of this midterm for you to rip out and use for reference as you work through the problem.

(a) Which of the listed constraints are unary constraints?

i ◯ ii ◯ iii ◯ iv ◯ v ◯

vi ◯ vii ◯ viii ◯ ix ◯ x ◯

(b) Rewrite implicit constraint viii. as an explicit constraint.

(c) How many edges are there in the constraint graph for this CSP?

(d) The table below shows the variable domains after all unary constraints have been enforced.

A		2	3	4	5	6
B		2	3			6
F	1		3		5	
J	1	2	3	4		
T	1	2	3	4	5	

Following the Minimum Remaining Values heuristic, which variable should we assign first? Break all ties alphabetically.

A ◯ B ◯ F ◯ J ◯ T ◯

(e) To decouple this from the previous question, assume that we choose to assign (F)araz first. In this question, we will choose which value to assign to using the Least Constraining Value method.

To determine the number of remaining values, enforce arc consistency to prune the domains. Then, count the total number of possible assignments (**not** the total number of remaining values). It may help you to enforce arc consistency twice, once before assigning values to (F)araz, and then again after assigning a value.

The domains after enforcing unary constraints are reproduced in each subquestion. The grids are provided as scratch space and **will not** be graded. Only numbers written in the blanks will be graded. The second grid is provided as a back-up in case you mess up on the first one. More grids are also provided on the second-to-last page of the exam.

(i) Assigning F = ______ results in ______ possible assignments.

A		2	3	4	5	6
B		2	3			6
F	1		3		5	
J	1	2	3	4		
T	1	2	3	4	5	

A		2	3	4	5	6
B		2	3			6
F	1		3		5	
J	1	2	3	4		
T	1	2	3	4	5	

(ii) Assigning F = ______ results in ______ possible assignments.

A		2	3	4	5	6
B		2	3			6
F	1		3		5	
J	1	2	3	4		
T	1	2	3	4	5	

A		2	3	4	5	6
B		2	3			6
F	1		3		5	
J	1	2	3	4		
T	1	2	3	4	5	

(iii) Assigning F = ______ results in ______ possible assignments.

A		2	3	4	5	6
B		2	3			6
F	1		3		5	
J	1	2	3	4		
T	1	2	3	4	5	

A		2	3	4	5	6
B		2	3			6
F	1		3		5	
J	1	2	3	4		
T	1	2	3	4	5	

(iv) Using the LCV method, which value should we assign to F? If there is a tie, choose the lower number. (e.g. If both 1 and 2 have the same value, then fill 1.)

1 ◯ 2 ◯ 3 ◯ 4 ◯ 5 ◯ 6 ◯

Q2. CSPs

In this question, you are trying to find a four-digit number satisfying the following conditions:

1. the number is odd,
2. the number only contains the digits 1, 2, 3, 4, and 5,
3. each digit (except the leftmost) is strictly larger than the digit to its left.

(a) CSPs

We will model this as a CSP where the variables are the four digits of our number, and the domains are the five digits we can choose from. The last variable only has 1, 3, and 5 in its domain since the number must be odd. The constraints are defined to reflect the third condition above. Thus before we start executing any algorithms, the domains are

1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 3 5

(i) Before assigning anything, enforce arc consistency. Write the values remaining in the domain of each variable after arc consistency is enforced.

(ii) With the domains you wrote in the previous part, which variable will the MRV (Minimum Remaining Value) heuristic choose to assign a value to first? If there is a tie, choose the leftmost variable.

- ○ The first digit (leftmost)
- ○ The second digit
- ○ The third digit
- ○ The fourth digit (rightmost)

(iii) Now suppose we assign to the leftmost digit first. Assuming we will continue filtering by enforcing arc consistency, which value will LCV (Least Constraining Value) choose to assign to the leftmost digit? **Break ties from large (5) to small (1).**

- ☐ 1
- ☐ 2
- ☐ 3
- ☐ 4
- ☐ 5

(iv) Now suppose we are running min-conflicts to try to solve this CSP. If we start with the number 1332, what will our number be after one interation of min-conflicts? Break variable selection ties from left to right, and **break value selection ties from small (1) to large (5)**.

____

(b) The following questions are completely unrelated to the above parts. Assume for these following questions, there are only binary constraints unless otherwise specified.

(i) [*true* or *false*] When enforcing arc consistency in a CSP, the set of values which remain when the algorithm terminates does not depend on the order in which arcs are processed from the queue.

(ii) [*true* or *false*] Once arc consistency is enforced as a pre-processing step, forward checking can be used during backtracking search to maintain arc consistency for all variables.

(iii) In a general CSP with n variables, each taking d possible values, what is the worst case time complexity of enforcing arc consistency using the AC-3 method discussed in class?

○ 0 ○ $O(1)$ ○ $O(nd^2)$ ○ $O(n^2d^3)$ ○ $O(d^n)$ ○ ∞

(iv) In a general CSP with n variables, each taking d possible values, what is the maximum number of times a backtracking search algorithm might have to backtrack (i.e. the number of the times it generates an assignment, partial or complete, that violates the constraints) before finding a solution or concluding that none exists?

○ 0 ○ $O(1)$ ○ $O(nd^2)$ ○ $O(n^2d^3)$ ○ $O(d^n)$ ○ ∞

(v) What is the maximum number of times a backtracking search algorithm might have to backtrack in a general CSP, if it is running arc consistency and applying the MRV and LCV heuristics?

○ 0 ○ $O(1)$ ○ $O(nd^2)$ ○ $O(n^2d^3)$ ○ $O(d^n)$ ○ ∞