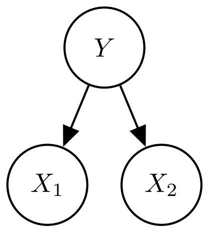
Q1. Naïve Bayes

You are given a naïve bayes model, shown below, with label Y and features X_1 and X_2. The conditional probabilities for the model are parametrized by p_1, p_2 and q.



X_1	Y	$P(X_1\|Y)$
0	0	p_1
1	0	$1-p_1$
0	1	$1-p_1$
1	1	p_1

X_2	Y	$P(X_2\|Y)$
0	0	p_2
1	0	$1-p_2$
0	1	$1-p_2$
1	1	p_2

Y	$P(Y)$
0	$1-q$
1	q

Note that some of the parameters are shared (e.g. $P(X_1 = 0|Y = 0) = P(X_1 = 1|Y = 1) = p_1$).

(a) Given a new data point with $X_1 = 1$ and $X_2 = 1$, what is the probability that this point has label $Y = 1$? Express your answer in terms of the parameters p_1, p_2 and q (you might not need all of them).

$P(Y = 1|X_1 = 1, X_2 = 1) =$ ______________________

The model is trained with the following data:

sample number	1	2	3	4	5	6	7	8	9	10
X_1	0	0	1	0	1	0	1	0	1	1
X_2	0	0	0	0	0	0	0	1	0	0
Y	0	0	0	0	0	0	0	1	1	1

(b) What are the maximum likelihood estimates for p_1, p_2 and q?

$p_1 =$ ______________________ $p_2 =$ ______________________ $q =$ ______________________

Q2. Machine Learning: Potpourri

(a) What it the **minimum** number of parameters needed to fully model a joint distribution $P(Y, F_1, F_2, ..., F_n)$ over label Y and n features F_i? Assume binary class where each feature can possibly take on k distinct values.

(b) Under the **Naive Bayes assumption**, what is the **minimum** number of parameters needed to model a joint distribution $P(Y, F_1, F_2, ..., F_n)$ over label Y and n features F_i? Assume binary class where each feature can take on k distinct values.

(c) You suspect that you are overfitting with your Naive Bayes with Laplace Smoothing. How would you adjust the strength k in Laplace Smoothing?

○ Increase k ○ Decrease k

(d) While using Naive Bayes with Laplace Smoothing, increasing the strength k in Laplace Smoothing can:

☐ Increase training error ☐ Decrease training error
☐ Increase validation error ☐ Decrease validation error

(e) It is possible for the perceptron algorithm to never terminate on a dataset that is linearly separable in its feature space.

○ True ○ False

(f) If the perceptron algorithm terminates, then it is guaranteed to find a max-margin separating decision boundary.

○ True ○ False

(g) In multiclass perceptron, every weight w_y can be written as a linear combination of the training data feature vectors.

○ True ○ False

(h) For binary class classification, logistic regression produces a linear decision boundary.

○ True ○ False

(i) In the binary classification case, logistic regression is exactly equivalent to a single-layer neural network with a sigmoid activation and the cross-entropy loss function.

○ True ○ False

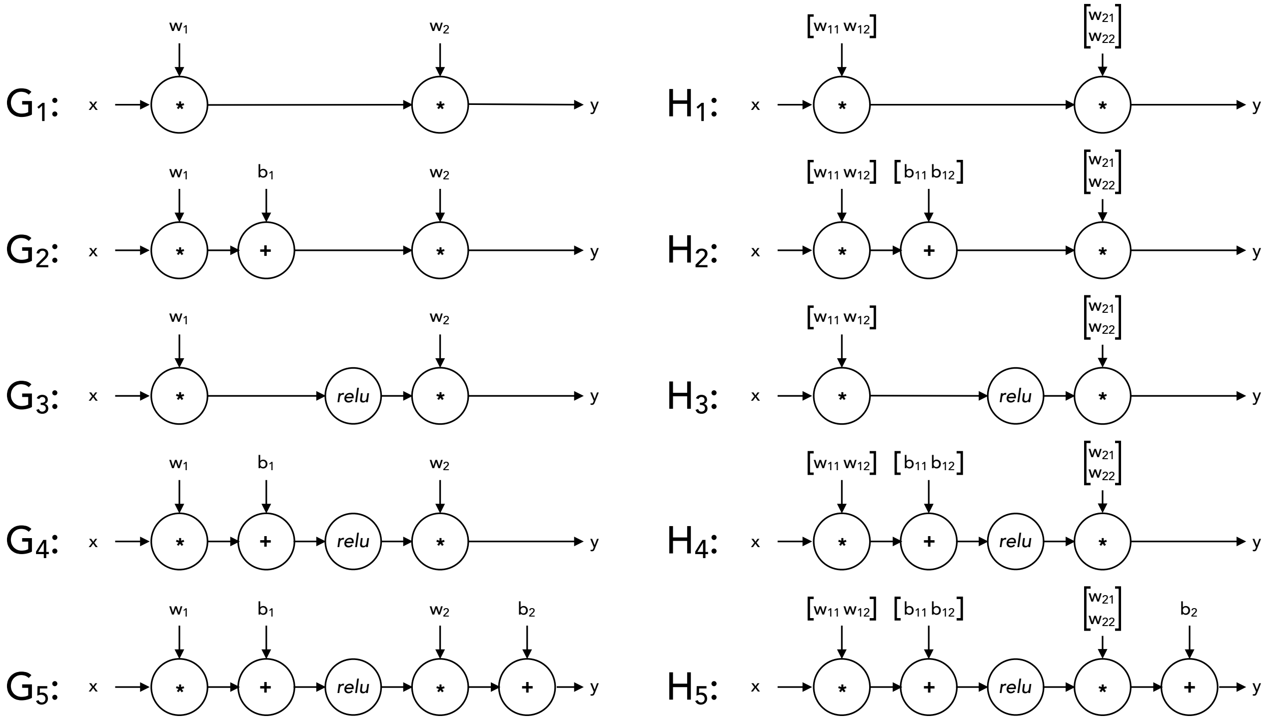
(j) **(i)** You train a linear classifier on 1,000 training points and discover that the training accuracy is only 50%. Which of the following, if done in isolation, has a good chance of improving your training accuracy?

☐ Add novel features ☐ Train on more data ☐ Train on less data

(ii) You now try training a neural network but you find that the training accuracy is still very low. Which of the following, if done in isolation, has a good chance of improving your training accuracy?

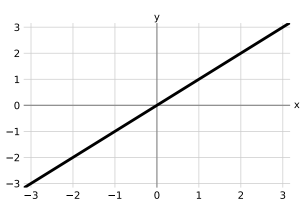
☐ Add more hidden layers ☐ Add more units to the hidden layers

Q3. Neural Networks: Representation

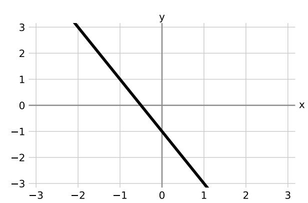


For each of the piecewise-linear functions below, mark all networks from the list above that can represent the function **exactly** on the range $x \in (-\infty, \infty)$. In the networks above, $relu$ denotes the element-wise ReLU nonlinearity: $relu(z) = max(0, z)$. The networks G_i use 1-dimensional layers, while the networks H_i have some 2-dimensional intermediate layers.

(a)

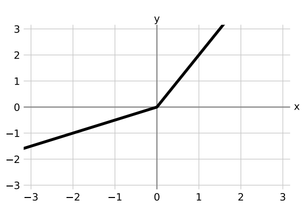


☐ G_1 ☐ H_1
☐ G_2 ☐ H_2
☐ G_3 ☐ H_3
☐ G_4 ☐ H_4
☐ G_5 ☐ H_5
○ None of the above

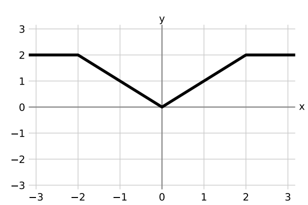


☐ G_1 ☐ H_1
☐ G_2 ☐ H_2
☐ G_3 ☐ H_3
☐ G_4 ☐ H_4
☐ G_5 ☐ H_5
○ None of the above

(b)



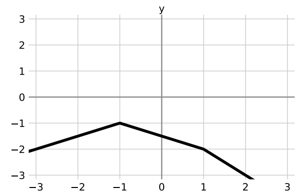
☐ G_1 ☐ H_1
☐ G_2 ☐ H_2
☐ G_3 ☐ H_3
☐ G_4 ☐ H_4
☐ G_5 ☐ H_5
○ None of the above



☐ G_1 ☐ H_1
☐ G_2 ☐ H_2
☐ G_3 ☐ H_3
☐ G_4 ☐ H_4
☐ G_5 ☐ H_5
○ None of the above

(c) 

☐ G_1 ☐ H_1
☐ G_2 ☐ H_2
☐ G_3 ☐ H_3
☐ G_4 ☐ H_4
☐ G_5 ☐ H_5
○ None of the above



☐ G_1 ☐ H_1
☐ G_2 ☐ H_2
☐ G_3 ☐ H_3
☐ G_4 ☐ H_4
☐ G_5 ☐ H_5
○ None of the above