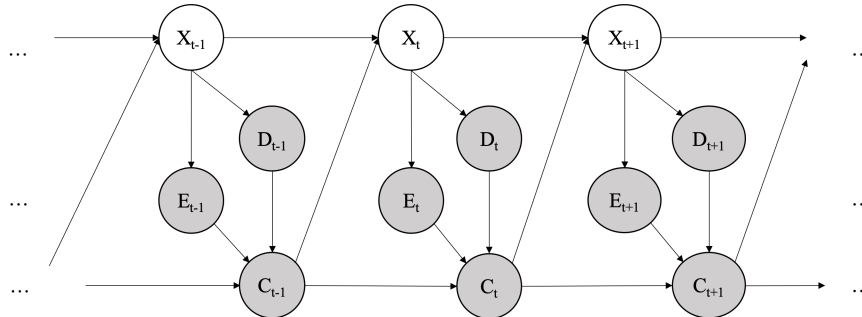


### Q1. We Are Getting Close...

Mesut is trying to remotely control a car, which has gone out of his view. The unknown state of the car is represented by the random variable  $X$ . While Mesut can't see the car itself, his high-tech sensors on the car provides two useful readings: an estimate ( $E$ ) of the distance to the car in front, and a detection model ( $D$ ) that detects if the car is headed into a wall. Using these two readings, Mesut applies the controls ( $C$ ), which determine the velocity of the car by changing the acceleration. The Dynamic Bayes Net below describes the setup.



(a) For the above DBN, complete the equations for performing updates. (Hint: think about the prediction update and observation update equations in the forward algorithm for HMMs.)

- Time elapse: (i) = (ii) (iii) (iv)  $P(x_{t-1}|e_{0:t-1}, d_{0:t-1}, c_{0:t-1})$
- (i)   $P(x_t)$                         $P(x_t|e_{0:t-1}, d_{0:t-1}, c_{0:t-1})$                         $P(e_t, d_t, c_t|e_{0:t-1}, d_{0:t-1}, c_{0:t-1})$
  - (ii)   $P(c_{0:t-1})$                         $P(x_{0:t-1}, c_{0:t-1})$                         $P(e_{0:t-1}, d_{0:t-1}, c_{0:t-1})$   
  $P(e_{0:t}, d_{0:t}, c_{0:t})$                        1
  - (iii)   $\sum_{x_{t-1}}$                         $\sum_{x_t}$                         $\max_{x_{t-1}}$                         $\max_{x_t}$                        1
  - (iv)   $P(x_{t-1}|x_{t-2})$                         $P(x_{t-1}, x_{t-2})$                         $P(x_t|e_{0:t-1}, d_{0:t-1}, c_{0:t-1})$   
  $P(x_t|x_{t-1})$                         $P(x_t, x_{t-1})$                         $P(x_t, e_{0:t-1}, d_{0:t-1}, c_{0:t-1})$   
  $P(x_t|x_{t-1}, c_{t-1})$                         $P(x_t, x_{t-1}, c_{t-1})$                        1

Update to incorporate new evidence at time  $t$ :

- $P(x_t|e_{0:t}, d_{0:t}, c_{0:t}) =$  (v) (vi) (vii) Your choice for (i)
- (v)   $(P(c_t|c_{0:t-1}))^{-1}$                         $(P(e_t|e_{0:t-1}) P(d_t|d_{0:t-1}) P(c_t|c_{0:t-1}))^{-1}$   
  $(P(e_t, d_t, c_t|e_{0:t-1}, d_{0:t-1}, c_{0:t-1}))^{-1}$                         $(P(e_{0:t-1}|e_t) P(d_{0:t-1}|d_t) P(c_{0:t-1}|c_t))^{-1}$   
  $(P(e_{0:t-1}, d_{0:t-1}, c_{0:t-1}|e_t, d_t, c_t))^{-1}$                        1
  - (vi)   $\sum_{x_{t-1}}$                         $\sum_{x_t}$                         $\sum_{x_{t-1}, x_t}$                         $\max_{x_{t-1}}$                         $\max_{x_t}$                        1
  - (vii)   $P(x_t|e_t, d_t, c_t)$                         $P(x_t, e_t, d_t, c_t)$   
  $P(x_t|e_t, d_t, c_t, c_{t-1})$                         $P(x_t, e_t, d_t, c_t, c_{t-1})$   
  $P(e_t, d_t|x_t)P(c_t|e_t, d_t, c_{t-1})$                         $P(e_t, d_t, c_t|x_t)$                        1

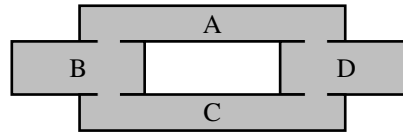
(viii) Suppose we want to do the above updates in one step and use normalization to reduce computation. Select all the terms that are not explicitly calculated in this implementation.

DO NOT include the choices if their values are 1.

- (ii)     (iii)     (iv)     (v)     (vi)     (vii)     None of the above

## Q2. HMMs: Help Your House Help You

Imagine you have a smart house that wants to track your location within itself so it can turn on the lights in the room you are in and make you food in your kitchen. Your house has 4 rooms ( $A, B, C, D$ ) in the floorplan below ( $A$  is connected to  $B$  and  $D$ ,  $B$  is connected to  $A$  and  $C$ ,  $C$  is connected to  $B$  and  $D$ , and  $D$  is connected to  $A$  and  $C$ ):



At the beginning of the day ( $t = 0$ ), your probabilities of being in each room are  $p_A, p_B, p_C$ , and  $p_D$  for rooms  $A, B, C$ , and  $D$ , respectively, and at each time  $t$  your position (following a Markovian process) is given by  $X_t$ . At each time, your probability of staying in the same room is  $q_0$ , your probability of moving clockwise to the next room is  $q_1$ , and your probability of moving counterclockwise to the next room is  $q_{-1} = 1 - q_0 - q_1$ .

(a) Initially, assume your house has no way of sensing where you are. What is the probability that you will be in room  $D$  at time  $t = 1$ ?

- $q_0 p_D$      
   $q_0 p_D + q_1 p_A + q_{-1} p_C + 2q_1 p_B$      
   $q_0 p_D + q_1 p_A + q_{-1} p_C$   
  $q_0 p_D + q_{-1} p_A + q_1 p_C$      
   $q_1 p_A + q_1 p_C + q_0 p_D$      
  None of these

Now assume your house contains a sensor  $M^A$  that detects motion ( $+m$ ) or no motion ( $-m$ ) in room  $A$ . However, the sensor is a bit noisy and can be tricked by movement in adjacent rooms, resulting in the conditional distributions for the sensor given in the table below. The prior distribution for the sensor's output is also given.

$M^A$	$P(M^A   X = A)$	$P(M^A   X = B)$	$P(M^A   X = C)$	$P(M^A   X = D)$	$M^A$	$P(M^A)$
$+m^A$	$1 - 2\gamma$	$\gamma$	0.0	$\gamma$	$+m^A$	0.5
$-m^A$	$2\gamma$	$1 - \gamma$	1.0	$1 - \gamma$	$-m^A$	0.5

(b) You decide to help your house to track your movements using a particle filter with three particles. At time  $t = T$ , the particles are at  $X^0 = A, X^1 = B, X^2 = D$ . What is the probability that the particles will be resampled as  $X^0 = X^1 = X^2 = A$  after time elapse? Select **all terms in the product**.

- $q_0$    
   $q_0^2$    
   $q_0^3$    
   $q_1$    
   $q_1^2$    
   $q_1^3$    
   $q_{-1}$    
   $q_{-1}^2$    
   $q_{-1}^3$    
  None of these

(c) Assume that the particles are actually resampled after time elapse as  $X^0 = D, X^1 = B, X^2 = C$ , and the sensor observes  $M^A = -m^A$ . What are the particle weights given the observation?

Particle	Weight							
$X^0 = D$	<input type="radio"/> $\gamma$	<input type="radio"/> $1 - \gamma$	<input type="radio"/> $1 - 2\gamma$	<input type="radio"/> 0.0	<input type="radio"/> 1.0	<input type="radio"/> $2\gamma$	<input type="radio"/> None of these	
$X^1 = B$	<input type="radio"/> $\gamma$	<input type="radio"/> $1 - \gamma$	<input type="radio"/> $1 - 2\gamma$	<input type="radio"/> 0.0	<input type="radio"/> 1.0	<input type="radio"/> $2\gamma$	<input type="radio"/> None of these	
$X^2 = C$	<input type="radio"/> $\gamma$	<input type="radio"/> $1 - \gamma$	<input type="radio"/> $1 - 2\gamma$	<input type="radio"/> 0.0	<input type="radio"/> 1.0	<input type="radio"/> $2\gamma$	<input type="radio"/> None of these	

Now, assume your house also contains sensors  $M^B$  and  $M^D$  in rooms  $B$  and  $D$ , respectively, with the conditional distributions of the sensors given below and the prior equivalent to that of sensor  $M^A$ .

$M^B$	$P(M^B   X = A)$	$P(M^B   X = B)$	$P(M^B   X = C)$	$P(M^B   X = D)$
$+m^B$	$\gamma$	$1 - 2\gamma$	$\gamma$	0.0
$-m^B$	$1 - \gamma$	$2\gamma$	$1 - \gamma$	1.0

$M^D$	$P(M^D   X = A)$	$P(M^D   X = B)$	$P(M^D   X = C)$	$P(M^D   X = D)$
$+m^D$	$\gamma$	0.0	$\gamma$	$1 - 2\gamma$
$-m^D$	$1 - \gamma$	1.0	$1 - \gamma$	$2\gamma$

- (d) Again, assume that the particles are actually resampled after time elapse as  $X^0 = D, X^1 = B, X^2 = C$ . The sensor readings are now  $M^A = -m^A, M^B = -m^B, M^D = +m^D$ . What are the particle weights given the observations?

Particle	Weight
$X^0 = D$	<input type="radio"/> $\gamma^2 - 2\gamma^3$ <input type="radio"/> $3 - 2\gamma$ <input type="radio"/> 0.0 <input type="radio"/> $\gamma - \gamma^2 + \gamma^3$ <input type="radio"/> $1 - 3\gamma + 2\gamma^2$ <input type="radio"/> $2 - \gamma$ <input type="radio"/> $1 - 2\gamma + \gamma^2$ <input type="radio"/> None of these
$X^1 = B$	<input type="radio"/> $\gamma^2 - 2\gamma^3$ <input type="radio"/> $3 - 2\gamma$ <input type="radio"/> 0.0 <input type="radio"/> $\gamma - \gamma^2 + \gamma^3$ <input type="radio"/> $1 - 3\gamma + 2\gamma^2$ <input type="radio"/> $2 - \gamma$ <input type="radio"/> $1 - 2\gamma + \gamma^2$ <input type="radio"/> None of these
$X^2 = C$	<input type="radio"/> $\gamma^2 - 2\gamma^3$ <input type="radio"/> $3 - 2\gamma$ <input type="radio"/> 0.0 <input type="radio"/> $\gamma - \gamma^2 + \gamma^3$ <input type="radio"/> $1 - 3\gamma + 2\gamma^2$ <input type="radio"/> $2 - \gamma$ <input type="radio"/> $1 - 2\gamma + \gamma^2$ <input type="radio"/> None of these

The sequence of observations from each sensor are expressed as the following:  $m_{0:t}^A$  are all measurements  $m_0^A, m_1^A, \dots, m_t^A$  from sensor  $M^A$ ,  $m_{0:t}^B$  are all measurements  $m_0^B, m_1^B, \dots, m_t^B$  from sensor  $M^B$ , and  $m_{0:t}^D$  are all measurements  $m_0^D, m_1^D, \dots, m_t^D$  from sensor  $M^D$ . Your house can get an accurate estimate of where you are at a given time  $t$  using the forward algorithm. The forward algorithm update step is shown here:

$$P(X_t | m_{0:t}^A, m_{0:t}^B, m_{0:t}^D) \propto P(X_t, m_{0:t}^A, m_{0:t}^B, m_{0:t}^D) \quad (1)$$

$$= \sum_{x_{t-1}} P(X_t, x_{t-1}, m_t^A, m_t^B, m_t^D, m_{0:t-1}^A, m_{0:t-1}^B, m_{0:t-1}^D) \quad (2)$$

$$= \sum_{x_{t-1}} \text{_____} P(X_t | x_{t-1}) P(x_{t-1}, m_{0:t-1}^A, m_{0:t-1}^B, m_{0:t-1}^D) \quad (3)$$

- (e) Which of the following expression(s) correctly complete the missing expression in line (3) above (regardless of whether they are available to the algorithm during execution)? Fill in **all** that apply.

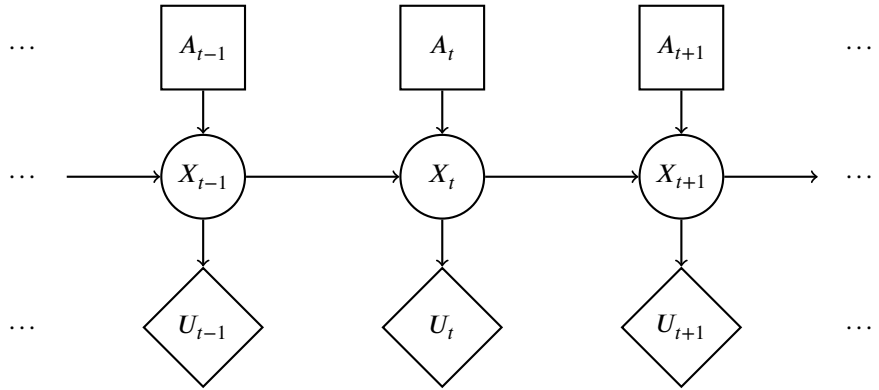
$P(m_t^A, m_t^B, m_t^D | X_t, x_{t-1})$       $P(m_t^A, m_t^B, m_t^D | x_{t-1})$       $P(m_t^A | x_{t-1})P(m_t^B | x_{t-1})P(m_t^D | x_{t-1})$

$P(m_t^A | m_{0:t-1}^A)P(m_t^B | m_{0:t-1}^B)P(m_t^D | m_{0:t-1}^D)$       $P(m_t^A, m_t^B, m_t^D | X_t, x_{t-1}, m_{0:t-1}^A, m_{0:t-1}^B, m_{0:t-1}^D)$

$P(m_t^A | X_t)P(m_t^B | X_t)P(m_t^D | X_t)$       $P(m_t^A, m_t^B, m_t^D | X_t)$      None of these

### Q3. Planning ahead with HMMs

Pacman is tired of using HMMs to estimate the location of ghosts. He wants to use HMMs to plan what actions to take in order to maximize his utility. Pacman uses the HMM (drawn to the right) of length  $T$  to model the planning problem. In the HMM,  $X_{1:T}$  is the sequence of hidden states of Pacman's world,  $A_{1:T}$  are actions Pacman can take, and  $U_t$  is the utility Pacman receives at the particular hidden state  $X_t$ . Notice that there are no evidence variables, and utilities are not discounted.



(a) The belief at time  $t$  is defined as  $B_t(X_t) = p(X_t|a_{1:t})$ . The forward algorithm update has the following form:

$$B_t(X_t) = \underline{\hspace{1cm}} \quad \text{(i)} \quad \underline{\hspace{1cm}} \quad \text{(ii)} \quad \underline{\hspace{1cm}} \quad B_{t-1}(x_{t-1}).$$

Complete the expression by choosing the option that fills in each blank.

- (i)        $\max_{x_{t-1}}$         $\sum_{x_{t-1}}$         $\max_{x_t}$         $\sum_{x_t}$        1
- (ii)       $p(X_t|x_{t-1})$       $p(X_t|x_{t-1})p(X_t|a_t)$       $p(X_t)$         $p(X_t|x_{t-1}, a_t)$      1
- None of the above combinations is correct

(b) Pacman would like to take actions  $A_{1:T}$  that maximizes the expected sum of utilities, which has the following form:

$$\text{MEU}_{1:T} = \underline{\hspace{1cm}} \quad \text{(i)} \quad \underline{\hspace{1cm}} \quad \text{(ii)} \quad \underline{\hspace{1cm}} \quad \text{(iii)} \quad \underline{\hspace{1cm}} \quad \text{(iv)} \quad \underline{\hspace{1cm}} \quad \text{(v)}$$

Complete the expression by choosing the option that fills in each blank.

- (i)        $\max_{a_{1:T}}$         $\max_{a_T}$         $\sum_{a_{1:T}}$         $\sum_{a_T}$        1
- (ii)       $\max_t$         $\prod_{t=1}^T$         $\sum_{t=1}^T$         $\min_t$        1
- (iii)      $\sum_{x_t, a_t}$         $\sum_{x_t}$         $\sum_{a_t}$         $\sum_{x_T}$        1
- (iv)      $p(x_t|x_{t-1}, a_t)$       $p(x_t)$         $B_t(x_t)$         $B_T(x_T)$        1
- (v)      $U_T$         $\frac{1}{U_t}$         $\frac{1}{U_T}$         $U_t$        1
- None of the above combinations is correct

(c) A greedy ghost now offers to tell Pacman the values of some of the hidden states. Pacman needs your help to figure out if the ghost's information is useful. Assume that the transition function  $p(x_t|x_{t-1}, a_t)$  is not deterministic. **With respect to the utility  $U_t$** , mark all that can be True:

- $\text{VPI}(X_{t-1}|X_{t-2}) > 0$       $\text{VPI}(X_{t-2}|X_{t-1}) > 0$       $\text{VPI}(X_{t-1}|X_{t-2}) = 0$       $\text{VPI}(X_{t-2}|X_{t-1}) = 0$
- None of the above

(d) Pacman notices that calculating the beliefs under this model is very slow using exact inference. He therefore decides to try out various particle filter methods to speed up inference. Order the following methods by how accurate their estimate of  $B_T(X_T)$  is? If different methods give an equivalently accurate estimate, mark them as the same number.

	Most accurate		Least accurate	
Exact inference	<input type="radio"/> 1	<input type="radio"/> 2	<input type="radio"/> 3	<input type="radio"/> 4
Particle filtering with no resampling	<input type="radio"/> 1	<input type="radio"/> 2	<input type="radio"/> 3	<input type="radio"/> 4
Particle filtering with resampling before every time elapse	<input type="radio"/> 1	<input type="radio"/> 2	<input type="radio"/> 3	<input type="radio"/> 4
Particle filtering with resampling before every other time elapse	<input type="radio"/> 1	<input type="radio"/> 2	<input type="radio"/> 3	<input type="radio"/> 4