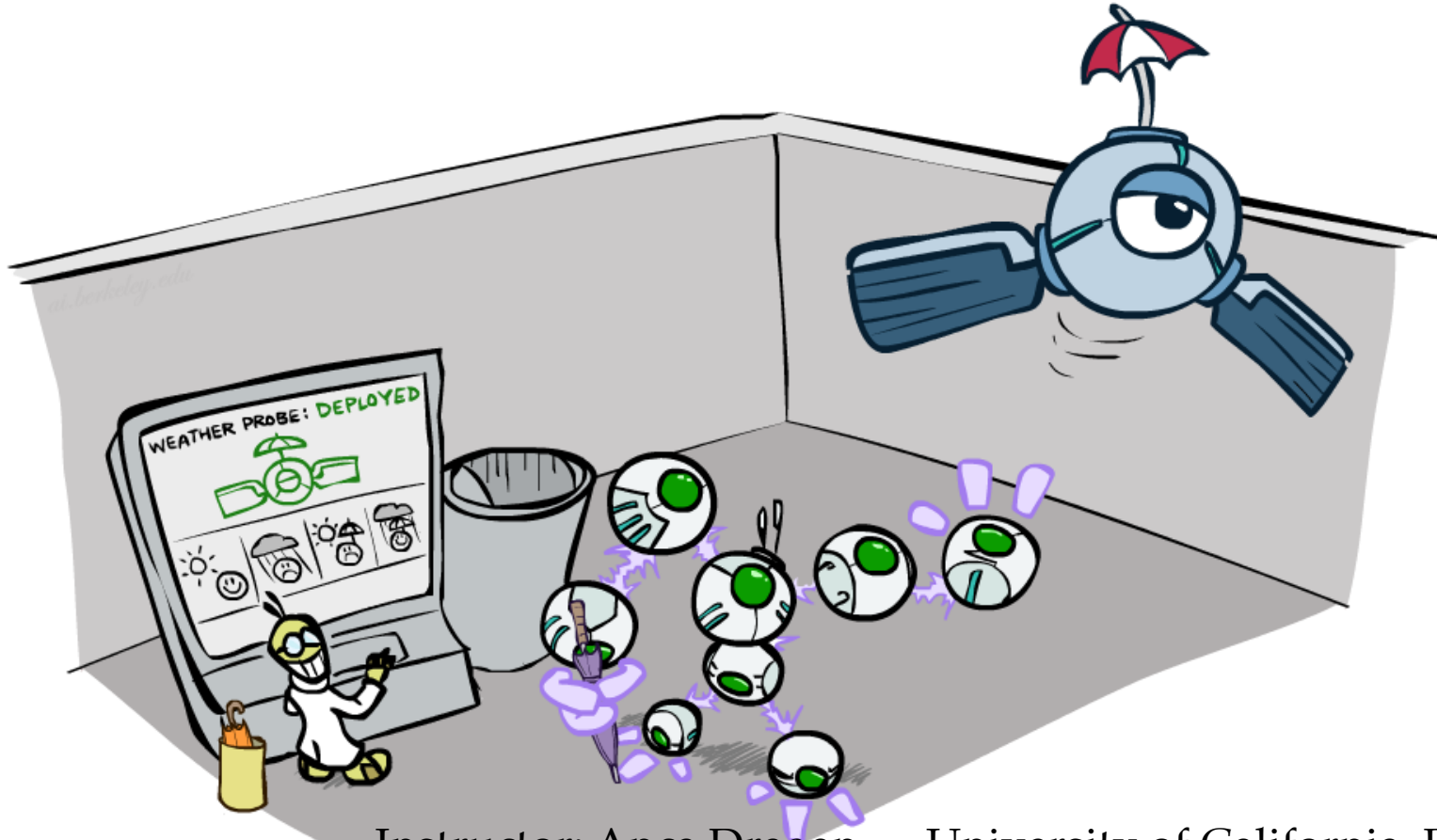


CS 188: Artificial Intelligence

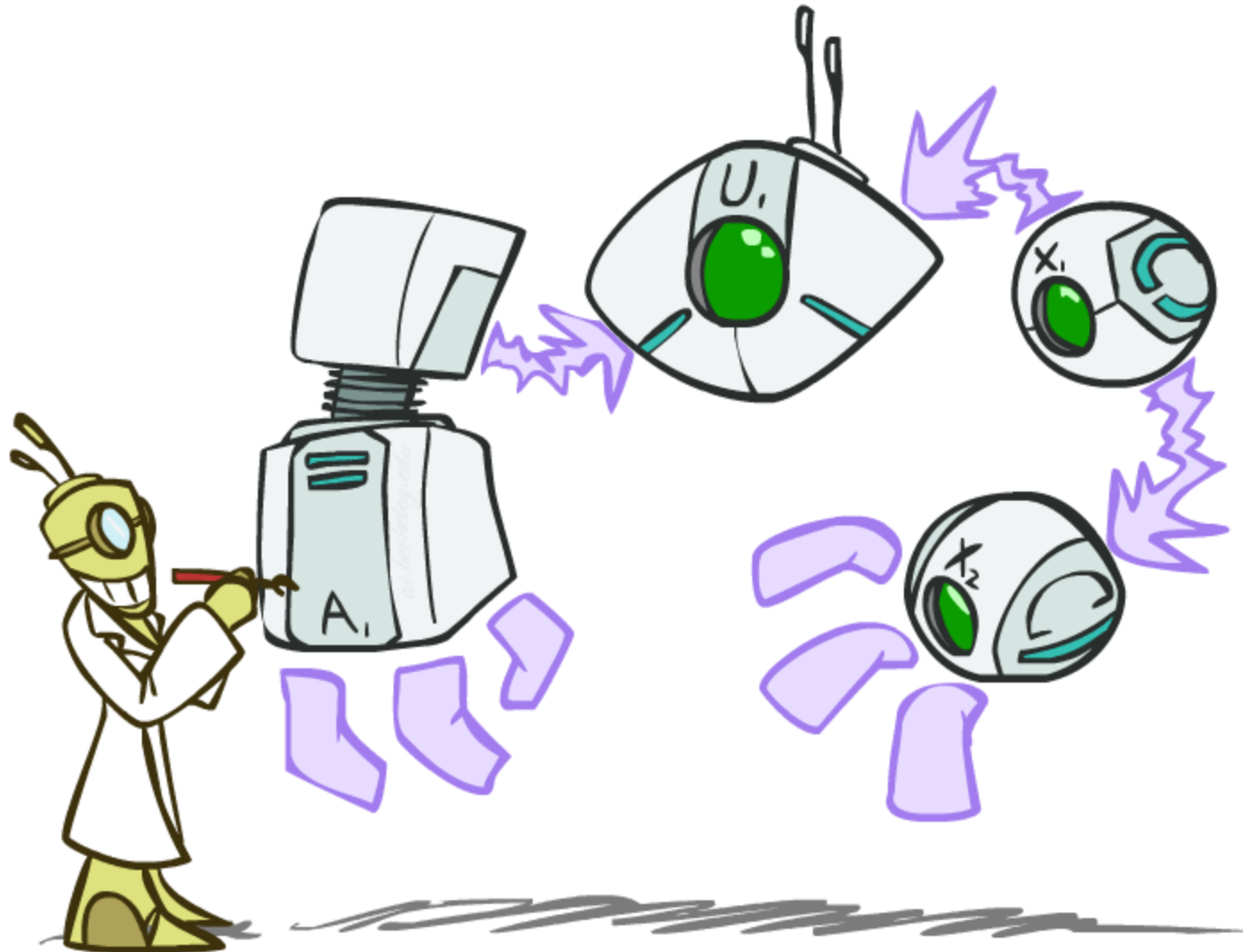
Decision Networks and Value of Perfect Information



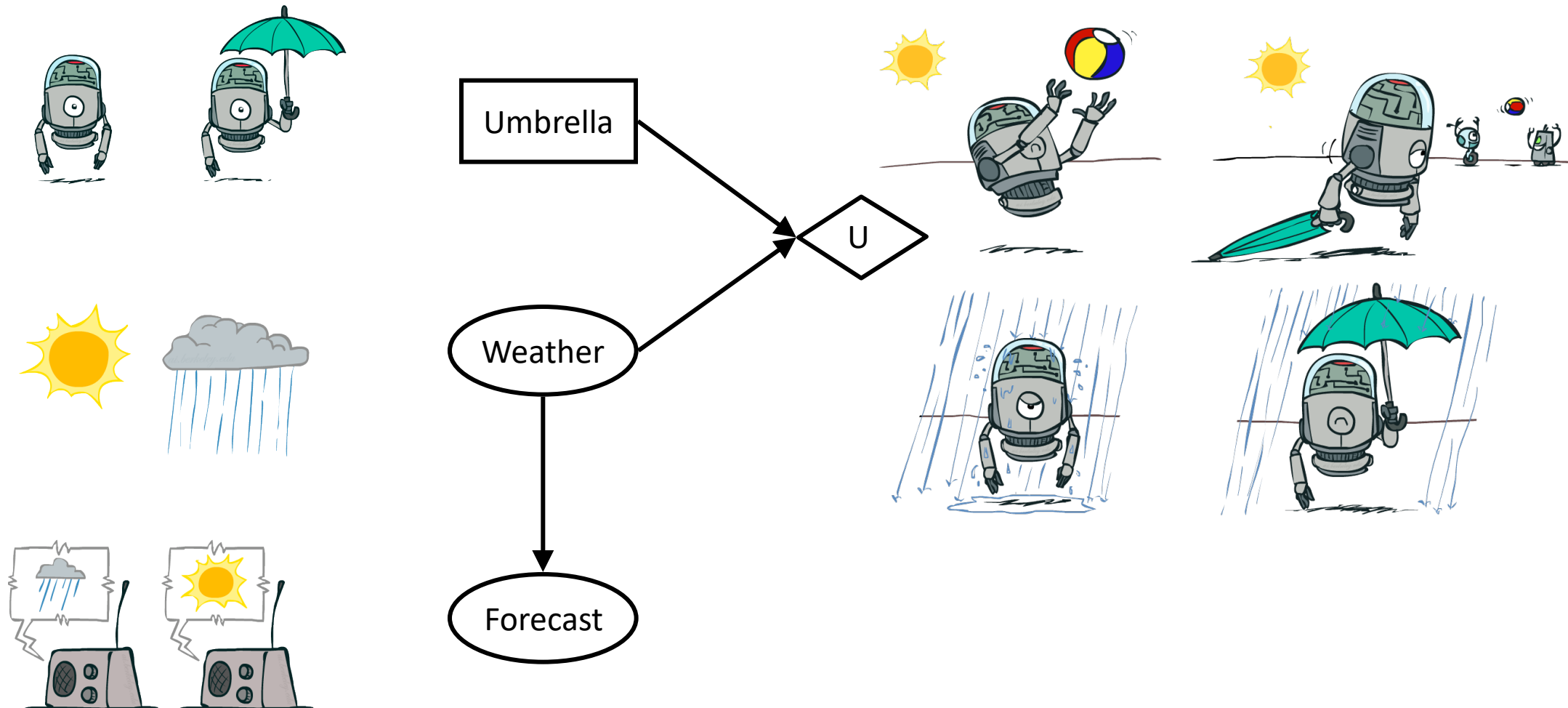
Instructor: Anca Dragan --- University of California, Berkeley

[These slides were created by Dan Klein, Pieter Abbeel, and Anca. <http://ai.berkeley.edu>.]

Decision Networks



Decision Networks



Decision Networks

- **MEU: choose the action which maximizes the expected utility given the evidence**

- Can directly operationalize this with decision networks
 - Bayes nets with nodes for utility and actions
 - Lets us calculate the expected utility for each action

- **New node types:**



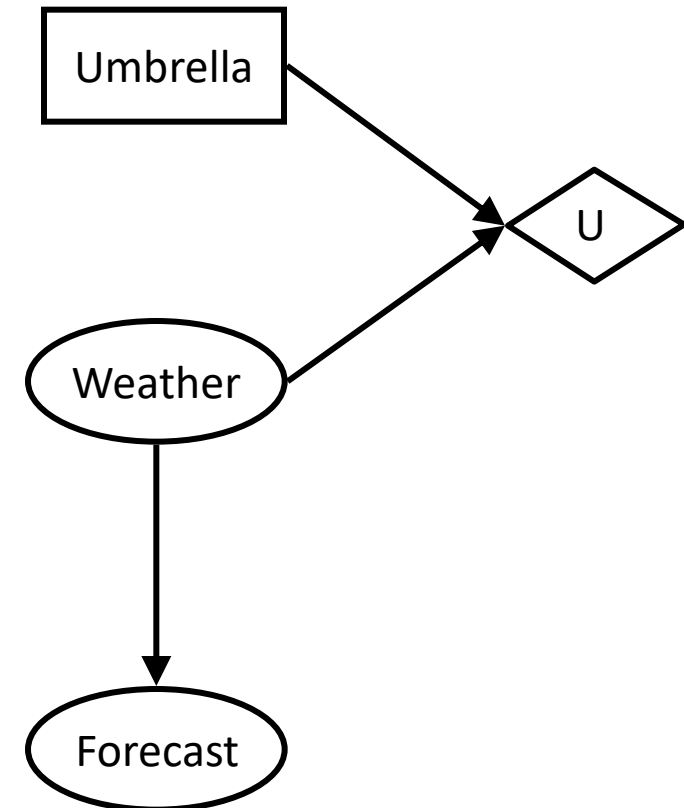
- Chance nodes (just like BNs)



- Actions (rectangles, cannot have parents, act as observed evidence)

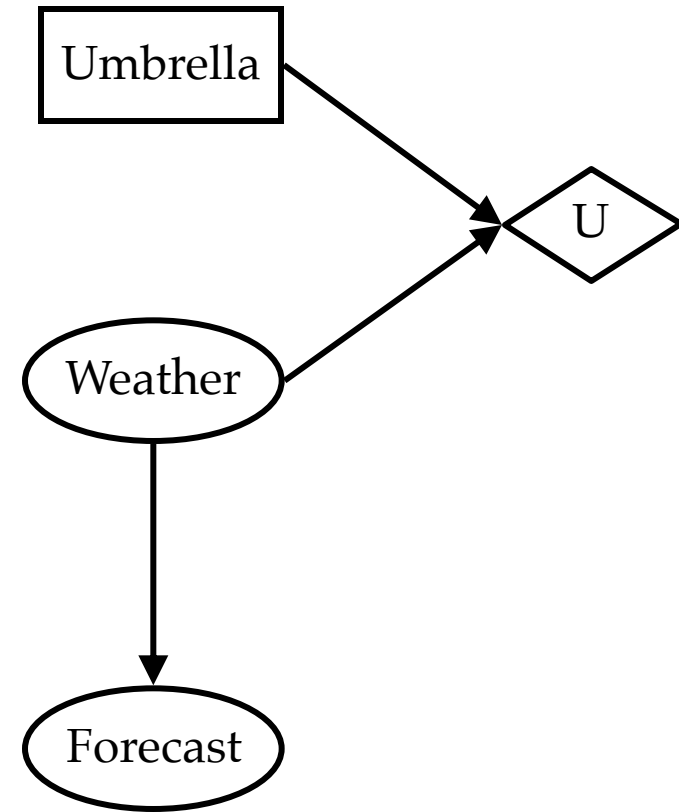


- Utility node (diamond, depends on action and chance nodes)



Decision Networks

- Action selection
 - Instantiate all evidence
 - Set action node(s) each possible way
 - Calculate posterior for all parents of utility node, given the evidence
 - Calculate expected utility for each action
 - Choose maximizing action



Maximum Expected Utility

Umbrella = leave

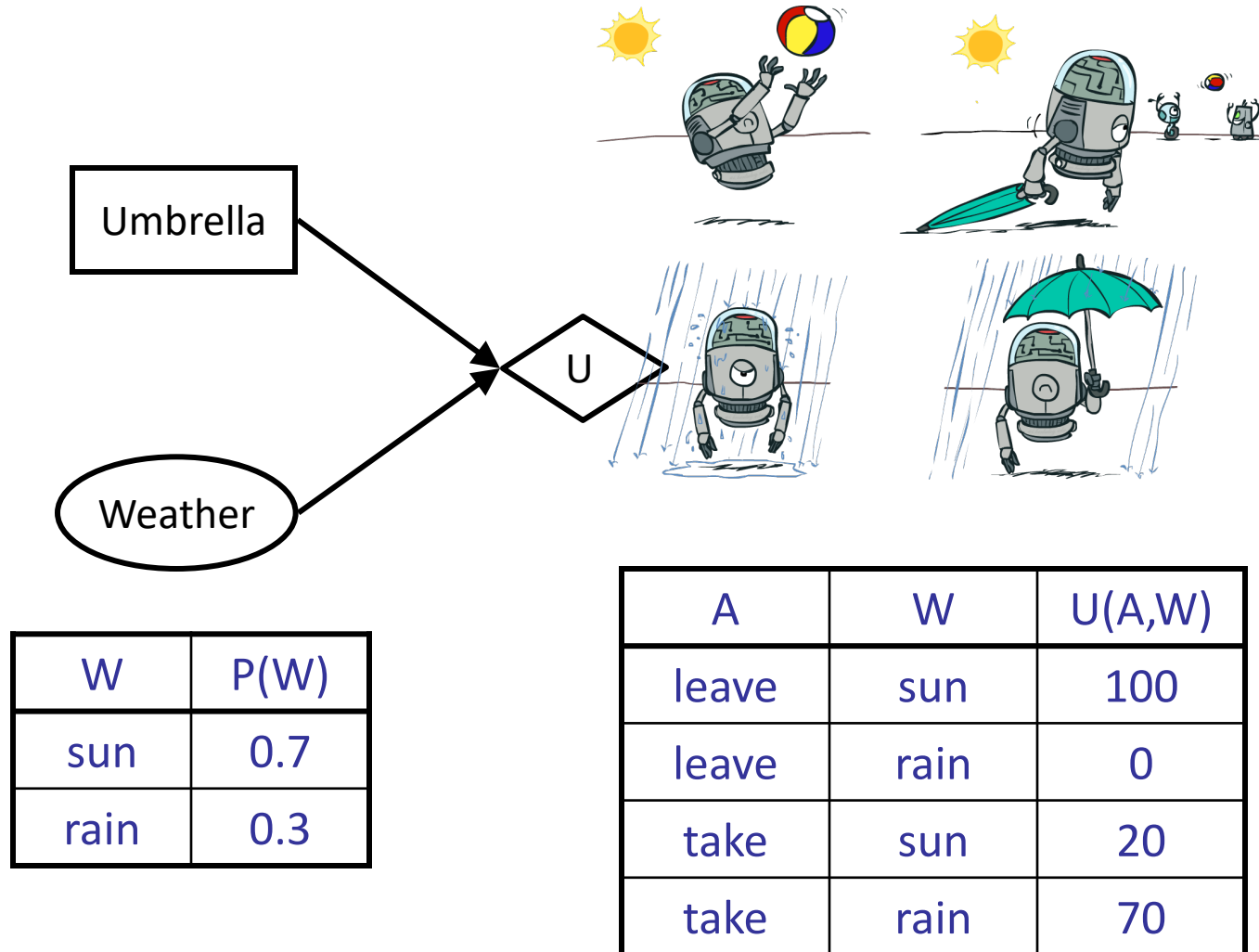
$$\begin{aligned} EU(\text{leave}) &= \sum_w P(w)U(\text{leave}, w) \\ &= 0.7 \cdot 100 + 0.3 \cdot 0 = 70 \end{aligned}$$

Umbrella = take

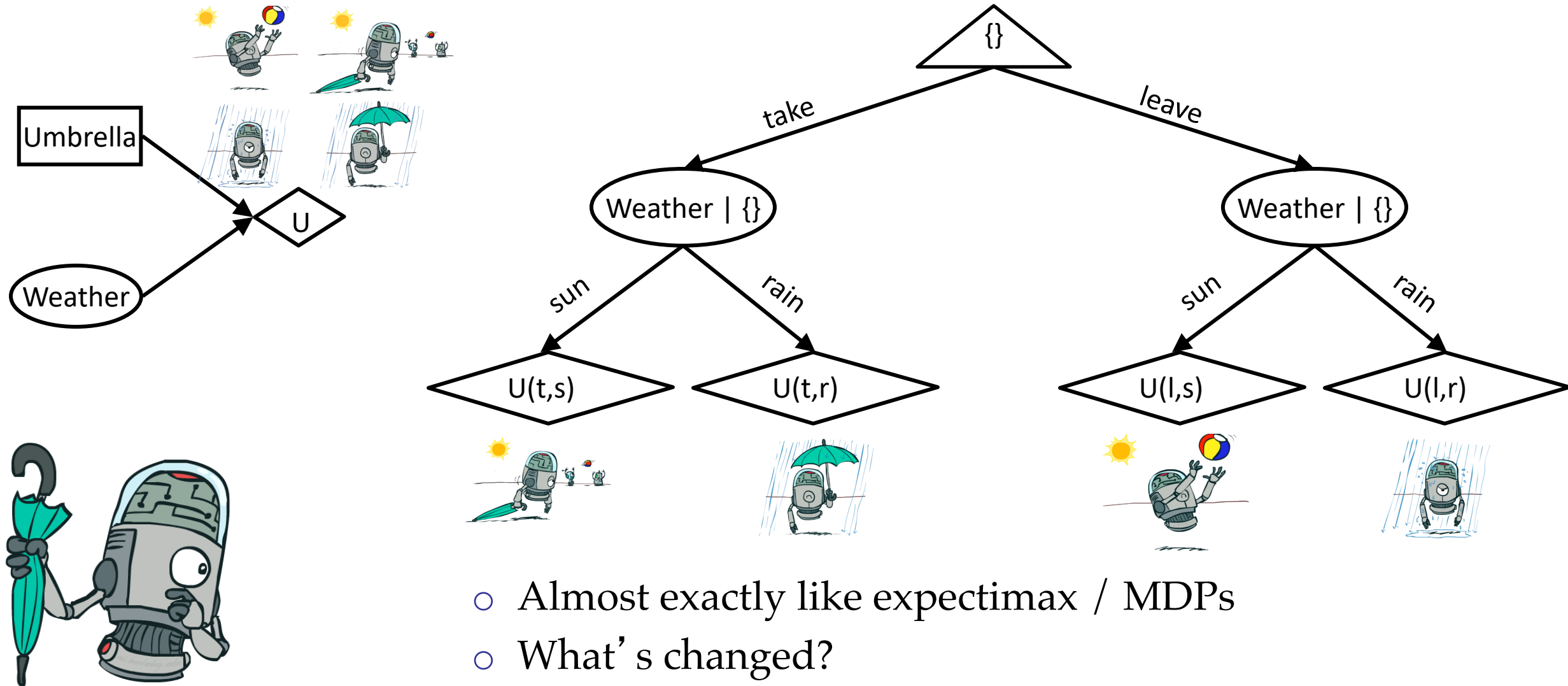
$$\begin{aligned} EU(\text{take}) &= \sum_w P(w)U(\text{take}, w) \\ &= 0.7 \cdot 20 + 0.3 \cdot 70 = 35 \end{aligned}$$

Optimal decision = leave

$$MEU(\emptyset) = \max_a EU(a) = 70$$



Decisions as Outcome Trees



Maximum Expected Utility

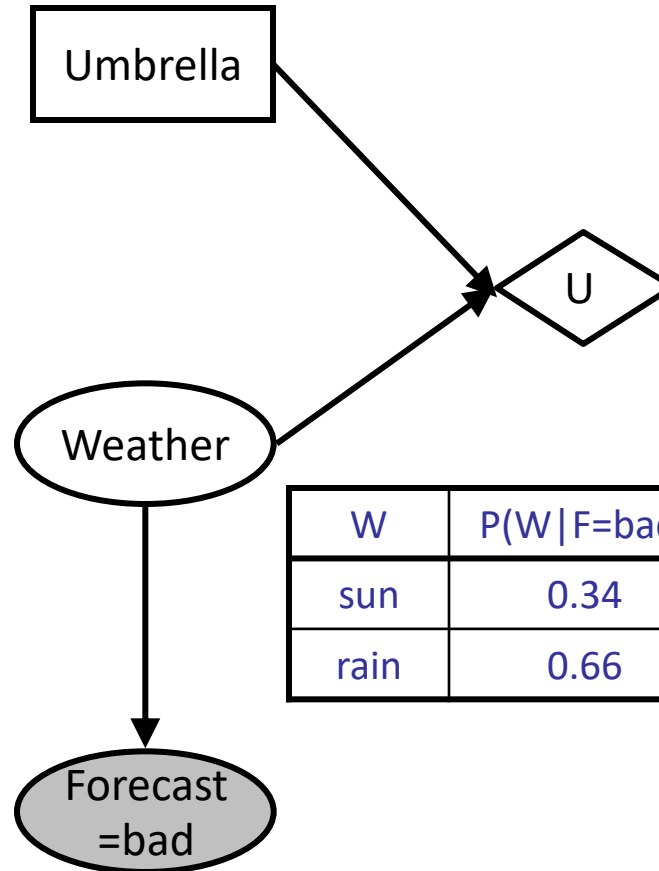
Umbrella = leave

$$EU(\text{leave}|\text{bad}) = \sum_w P(w|\text{bad})U(\text{leave}, w)$$

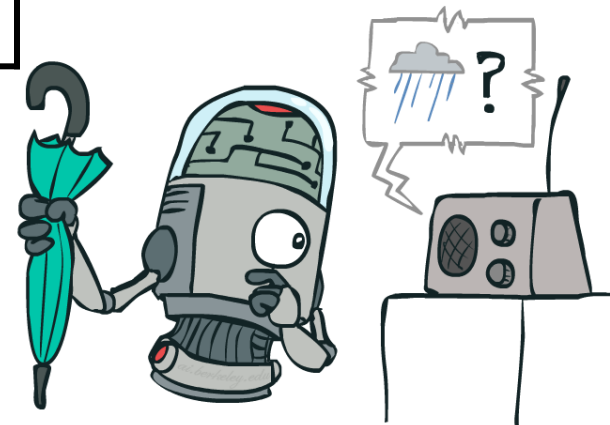
$$P(W) \quad P(F|W)$$

$$P(W|F) = \frac{P(W, F)}{\sum_w P(w, F)}$$

$$= \frac{P(F|W)P(W)}{\sum_w P(F|w)P(w)}$$



A	W	U(A,W)
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70



Maximum Expected Utility

Umbrella = leave

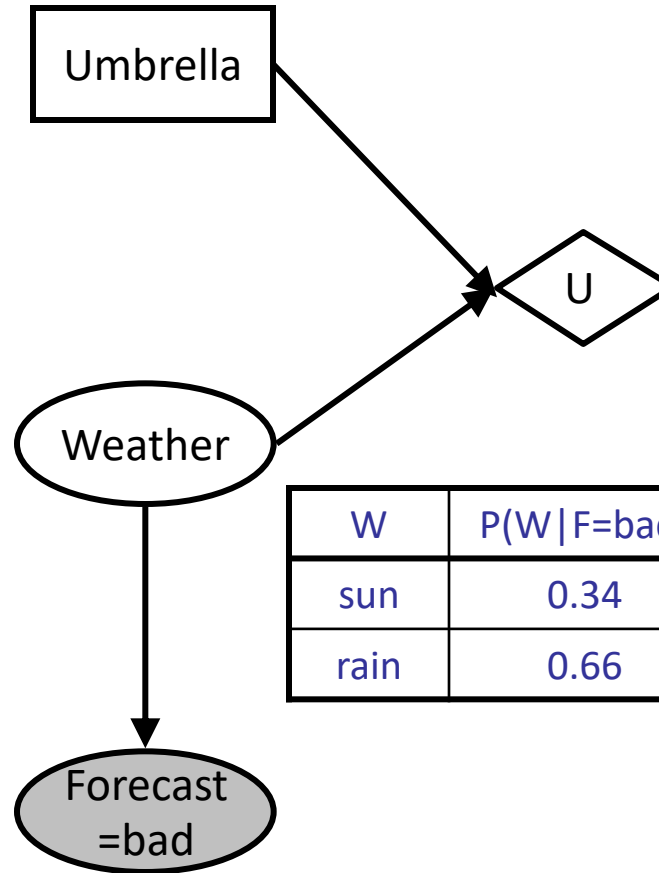
$$\begin{aligned} EU(\text{leave}|\text{bad}) &= \sum_w P(w|\text{bad})U(\text{leave}, w) \\ &= 0.34 \cdot 100 + 0.66 \cdot 0 = 34 \end{aligned}$$

Umbrella = take

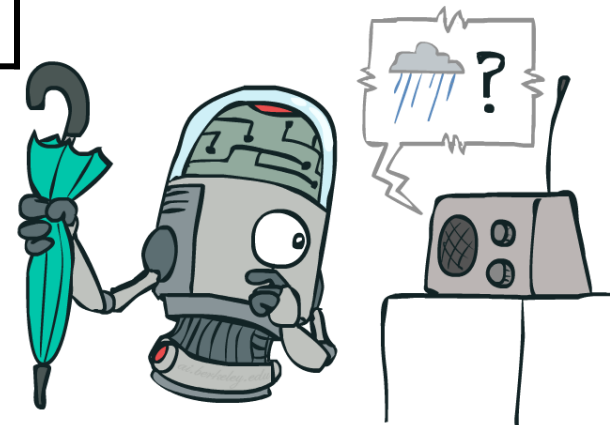
$$\begin{aligned} EU(\text{take}|\text{bad}) &= \sum_w P(w|\text{bad})U(\text{take}, w) \\ &= 0.34 \cdot 20 + 0.66 \cdot 70 = 53 \end{aligned}$$

Optimal decision = take

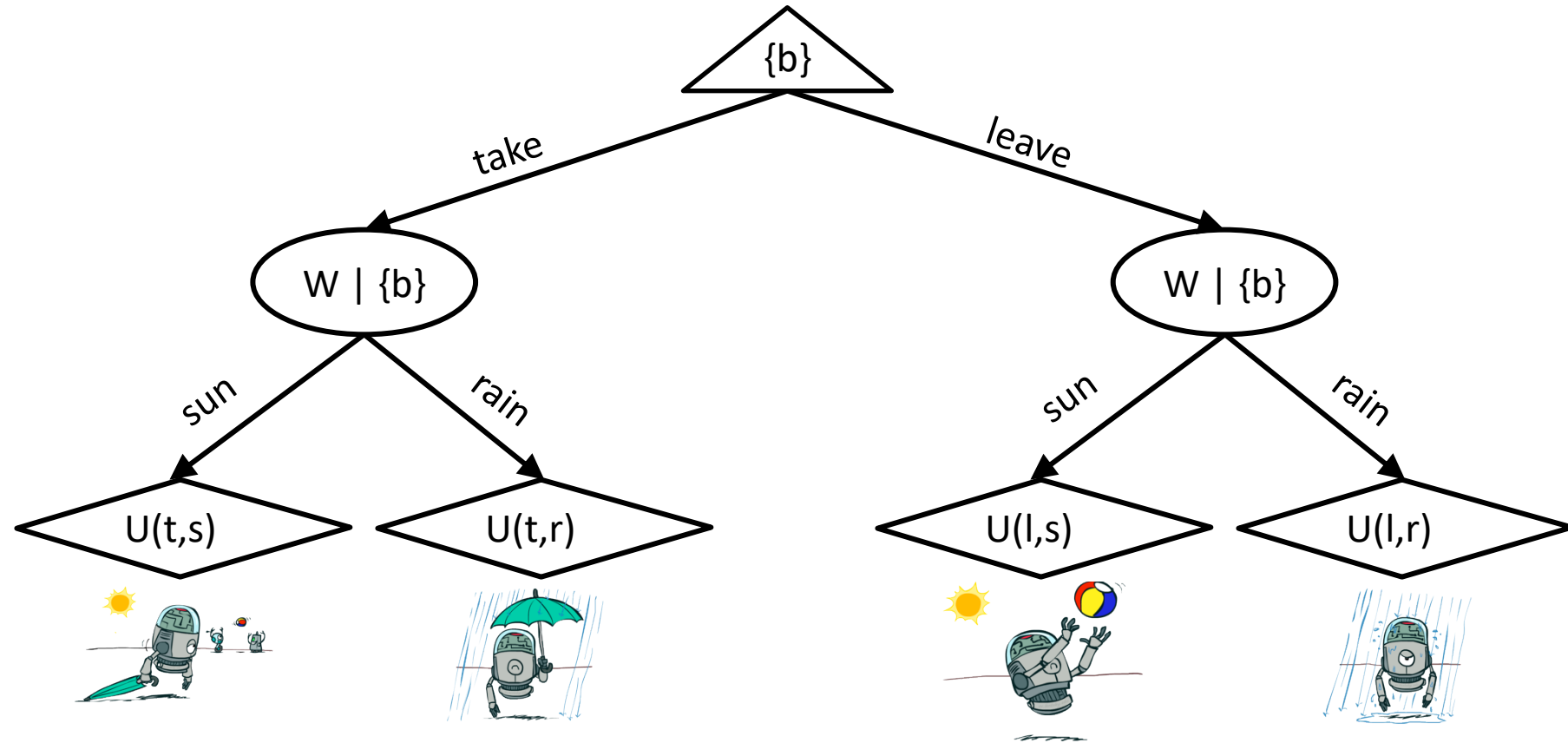
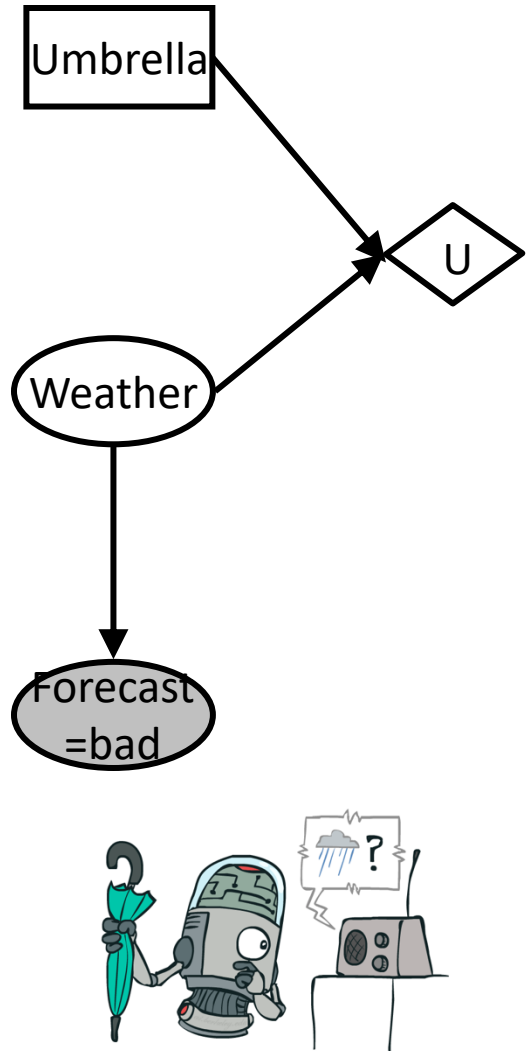
$$MEU(F = \text{bad}) = \max_a EU(a|\text{bad}) = 53$$



A	W	U(A,W)
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70



Decisions as Outcome Trees

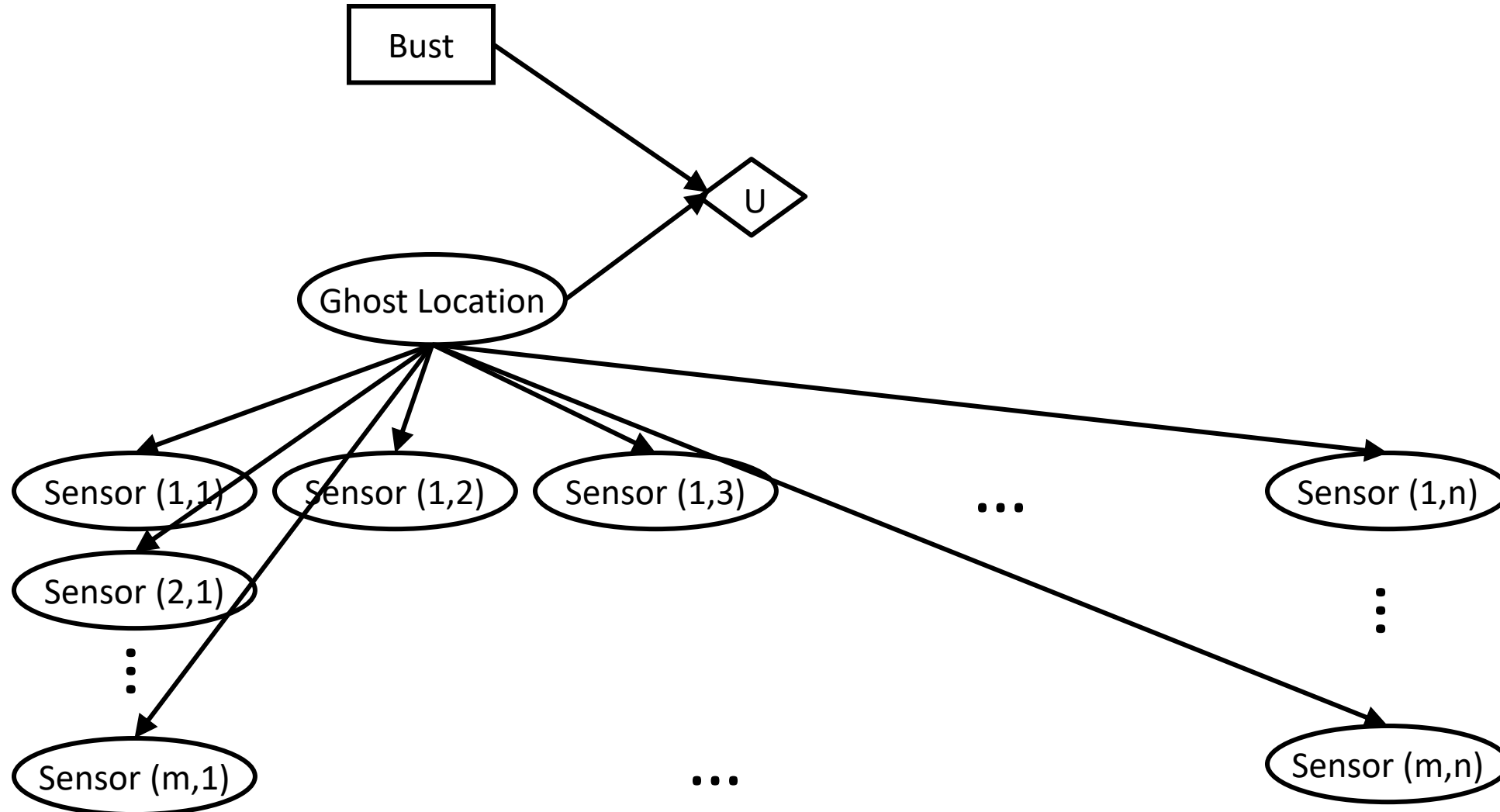


Video of Demo Ghostbusters with Probability

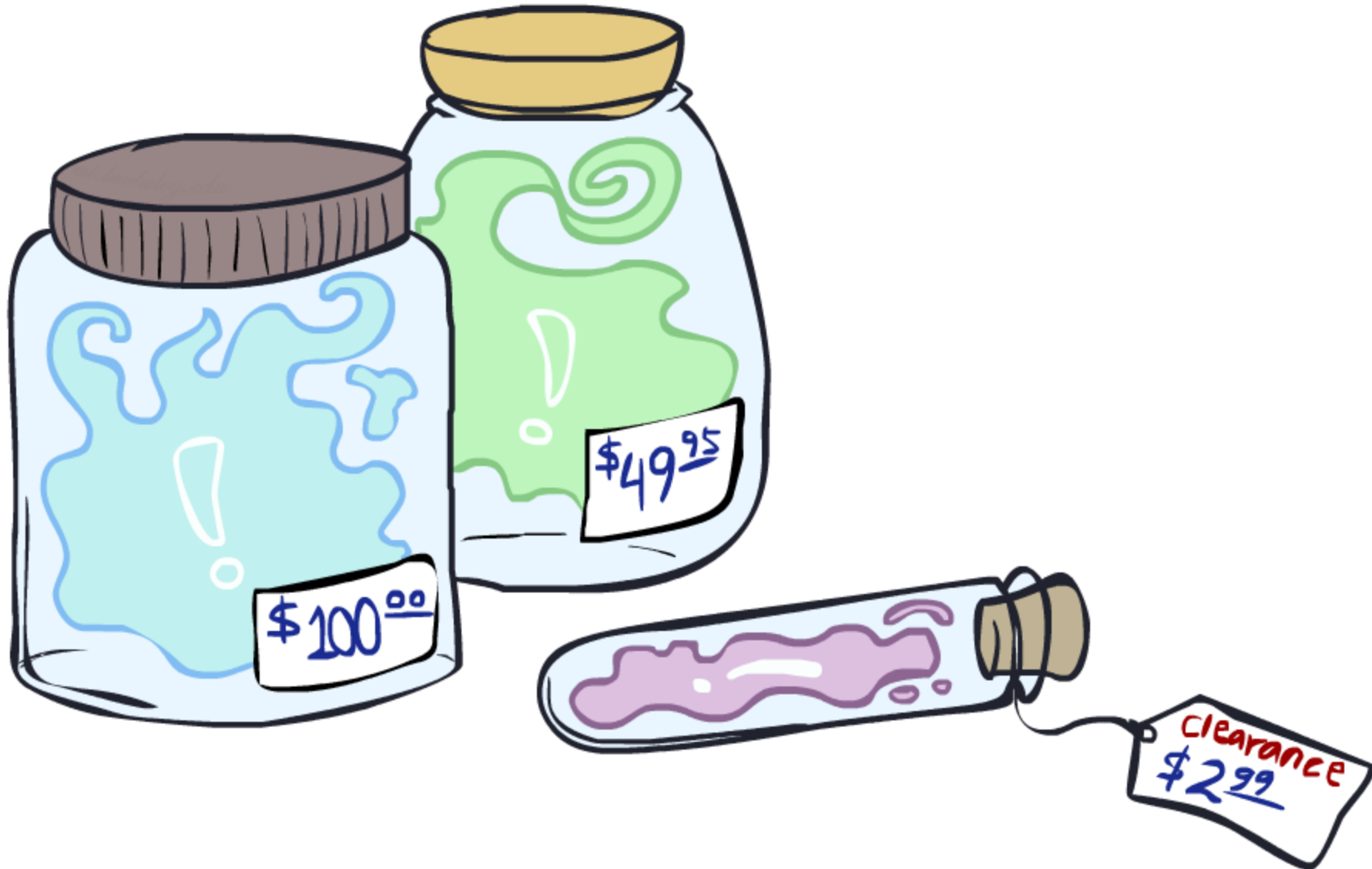


Ghostbusters Decision Network

Demo: Ghostbusters with probability

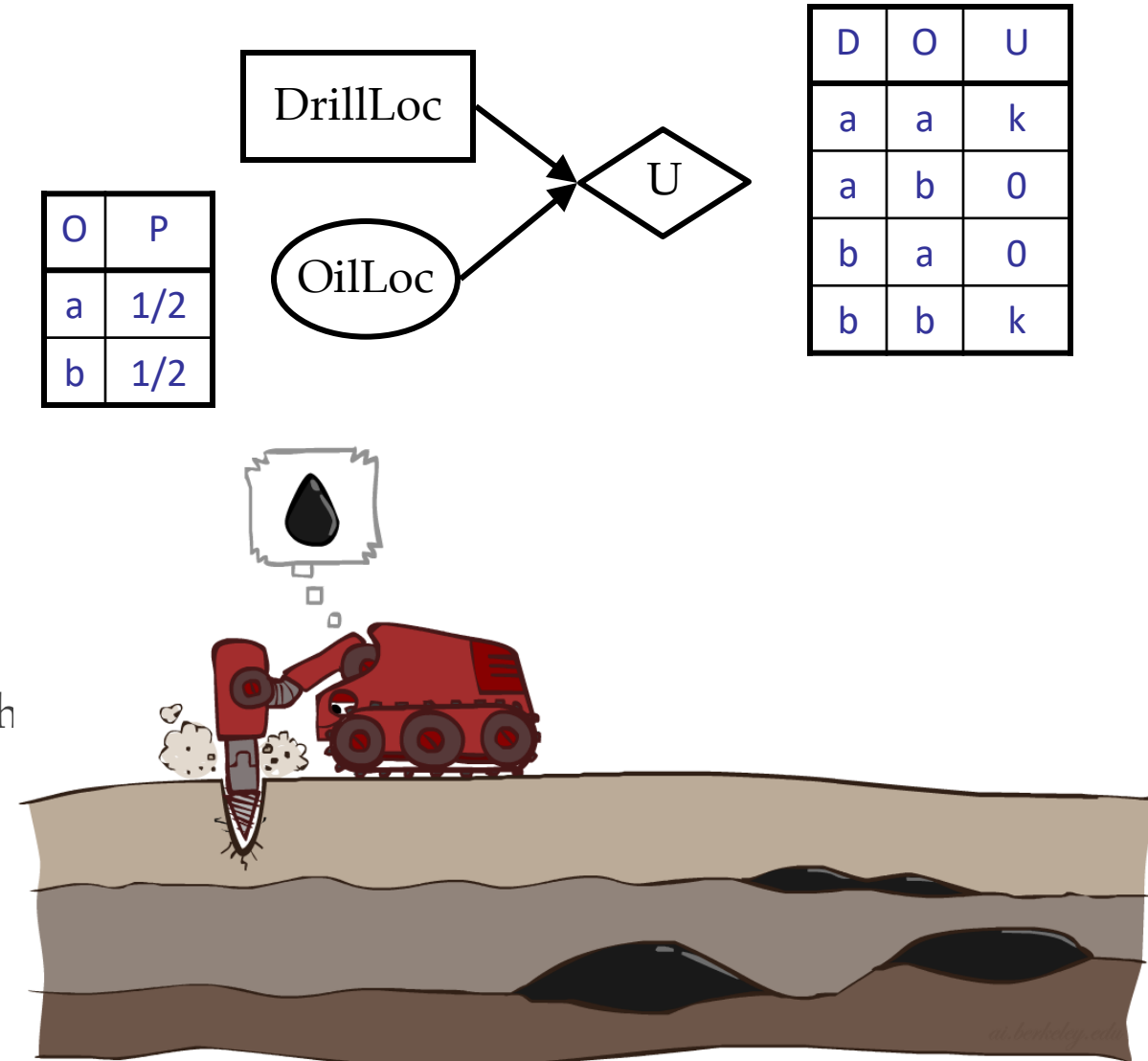


Value of Information



Value of Information

- Idea: compute value of acquiring evidence
 - Can be done directly from decision network
- Example: buying oil drilling rights
 - Two blocks A and B, exactly one has oil, worth k
 - You can drill in one location
 - Prior probabilities 0.5 each, & mutually exclusive
 - Drilling in either A or B has $EU = k/2$, $MEU = k/2$
- Question: what's the **value of information** of O?
 - Value of knowing which of A or B has oil
 - Value is expected gain in MEU from new info
 - Survey may say "oil in a" or "oil in b," prob 0.5 each
 - If we know OilLoc, MEU is k (either way)
 - Gain in MEU from knowing OilLoc?
 - $VPI(OilLoc) = k/2$
 - Fair price of information: $k/2$



Value of Perfect Information

MEU with no evidence

$$\text{MEU}(\emptyset) = \max_a \text{EU}(a) = 70$$

MEU if forecast is bad

$$\text{MEU}(F = \text{bad}) = \max_a \text{EU}(a|\text{bad}) = 53$$

MEU if forecast is good

$$\text{MEU}(F = \text{good}) = \max_a \text{EU}(a|\text{good}) = 95$$

Forecast distribution

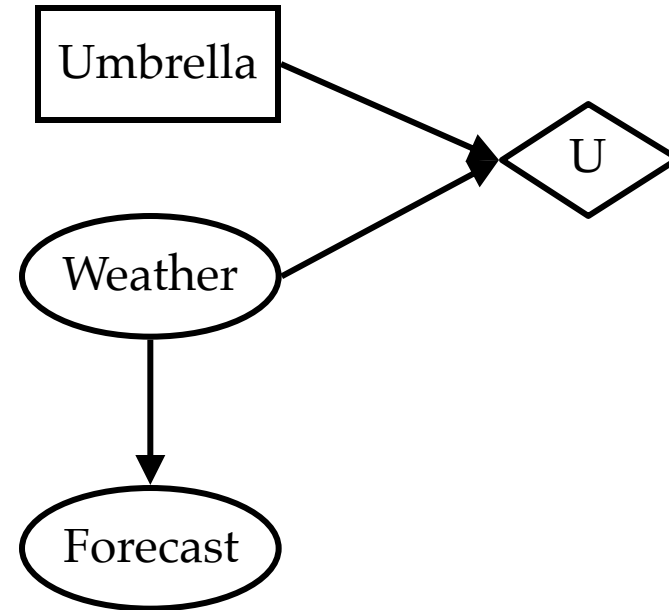
F	P(F)
good	0.59
bad	0.41



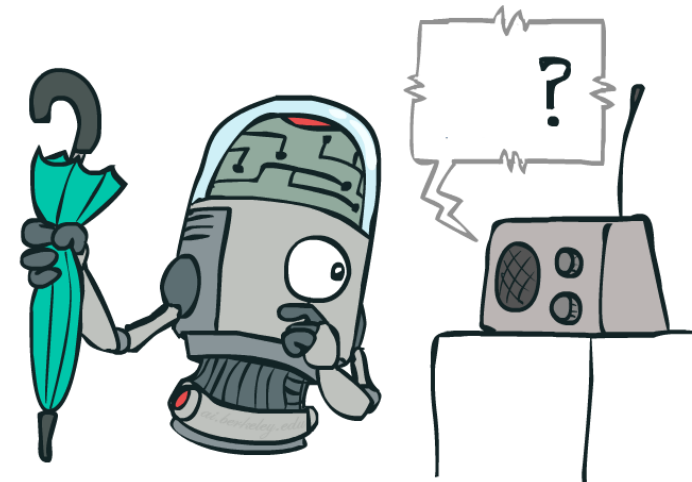
$$0.59 \cdot (95) + 0.41 \cdot (53) - 70$$

$$77.8 - 70 = 7.8$$

$$\text{VPI}(E'|e) = \left(\sum_{e'} P(e'|e) \text{MEU}(e, e') \right) - \text{MEU}(e)$$



A	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70



Value of Information

- Assume we have evidence $E=e$. Value if we act now:

$$\text{MEU}(e) = \max_a \sum_s P(s|e) U(s, a)$$

- Assume we see that $E' = e'$. Value if we act then:

$$\text{MEU}(e, e') = \max_a \sum_s P(s|e, e') U(s, a)$$

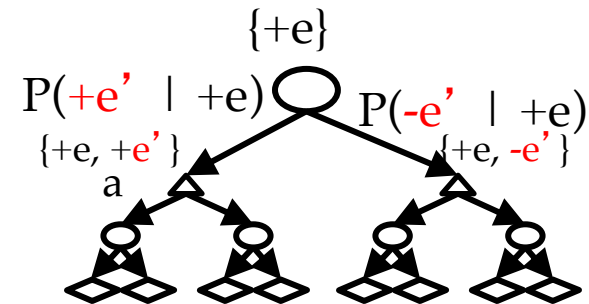
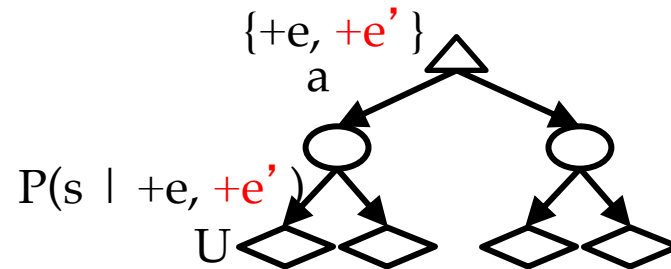
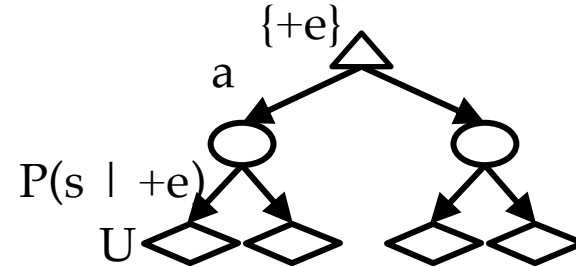
- BUT E' is a random variable whose value is **unknown**, so we don't know what e' will be

- Expected value if E' is revealed and then we act:

$$\text{MEU}(e, E') = \sum_{e'} P(e'|e) \text{MEU}(e, e')$$

- Value of information: how much MEU goes up by revealing E' first then acting, over acting now:

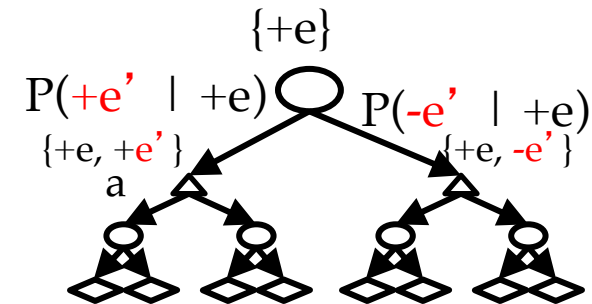
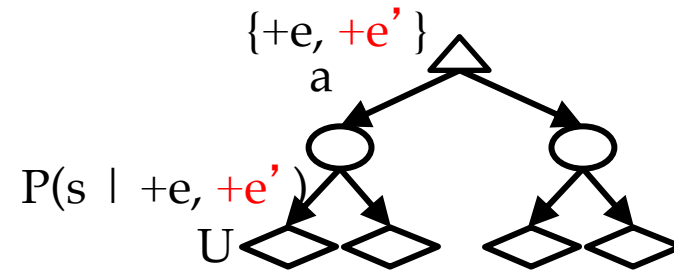
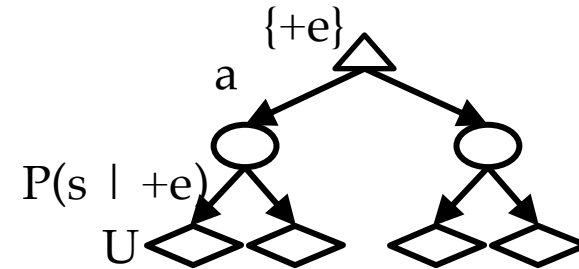
$$\text{VPI}(E'|e) = \text{MEU}(e, E') - \text{MEU}(e)$$



Value of Information

$$\begin{aligned}\text{MEU}(e, E') &= \sum_{e'} P(e'|e) \text{MEU}(e, e') \\ &= \sum_{e'} P(e'|e) \max_a \sum_s P(s|e, e') U(s, a)\end{aligned}$$

$$\begin{aligned}\text{MEU}(e) &= \max_a \sum_s P(s|e) U(s, a) \\ &= \max_a \sum_{e'} P(e|e') \sum_s P(s|e, e') U(s, a)\end{aligned}$$



VPI Properties

- Nonnegative

$$\forall E', e : \text{VPI}(E'|e) \geq 0$$



- Nonadditive

(think of observing E_j twice)

$$\text{VPI}(E_j, E_k|e) \neq \text{VPI}(E_j|e) + \text{VPI}(E_k|e)$$



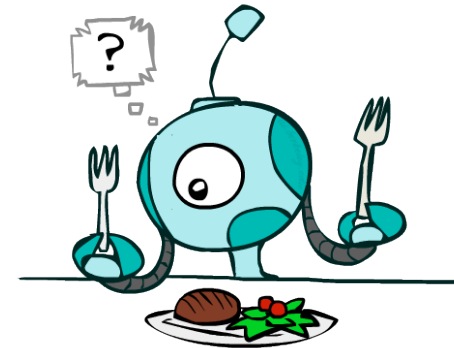
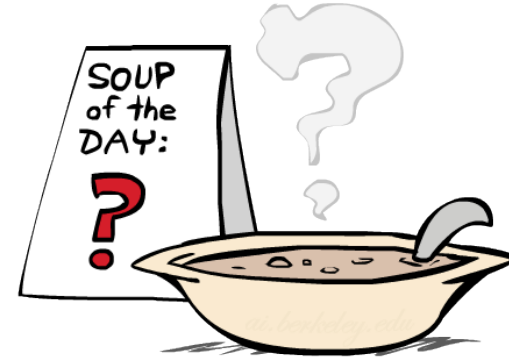
- Order-independent

$$\begin{aligned} \text{VPI}(E_j, E_k|e) &= \text{VPI}(E_j|e) + \text{VPI}(E_k|e, E_j) \\ &= \text{VPI}(E_k|e) + \text{VPI}(E_j|e, E_k) \end{aligned}$$



Quick VPI Questions

- The soup of the day is either clam chowder or split pea, but you wouldn't order either one. What's the value of knowing which it is?
- There are two kinds of plastic forks at a picnic. One kind is slightly sturdier. What's the value of knowing which?
- You're playing the lottery. The prize will be \$0 or \$100. You can play any number between 1 and 100 (chance of winning is 1%). What is the value of knowing the winning number?



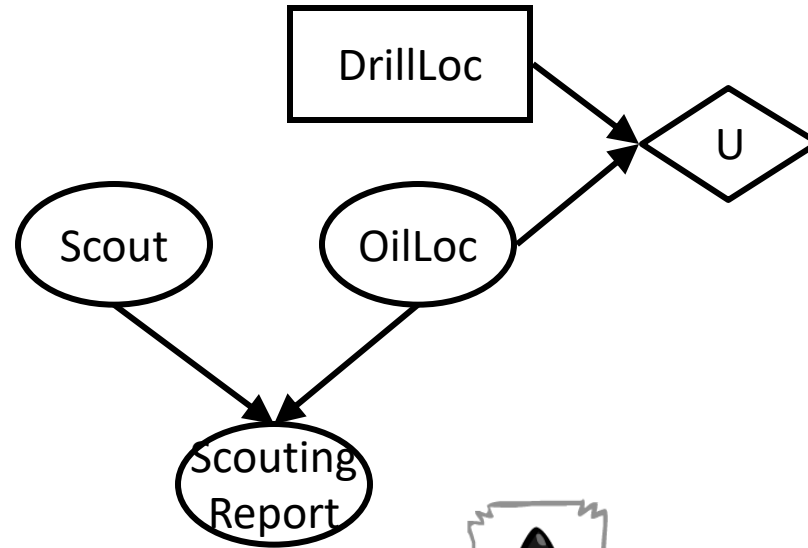
Value of Imperfect Information?



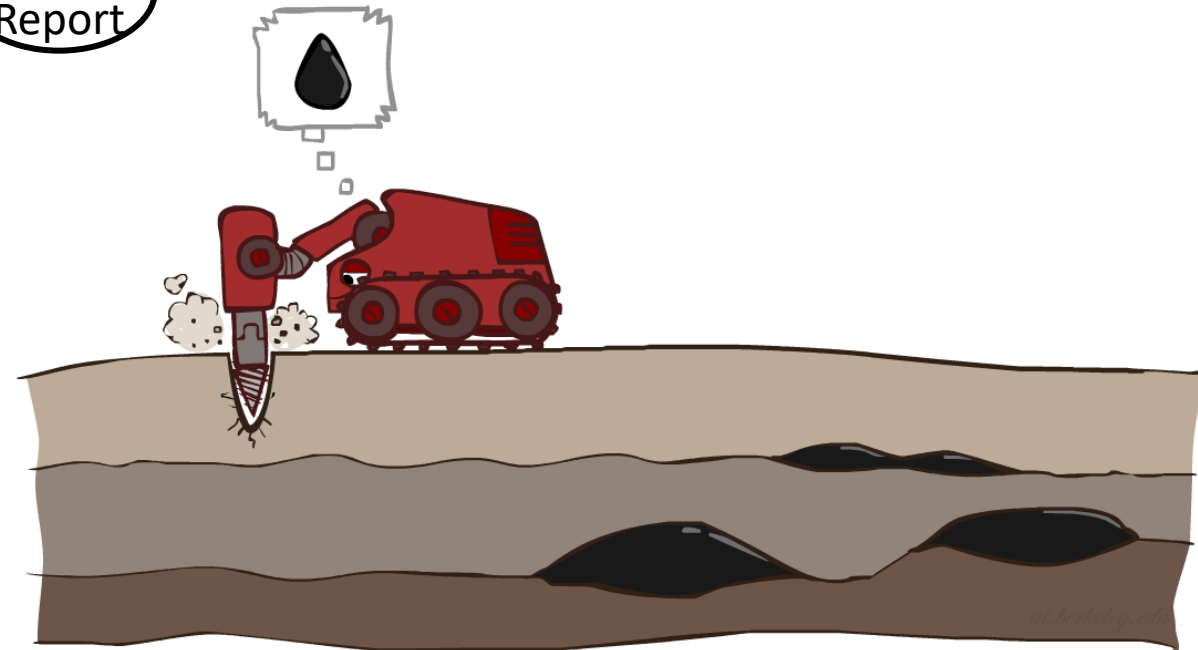
- No such thing
- Information corresponds to the observation of a node in the decision network
- If data is “noisy” that just means we don’t observe the original variable, but another variable which is a noisy version of the original one

VPI Question

- VPI(OilLoc) ?
- VPI(ScoutingReport) ?
- VPI(Scout) ?
- VPI(Scout | ScoutingReport) ?



- Generally:
If $\text{Parents}(U) \perp\!\!\!\perp Z \mid \text{CurrentEvidence}$
Then $\text{VPI}(Z \mid \text{CurrentEvidence}) = 0$

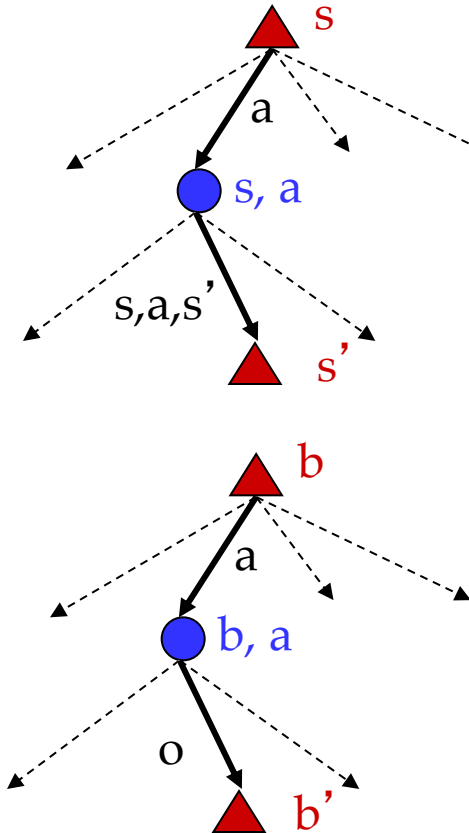


POMDPs



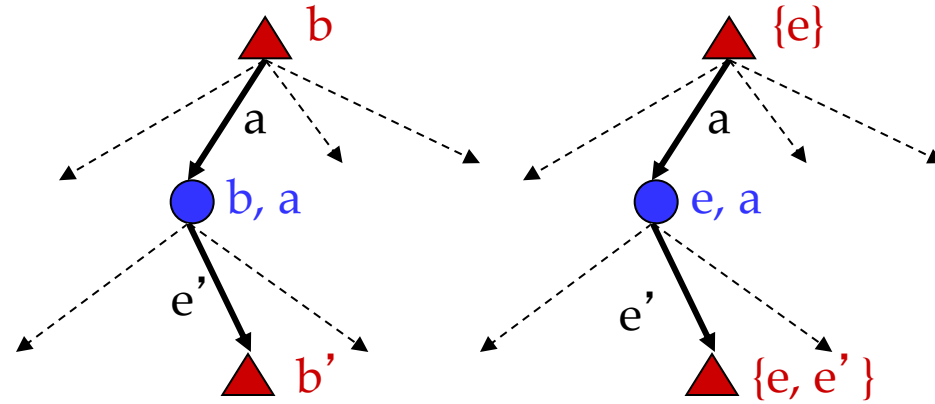
POMDPs

- MDPs have:
 - States S
 - Actions A
 - Transition function $P(s' | s, a)$ (or $T(s, a, s')$)
 - Rewards $R(s, a, s')$
- POMDPs add:
 - Observations O
 - Observation function $P(o | s)$ (or $O(s, o)$)
- POMDPs are MDPs over belief states b (distributions over S)

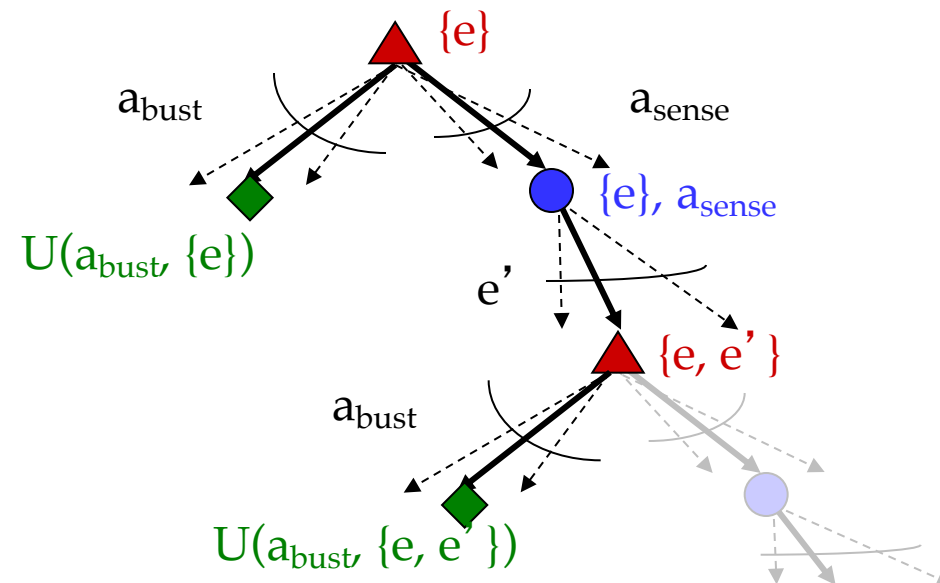


Example: Ghostbusters

- In (static) Ghostbusters:
 - Belief state determined by evidence to date $\{e\}$
 - Tree really over evidence sets
 - Probabilistic reasoning needed to predict new evidence given past evidence



- Solving POMDPs
 - One way: use truncated expectimax to compute approximate value of actions
 - What if you only considered busting or one sense followed by a bust?
 - You get a VPI-based agent!



Video of Demo Ghostbusters with VPI



More Generally*

- General solutions map belief functions to actions
 - Can divide regions of belief space (set of belief functions) into policy regions (gets complex quickly)
 - Can build approximate policies using discretization methods
 - Can factor belief functions in various ways
- Overall, POMDPs are very (actually PSACE-) hard
- Most real problems are POMDPs, but we can rarely solve them in general!

