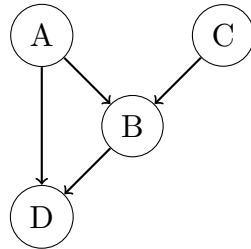


Q1. Bayes Net Inference



Consider the Bayes net graph depicted above.

- (a) (i) Select all conditional independences that are enforced by this Bayes net graph.
- $A \perp\!\!\!\perp B$    
   $A \perp\!\!\!\perp C | B$    
   $D \perp\!\!\!\perp C | A, B$    
   $D \perp\!\!\!\perp C$   
  $A \perp\!\!\!\perp C$    
   $A \perp\!\!\!\perp C | D$    
   $A \perp\!\!\!\perp C | B, D$    
   $D \perp\!\!\!\perp C | B$
- (ii) Because of these conditional independences, there are some distributions that cannot be represented by this Bayes net. What is the minimum set of edges that would need to be added such that the resulting Bayes net could represent any distribution?
- $A \rightarrow C$    
   $C \rightarrow A$    
   $C \rightarrow D$    
   $D \rightarrow C$   
  $D \rightarrow A$    
   $D \rightarrow B$    
   $B \rightarrow C$    
   $B \rightarrow A$
- (b) For the rest of this Q2, we use the **original, unmodified** Bayes net depicted at the beginning of the problem statement. Here are some partially-filled conditional probability tables on  $A, B, C$ , and  $D$ . Note that these are not necessarily factors of the Bayes net. Fill in the six blank entries such that this distribution can be represented by the Bayes net.

$A$	$C$	$P(C   A)$
$+a$	$+c$	0.8
$+a$	$-c$	0.2
$-a$	$+c$	0.8
$-a$	$-c$	0.2

$A$	$B$	$D$	$P(D   A, B)$
$+a$	$+b$	$+d$	0.60
$+a$	$+b$	$-d$	0.40
$+a$	$-b$	$+d$	0.10
$+a$	$-b$	$-d$	0.90
$-a$	$+b$	$+d$	0.20
$-a$	$+b$	$-d$	0.80
$-a$	$-b$	$+d$	0.50
$-a$	$-b$	$-d$	0.50

$A$	$B$	$C$	$P(C   A, B)$
$+a$	$+b$	$+c$	0.50
$+a$	$+b$	$-c$	0.50
$+a$	$-b$	$+c$	0.20
$+a$	$-b$	$-c$	0.80
$-a$	$+b$	$+c$	0.90
$-a$	$+b$	$-c$	0.10
$-a$	$-b$	$+c$	0.40
$-a$	$-b$	$-c$	0.60

$C$	$P(C)$
$+c$	(i)
$-c$	(ii)

$A$	$B$	$C$	$D$	$P(D, C   A, B)$
$+a$	$+b$	$+c$	$+d$	(iii)
$+a$	$+b$	$-c$	$-d$	(iv)
$+a$	$-b$	$+c$	$+d$	(v)
$+a$	$-b$	$-c$	$-d$	(vi)
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

(i):

(ii):

(iii):

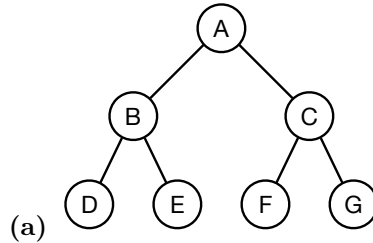
(iv):

(v):

(vi):

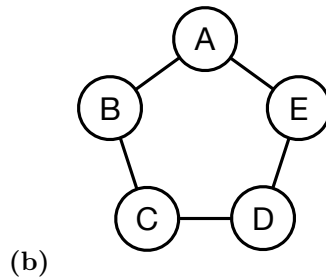
## Q2. Independence

In each part of this question, you are given a Bayes' net where the edges do not have a direction. Assign a direction to every edge (by adding an arrowhead at one end of each edge) to ensure that the Bayes' Net structure implies the assumptions provided. You cannot add new edges. The Bayes' nets can imply more assumptions than listed, but they *must* imply the ones listed. There may be more than one correct solution.



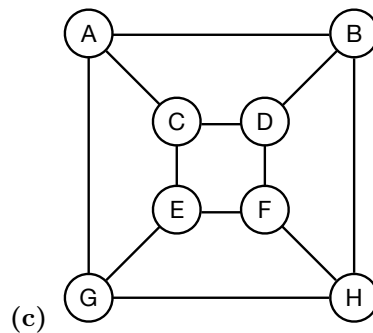
**Assumptions:**

- $A \perp\!\!\!\perp G$
- $D \perp\!\!\!\perp E$
- $E \perp\!\!\!\perp F$
- $F \perp\!\!\!\perp G \mid C$



**Assumptions:**

- $B \perp\!\!\!\perp E$
- $E \perp\!\!\!\perp C \mid D$

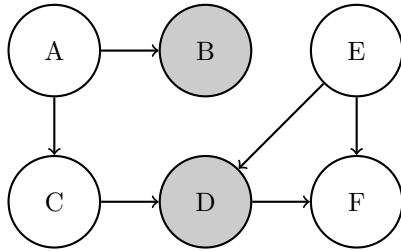


**Assumptions:**

- $F \perp\!\!\!\perp G$
- $F \perp\!\!\!\perp B \mid G$
- $D \perp\!\!\!\perp E \mid F$

# Q3. Bayes Nets and Sampling

You are given a bayes net with the following probability tables:



E	D	F	$P(F E, D)$
0	0	0	0.6
0	0	1	0.4
0	1	0	0.7
0	1	1	0.3
1	0	0	0.2
1	0	1	0.8
1	1	0	0.7
1	1	1	0.3

A	$P(A)$
0	0.75
1	0.25

A	B	$P(B A)$
0	0	0.1
0	1	0.9
1	0	0.5
1	1	0.5

A	C	$P(C A)$
0	0	0.3
0	1	0.7
1	0	0.7
1	1	0.3

E	$P(E)$
0	0.1
1	0.9

E	C	D	$P(D E, C)$
0	0	0	0.5
0	0	1	0.5
0	1	0	0.2
0	1	1	0.8
1	0	0	0.5
1	0	1	0.5
1	1	0	0.2
1	1	1	0.8

You want to know  $P(C = 0|B = 1, D = 0)$  and decide to use sampling to approximate it.

(a) With prior sampling, what would be the likelihood of obtaining the sample  $[A=1, B=0, C=0, D=0, E=1, F=0]$ ?

- $0.25 \cdot 0.1 \cdot 0.3 \cdot 0.9 \cdot 0.8 \cdot 0.7$
- $0.75 \cdot 0.1 \cdot 0.3 \cdot 0.9 \cdot 0.5 \cdot 0.8$
- $0.25 \cdot 0.9 \cdot 0.7 \cdot 0.1 \cdot 0.5 \cdot 0.6$
- Other \_\_\_\_\_
- $0.25 \cdot 0.5 \cdot 0.7 \cdot 0.5 \cdot 0.9 \cdot 0.2$
- $0.25 \cdot 0.5 \cdot 0.3 \cdot 0.2 \cdot 0.9 \cdot 0.2$
- $0.75 \cdot 0.1 \cdot 0.3 \cdot 0.9 \cdot 0.5 \cdot 0.2 + 0.25 \cdot 0.5 \cdot 0.7 \cdot 0.5 \cdot 0.9 \cdot 0.2$

(b) Assume you obtained the sample  $[A = 1, B=1, C=0, D=0, E=1, F=1]$  through likelihood weighting. What is its weight?

- $0.25 \cdot 0.5 \cdot 0.7 \cdot 0.5 \cdot 0.9 \cdot 0.8$
- $0.25 \cdot 0.7 \cdot 0.9 \cdot 0.8 + 0.75 \cdot 0.3 \cdot 0.9 \cdot 0.8$
- $0.25 \cdot 0.5 \cdot 0.7 \cdot 0.5 \cdot 0.8$
- Other \_\_\_\_\_
- 0
- $0.5 \cdot 0.5$
- $0.9 \cdot 0.5 + 0.1 \cdot 0.5$

(c) You decide to use Gibb's sampling instead. Starting with the initialization  $[A = 1, B=1, C=0, D=0, E=0, F=0]$ , suppose you resample F first, what is the probability that the next sample drawn is  $[A = 1, B=1, C=0, D=0, E=0, F=1]$ ?

0.4

0.6

$0.6 \cdot 0.1 \cdot 0.5$

0

$0.25 \cdot 0.5 \cdot 0.7 \cdot 0.5 \cdot 0.1 \cdot 0.3$

$0.9 \cdot 0.5 + 0.1 \cdot 0.5$

Other \_\_\_\_\_